C1, IYGB, PAPER O

1. a) $\frac{2\sqrt{3}-1}{2-\sqrt{3}} = \frac{(2\sqrt{3}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4\sqrt{3}+6-2-\sqrt{3}}{4+\cancel{2\sqrt{3}}-\cancel{2\sqrt{3}}-3} = \frac{3\sqrt{3}+4}{1}$

$= 4+3\sqrt{3}$

b) $2^{x+2} = 4\sqrt{2}$

$2^{x+2} = 2^2 \times 2^{\frac{1}{2}}$

$2^{x+2} = 2^{\frac{5}{2}}$

$x+2 = \frac{5}{2}$

$x = \frac{1}{2}$

2. $\boxed{\begin{aligned} y &= 4\sqrt{x} \\ y &= 4x^{\frac{1}{2}} \\ \frac{dy}{dx} &= 2x^{-\frac{1}{2}} \\ \frac{d^2y}{dx^2} &= -x^{-\frac{3}{2}} \end{aligned}}$

THUS $\frac{d^2y}{dx^2} + \frac{8}{y^2}\frac{dy}{dx}$

$= -x^{-\frac{3}{2}} + \frac{8}{(4x^{\frac{1}{2}})^2}\left(2x^{-\frac{1}{2}}\right)$

$= -\frac{1}{x^{\frac{3}{2}}} + \frac{\cancel{16}x^{-\frac{1}{2}}}{\cancel{16}x}$

$= -\frac{1}{x^{\frac{3}{2}}} + \frac{1}{x^{\frac{3}{2}}}$

$= 0$ AS REQUIRED

3. a) • $a_1 = k$

$a_2 = (a_1)^2 - 4 = k^2 - 4$

$a_3 = (a_2)^2 - 4 = (k^2-4)^2 - 4 = k^4 - 8k^2 + 16 - 4 = k^4 - 8k^2 + 12$

b) • $a_2 + a_3 = 26$

$\Rightarrow (k^2-4) + (k^4 - 8k^2 + 12) = 26$

$\Rightarrow k^4 - 7k^2 + 8 = 26$

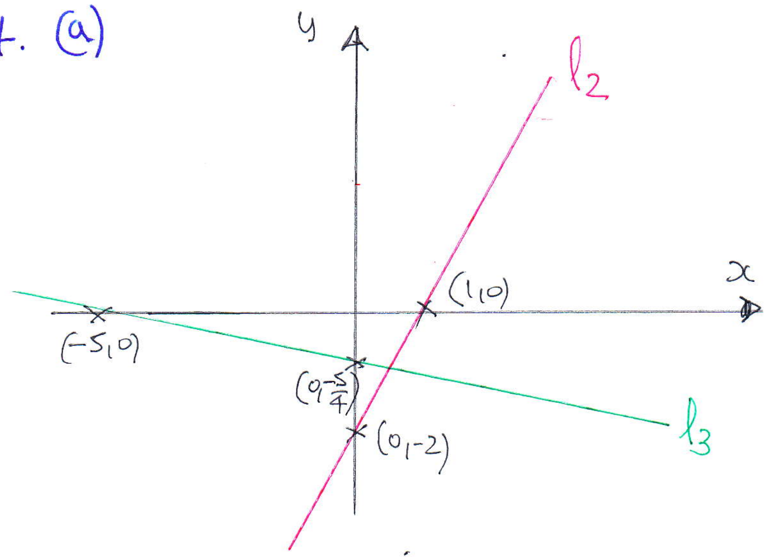
$\Rightarrow k^4 - 7k^2 - 18 = 0$

$\Rightarrow (k^2-9)(k^2+2) = 0$

$\Rightarrow k^2 = \begin{cases} 9 \\ \cancel{-2} \end{cases}$

$\Rightarrow k = \pm 3$

4. (a)



$l_1: x+4y+5=0$

$x=0 \quad y=-\frac{5}{4}$

$y=0 \quad x=-5$

$l_2: y=2x-2$

$x=0 \quad y=-2$

$y=0 \quad x=1$

b) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} x+4y+5=0 \\ y=2x-2 \end{array}\right\} \Rightarrow \begin{array}{l} x+4(2x-2)+5=0 \\ x+8x-8+5=0 \\ 9x=3 \\ \boxed{x=\frac{1}{3}} \end{array}$$

& $y=2\times\frac{1}{3}-2$

$y=\frac{2}{3}-2$

$\boxed{y=-\frac{4}{3}}$

$\therefore P\left(\frac{1}{3},-\frac{4}{3}\right)$

c) $x+4y+5=0$

$4y=-x-5$

$y=-\frac{1}{4}x-\frac{5}{4}$

GRAD l_1 IS $-\frac{1}{4}$

REQUIRED GRAD IS 4
(PERPENDICULAR)

- $y-y_0=m(x-x_0)$

$y+\frac{4}{3}=4\left(x-\frac{1}{3}\right)$

$y+\frac{4}{3}=4x-\frac{4}{3}$

$3y+4=12x-4$

$3y-12x=-8$

$12x-3y=8$

5. $x^2+(2k+1)+k^2=2$

$\Rightarrow x^2+(2k+1)+k^2-2=0$

- REAL ROOTS $b^2-4ac \geqslant 0$

$(2k+1)^2-4\times1(k^2-2) \geqslant 0$

$\cancel{4k^2}+4k+1-\cancel{4k^2}+8 \geqslant 0$

$4k \geqslant -9$

$k \geqslant -\frac{9}{4}$

6. (a)(b) $6, 11, 16, \ldots$

$a=6$
$d=5$

- $u_n = a+(n-1)d$

$u_{10} = 6+9\times5$

$u_{10} = 6+45$

$u_{10} = 51$

- $S_n = \frac{n}{2}[a+L]$

$S_{10} = \frac{10}{2}[6+51]$

$S_{10} = 5\times57$

$S_{10} = 250+35$

$S_{10} = 285$

(c) $S_k \leqslant 1200$

$\frac{k}{2}[2a+(k-1)d] \leqslant 1200$

$\frac{k}{2}[12+(k-1)\times5] \leqslant 1200$

$\frac{k}{2}(12+5k-5) \leqslant 1200$

$k(5k+7) \leqslant 2400$

AS REQUIRED

(d) • IF $k=20$

$20\times107=2140$

$21\times112=2352$

$22\times117=2574$

$\therefore k=21$

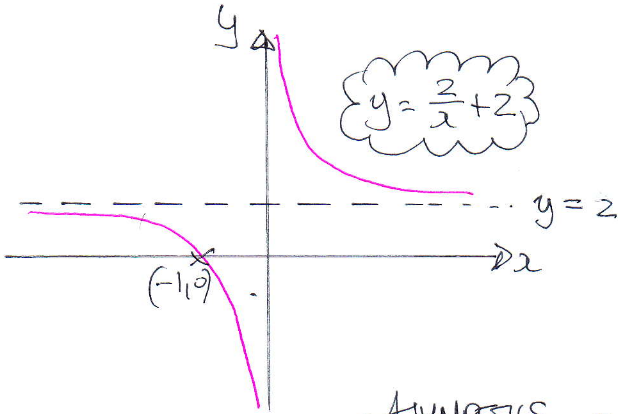
C1, IYGB, PAPER O

7. a) THIS IS $f(x-2)$

SO IT IS A TRANSLATION, 2 UNITS TO THE "RIGHT"

OR BY VECTOR $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

b)



$y = 0$

$0 = \frac{2}{x} + 2$

$-2 = \frac{2}{x}$

$-2x = 2$

$x = -1$

ASYMPTOTES

$y = 2$

$x = 0$, OR y AXIS

c) $\left.\begin{array}{l} y = \frac{2}{x-2} \\ y = \frac{2}{x} + 2 \end{array}\right\} \Rightarrow \frac{2}{x-2} = \frac{2}{x} + 2$ (DIVIDE BY 2)

$\Rightarrow \frac{1}{x-2} = \frac{1}{x} + 1$ (MULTIPLY BY x)

$\Rightarrow \frac{x}{x-2} = 1 + x$ (MULTIPLY BY $x-2$)

$\Rightarrow x = 1(x-2) + x(x-2)$

$\Rightarrow \cancel{x} = \cancel{x} - 2 + x^2 - 2x$

$\Rightarrow 0 = x^2 - 2x - 2$

AS REQUIRED

d) $x^2 - 2x - 2 = 0$

$\Rightarrow (x-1)^2 - 1 - 2 = 0$

$\Rightarrow (x-1)^2 = 3$

$\Rightarrow (x-1) = \pm\sqrt{3}$

$\Rightarrow x = 1 \pm \sqrt{3}$

8. a) $y = 2x^3 - 6x^2 + 3x + 5$

- $\frac{dy}{dx} = 6x^2 - 12x + 3$
- $\left.\frac{dy}{dx}\right|_{x=2} = 6\times2^2 - 12\times2 + 3$

$= 24 - 24 + 3$

$= 3$

EQUATION OF TANGENT

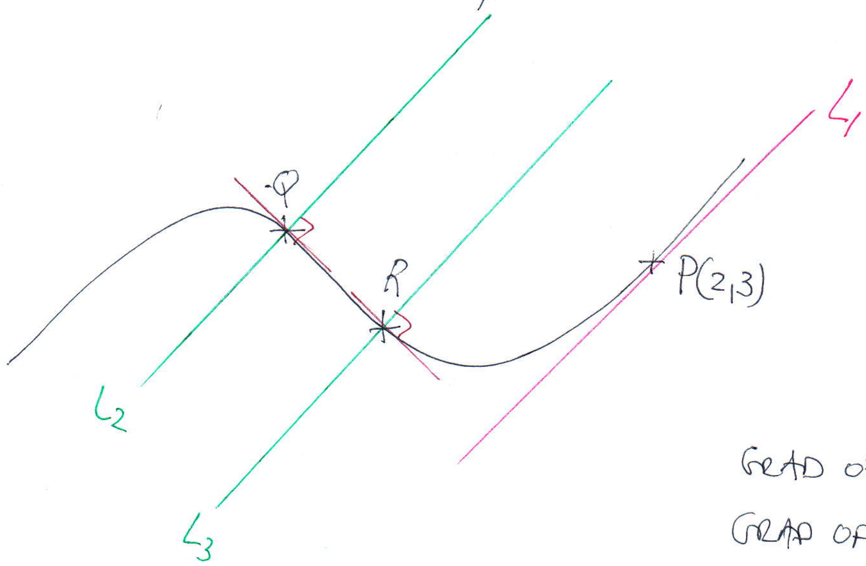
$y - y_0 = m(x - x_0)$

$y - 3 = 3(x - 2)$

$y - 3 = 3x - 6$

$y = 3x - 3$

b)



GRAD OF L_1 = 3

GRAD OF L_2 / L_3 = 3

GRAD AT Q & R $= -\frac{1}{3}$

$\frac{dy}{dx} = -\frac{1}{3}$

$6x^2 - 12x + 3 = -\frac{1}{3}$

$18x^2 - 36x + 9 = -1$

$18x^2 - 36x + 10 = 0$

$9x^2 - 18x + 5 = 0$

$(3x - 5)(3x - 1) = 0$

$x = \frac{1}{3}, \ \frac{5}{3}$

9. (a) $f'(x) = 3x^2 + 4x + k$

$y = \int 3x^2 + 4x + k \, dx$

$\boxed{y = x^3 + 2x^2 + kx + C}$

$(-2,-1) \Rightarrow -1 = (-2)^3 + 2(-2)^2 + k(-2) + C$
$(1,-4) \Rightarrow -4 = 1 + 2 + k + C$
$\Rightarrow -1 = \cancel{-8+8} - 2k + C$
$-7 = k + C$

$k + C = -7$
$-2k + C = -1$

SUBTRACT $3k = -6$
$\boxed{k = -2}$

& $k + C = -7$
$-2 + C = -7$
$\boxed{C = -5}$

$\therefore y = x^3 + 2x^2 - 2x - 5$

b) $y = x^3 + 2x^2 - 2x - 5$
$y = -3x - 5$

$\Rightarrow x^3 + 2x^2 - 2x \cancel{-5} = -3x \cancel{-5}$

$\Rightarrow x^3 + 2x^2 + x = 0$

$\Rightarrow x(x^2 + 2x + 1) = 0$

$\Rightarrow x(x+1)^2 = 0$

$\Rightarrow x = 0$ or -1 (REPEATED)

$\therefore$ TANGENT AT $x = -1$

$\therefore (-1, -2)$

"$y = -3x - 5$"