C1, IYGB, PAPER J

1. $\int 6x + 9\sqrt{x} - \frac{4}{x^2}\,dx = \int 6x + 9x^{\frac{1}{2}} - 4x^{-2}\,dx$

$= 3x^2 + 6x^{\frac{3}{2}} + 4x^{-1} + C$

$= 3x^2 + 6x^{\frac{3}{2}} + \frac{4}{x} + C$

$\left(\frac{9}{\frac{1}{2}} = \frac{18}{3}\right)$

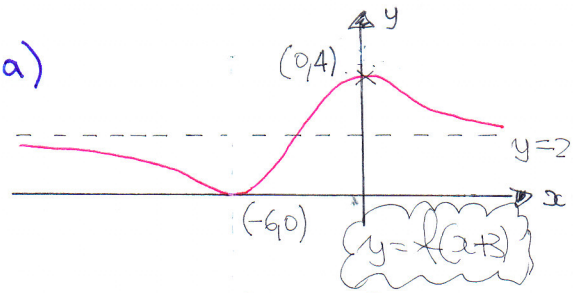
2. a) $(\sqrt{3} - \sqrt{2})^2 = (\sqrt{3})^2 - 2\sqrt{3}\sqrt{2} + (\sqrt{2})^2 = 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6}$

b) $\sqrt{14}\sqrt{42} = \sqrt{14}(\sqrt{14}\sqrt{3}) = 14\sqrt{3}$

OR

$\sqrt{7}\sqrt{2} \times \sqrt{6}\sqrt{7} = 7\sqrt{12} = 7\sqrt{4}\sqrt{3} = 14\sqrt{3}$

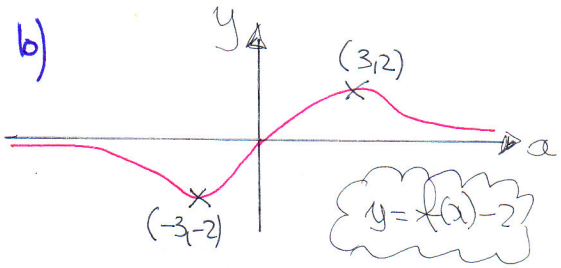
3. a)



$y = f(x+3)$

TRANSLATION, 3 UNITS TO THE RIGHT

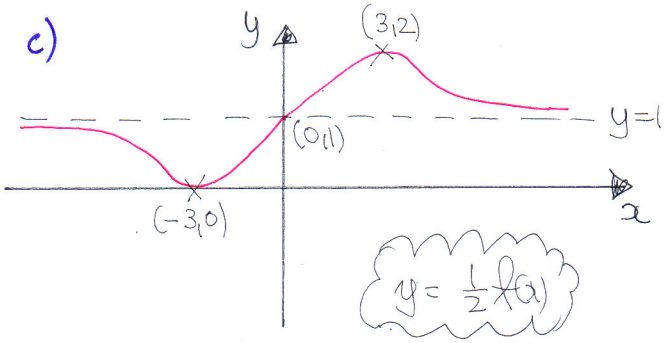
b)



$y = f(x) - 2$

TRANSLATION, DOWNWARDS BY 2 UNITS

c)



$y = \frac{1}{2}f(x)$

VERTICAL STRETCH, BY SCALE FACTOR OF $\frac{1}{2}$

4. (a) $6 - 2(x+2) < 10$

$6 - 2x - 4 < 10$

$2 - 2x < 10$

$-2x < 8$

$x > -4$

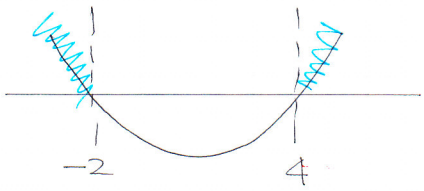
b) $(x+1)^2 \geqslant 4x+9$

$x^2+2x+1 \geqslant 4x+9$

$x^2-2x-8 \geqslant 0$

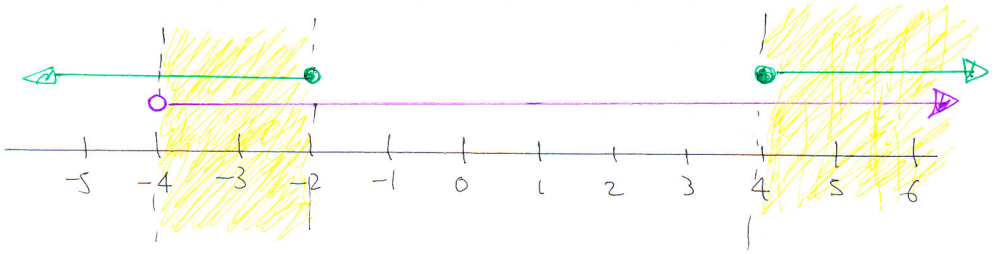
$(x+2)(x-4) \geqslant 0$

C.V = 4, -2



$x \leqslant -2$ OR $x \geqslant 4$

c)



$-4 < x \leqslant -2$ OR $x \geqslant 4$

5. $y = k(x+4)(x-2)(x-4) = k(x+4)(x^2-6x+8)$

THIS IS TO ADJUST THE y INTERCEPT

$= k[x^3-6x^2+8x + 4x^2-24x+32]$

$= k(x^3-2x^2-16x+32)$

BY INSPECTION $k = \frac{1}{2}$

$\therefore y = \frac{1}{2}x^3-x^2-8x+16$

6. (a) GRAD PQ $= \frac{y_2-y_1}{x_2-x_1} = \frac{6+2}{6+6} = \frac{8}{12} = \frac{2}{3}$

GRADIENT QR IS $-\frac{3}{2}$

$y-y_0 = m(x-x_0)$

$y-6 = -\frac{3}{2}(x-6)$

$2y-12 = -3x+18$

$\therefore 3x+2y = 30$

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b) Q MUST BE THE MIDPOINT OF R & T

$Q(6,6)$ $T(10,0)$ ←

$3x+2y=30$
$y=0$
$3x=30$
$x=10$

∴ BY INSPECTION $R(2,12)$

OR $\left(\frac{x+10}{2}, \frac{y+0}{2}\right) = (6,6)$

$(x+10, y) = (12,12)$

$(x,y) = (2,12)$

7. a) $y=x^2-4x-5$ and $y=2x-14$

$\Rightarrow x^2-4x-5=2x-14$

$\Rightarrow x^2-6x+9=0$

$\Rightarrow (x-3)^2=0$

$\Rightarrow \boxed{x=3}$ & $y=2x-14$

$\boxed{y=-8}$

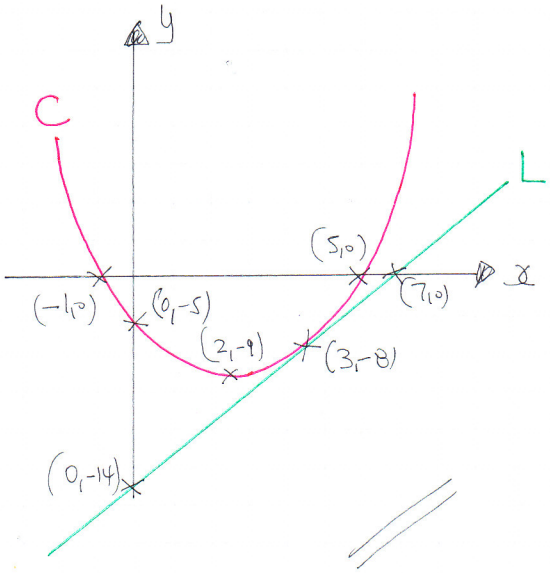
∴ $(3,-8)$

b)
- $y=x^2-4x-5$
- $y=(x-5)(x+1)$
- $y=(x-2)^2-4-5$
- $y=(x-2)^2-9$ (NOT ACTUALLY NEEDED)

$(5,0)$, $(-1,0)$, $(0,-5)$ MIN AT $(2,-9)$

- $y=2x-14$

$(0,-14)$ $(7,0)$



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8. a) $y=(x-1)(x^2+4x+5)$ $\quad y=0$

EITHER $x=1$ OR $x^2+4x+5=0$

- $b^2-4ac\ldots$
- $=4^2-4\times1\times5$
- $=16-20$
- $=-4<0$

NO MORE SOLUTIONS

ONLY POINT IS $(1,0)$

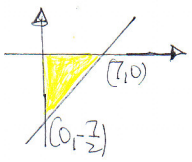
b)
- MULTIPLY OUT

$$y=(x-1)(x^2+4x+5)=x^3+4x^2+5x-x^2-4x-5$$
$$=x^3+3x^2+x-5$$

- $\dfrac{dy}{dx}=3x^2+6x+1$
- $\left.\dfrac{dy}{dx}\right|_{x=-1}=3-6+1=-2$
- WHEN $x=-1$ $\quad y=(-1-1)(1-4+5)=(-2)(2)=-4$ $\quad$ i.e. $(-1,-4)$
- NORMAL GRADIENT IS $\frac{1}{2}$
- $y-y_0=m(x-x_0)$
- $y+4=\frac{1}{2}(x+1)$
- $2y+8=x+1$
- $2y=x-7$

c)
- WHEN $x=0$ $\quad y=-\frac{7}{2}$ $\quad (0,-\frac{7}{2})$
- WHEN $y=0$ $\quad x=7$ $\quad (7,0)$

$$\text{AREA}=\frac{1}{2}\times\frac{7}{2}\times7=\frac{49}{4}=12\frac{1}{4}$$



9. a) $4x^2+(16-p)x+(13-p)=0$

EQUAL ROOTS: $b^2-4ac=0$

$(16-p)^2-4\times4\times(13-p)=0$

$256-32p+p^2-16(13-p)=0$

$p^2-32p+256+16p-208=0$

$p^2-16p+48=0$

$(p-4)(p-12)=0$

$p=4,\ 12$

16 × 13: 48 + 160 = 208

b) IF $p=4$

$4x^2+12x+9=0$

$(2x+3)^2=0$

$x=-\frac{3}{2}$

IF $p=12$

$4x^2+4x+1=0$

$(2x+1)^2=0$

$x=-\frac{1}{2}$

10. (a) $u_n=a+(n-1)d$

$u_{30}=19+29\times4$

$u_{30}=19+80+36$

$u_{30}=135$

b) $S_n=\frac{n}{2}(a+L)$

$S_{30}=\frac{30}{2}[19+135]$

$S_{30}=15\times154$

$S_{30}=1540+770$

$S_{30}=2310$

OR $S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{30}=\frac{30}{2}[2\times19+29\times4]$

$S_{30}=15[38+116]$

$S_{30}=15\times154$

$S_{30}=2310$

(c) $u_n<250$

$19+(n-1)\times4<250$

$4n-4<231$

$4n<235$

$n<\frac{235}{4}=\frac{200+32+3}{4}$

$n<50+8+\frac{3}{4}$

$\therefore n<58.75$

$n=58$

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(d) $S_4 > 4000$

$$\frac{n}{2}\left[2a+(n-1)d\right] > 4000$$

$$\frac{n}{2}\left[2\times 19+(n-1)\times 4\right] > 4000$$

$$n\left[19+2(n-1)\right] > 4000$$

$$n\left(2n-2+19\right) > 4000$$

$$\boxed{n\left(2n+17\right) > 4000}$$

now $S_{30} = 2310$ TRY HIGHER

$n=40$ $\quad 40\times(2\times 40+17) = 40\times 97 = 4000-120 = 3880 < 4000$

$n=41$ $\quad 41\times(2\times 41+17) = 41\times 99 = 4100-41 = 4059 > 4000$

$\therefore n = 41$

OR $k=41$