C1, IYGB, PAPER X

1. $x^2-2x-4>0$

IT DOES NOT FACTORIZE NICELY SO TREAT IT AS A QUADRATIC EQUATION & SEEK SOLUTIONS

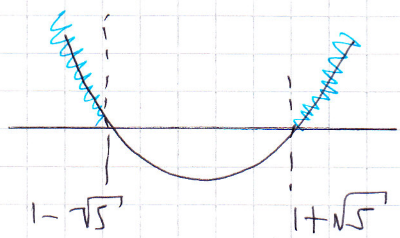
$x^2-2x-4=0$

$(x-1)^2-1^2-4=0$

$(x-1)^2=5$

$x-1=\pm\sqrt{5}$

$x=1\pm\sqrt{5}$



$x<1-\sqrt{5}$ OR $x>1+\sqrt{5}$

2. $(1+\sqrt{3})^4=\left[(1+\sqrt{3})^2\right]^2=(1+2\sqrt{3}+3)^2=(4+2\sqrt{3})^2$

$=16+16\sqrt{3}+(4\times3)=28+16\sqrt{3}$

3. $f'(x)=5-\frac{8}{x^2}$

$f(x)=\int 5-8x^{-2}\,dx$

$f(x)=5x+8x^{-1}+C$

$f(x)=5x+\frac{8}{x}+C$

NOW $2f(1)=4+f(2)$

$2[5+8+C]=4+[10+4+C]$

$26+2C=18+C$

$\boxed{C=-8}$

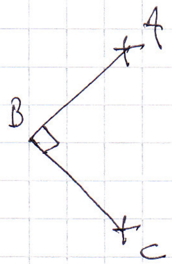
$\therefore\ f(x)=5x+\frac{8}{x}-8$

$f(4)=20+2-8$

$f(4)=14$

4. a) GRAD BC $=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-y}{2-(-2)}=\frac{-3-y}{4}=-\frac{y+3}{4}$

b) GRAD AB $=\frac{y_2-y_1}{x_2-x_1}=\frac{y-5}{-2-1}=\frac{y-5}{-3}$



IF PERPENDICULAR, $m_1\times m_2=-1$ OR $m_1=-\frac{1}{m_2}$

$-\frac{y+3}{4}\times\frac{y-5}{-3}=-1$

$\frac{y+3}{4}\times\frac{y-5}{3}=-1$

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$$\Rightarrow \frac{(y+3)(y-5)}{12} = -1$$

$$\Rightarrow y^2 - 2y - 15 = -12$$

$$\Rightarrow y^2 - 2y - 3 = 0$$

$$\Rightarrow (y+1)(y-3) = 0$$

$$\therefore y = \begin{cases} -1 \\ 3 \end{cases}$$

5. a) $u_n = a + (n-1)d$

$36000 = 18000 + (n-1) \times 1800$

$18000 = 1800(n-1)$

$10 = n - 1$

$n = 11$

b) $S_n = \frac{n}{2}[a + L]$

$S_{11} = \frac{11}{2}[18000 + 36000]$

$S_{11} = \frac{11}{2} \times 54000$

$S_{11} = 11 \times 27000$

$S_{11} = 297000$

IE £297 000

c) FIRSTLY FIND A

$u_n = a + (n-1)d$

$36000 = A + (15-1) \times 1000$

$36000 = A + 14000$

$A = 22000$

THUS

OSAMA OBAMA

$18000 + (n-1) \times 1800 = 22000 + (n-1) \times 1000$

$800(n-1) = 4000$

$(n-1) = 5$

$n = 6$

d) OSAMA

FIRST 11 YEARS £297000

+

4 YEARS AT £36000

(£144000)

TOTAL £441 000

OBAMA

$S_n = \frac{n}{2}[a + L]$

$S_{15} = \frac{15}{2}[22000 + 36000]$

$S_{15} = \frac{15}{2} \times 58000$

$S_{15} = 15 \times 29000$

$S_{15} = 435000$

$\therefore$ DIFFERENCE $= 441000 - 435000$

$= £6000$

6. a) $y=(1+\sqrt{x})^2 = 1+2\sqrt{x}+x = 1+2x^{\frac{1}{2}}+x$

$\therefore \frac{dy}{dx} = x^{-\frac{1}{2}}+1$

b) $2y=3x+6$

$y=\frac{3}{2}x+3$

TANGENT GRAD

$x^{-\frac{1}{2}}+1=\frac{3}{2}$

$x^{-\frac{1}{2}}=\frac{1}{2}$

$\frac{1}{\sqrt{x}}=\frac{1}{2}$

$\sqrt{x}=2$

$\boxed{x=4}$ $\quad y=(1+\sqrt{4})^2=9$

$\therefore P(4,9)$

7. a) $\boxed{C_{t+1} = a + bC_t}$

$\left.\begin{aligned} C_4 &= a+bC_3 \\ C_5 &= a+bC_4 \end{aligned}\right\} \Rightarrow$

$\left.\begin{aligned} 76 &= a+b\times 88 \\ 70 &= a+b\times 76 \end{aligned}\right\} \Rightarrow$ SUBTRACT

$6 = 12b$

$b=\frac{1}{2}$

$76=a+\frac{1}{2}\times 88$

$76=a+44$

$a=32$

b) REARRANGE RELATION

$C_{t+1}=32+\frac{1}{2}C_t$

$2C_{t+1}=64+C_t$

$\boxed{C_t = 2C_{t+1}-64}$

- $C_2=2C_3-64$

  $C_2=2\times 88-64$

  $C_2=176-64$

  $C_2=112$

- $C_1=2C_2-64$

  $C_1=2\times 112-64$

  $C_1=224-64$

  $C_1=160$

c) LET THE LIMIT BE L

$C_{t+1}=32+\frac{1}{2}C_t$

$L=32+\frac{1}{2}L$

$\frac{1}{2}L=32$

$L=64$

8. • LET THE LINE ~~HAVE~~ EQUATION $y = mx$, $m > 0$

$$\left.\begin{array}{l} y = mx \\ y = \sqrt{2x-4} \end{array}\right\} \Rightarrow mx = \sqrt{2x-4}$$

$$\Rightarrow m^2x^2 = 2x - 4$$

$$\Rightarrow m^2x^2 - 2x + 4 = 0$$

BUT THE LINE IS A TANGENT, SO WE SEEK REPEATED ROOTS

$$b^2 - 4ac = 0$$

$$(-2)^2 - 4m^2 \times 4 = 0$$

$$4 - 16m^2 = 0$$

$$4 = 16m^2$$

$$m^2 = \frac{1}{4}$$

$$m = \frac{1}{2}$$

$m \neq -\frac{1}{2}$
GRAD MUST BE POSITIVE FROM GRAPH

USING $m = \frac{1}{2} \Rightarrow \frac{1}{4}x^2 - 2x + 4 = 0$

$$\Rightarrow x^2 - 8x + 16 = 0$$

$$\Rightarrow (x-4)^2 = 0$$

$$\Rightarrow \boxed{x = 4} \quad \boxed{y = 2} \quad \therefore P(4,2)$$

$(y = \frac{1}{2}x)$

9. $2\sqrt{3}(x^2+1) = 7x$

$$\Rightarrow 2\sqrt{3}x^2 + 2\sqrt{3} - 7x = 0$$

$$\Rightarrow 2\sqrt{3}x^2 - 7x + 2\sqrt{3} = 0$$

BY QUADRATIC FORMULA

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 4(2\sqrt{3})(2\sqrt{3})}}{2 \times 2\sqrt{3}}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 4 \times 3 \times 4}}{2 \times 2\sqrt{3}} = \frac{7 \pm \sqrt{1}}{4\sqrt{3}} = \frac{7 \pm 1}{4\sqrt{3}}$$

$$\Rightarrow x = \begin{cases} \dfrac{8}{4\sqrt{3}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3} = \dfrac{2}{3}\sqrt{3} \\[2ex] \dfrac{6}{4\sqrt{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{3\sqrt{3}}{2 \times 3} = \dfrac{1}{2}\sqrt{3} \end{cases}$$

6. $f(x) \longmapsto \dots \longmapsto \boxed{8x^2 - 22x + 10}$

TRANSLATION BY $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x \longmapsto (x-1)$

HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

$x \longmapsto 2x$

REVERSING THE TRANSFORMATIONS

$x \longmapsto \frac{1}{2}x$, $y = 8\left(\frac{1}{2}x\right)^2 - 22\left(\frac{1}{2}x\right) + 10$

$y = 8\left(\frac{1}{4}x^2\right) - 11x + 10$

$y = 2x^2 - 11x + 10$

$x \longmapsto (x+1)$ $y = 2(x+1)^2 - 11(x+1) + 10$

$y = 2(x^2 + 2x + 1) - 11x - 11 + 10$

$y = 2x^2 + 4x + 2 - 11x - 1$

$y = 2x^2 - 7x + 1$

AS REQUIRED

ALTERNATIVE REVERSE $g(x) \longmapsto g(2x) \longmapsto g\left((x+1)\right) = g(x + \tfrac{1}{2})$

THUS $y = 8\left(\frac{1}{2}x + \frac{1}{2}\right)^2 - 22\left(\frac{1}{2}x + \frac{1}{2}\right) + 10$

$y = 8\left(\frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}\right) - 11x - 11 + 10$

$y = 2x^2 + 4x + 2 - 11x - 11 + 10$

$y = 2x^2 - 7x + 1$