# C1, IYGB, PAPER Z

**1.**

$$\frac{dy}{dx} = 3x^2 + kx + 7$$

$$y = \int 3x^2 + kx + 7 \, dx$$

$$\boxed{y = x^3 + \tfrac{1}{2}kx^2 + 7x + C}$$

$$(-1,-9) \Rightarrow -9 = -1 + \tfrac{1}{2}k - 7 + C$$

$$(2,6) \Rightarrow 6 = 8 + 2k + 14 + C$$

$$-1 = \tfrac{1}{2}k + C$$

$$-16 = 2k + C$$

$$\left.\begin{array}{l} C = -1 - \tfrac{1}{2}k \\ C = -16 - 2k \end{array}\right\} \Rightarrow$$

$$-1 - \tfrac{1}{2}k = -16 - 2k$$

$$-2 - k = -32 - 4k$$

$$3k = -30$$

$$k = -10$$

$$\therefore C = -2(-10) - 16$$

$$C = 4$$

$$\therefore y = x^3 - 5x^2 + 7x + 4$$

**2. a)**

$$\text{GRAD AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{10-6} = \frac{8}{4} = 2$$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = 2(x - 6)$$

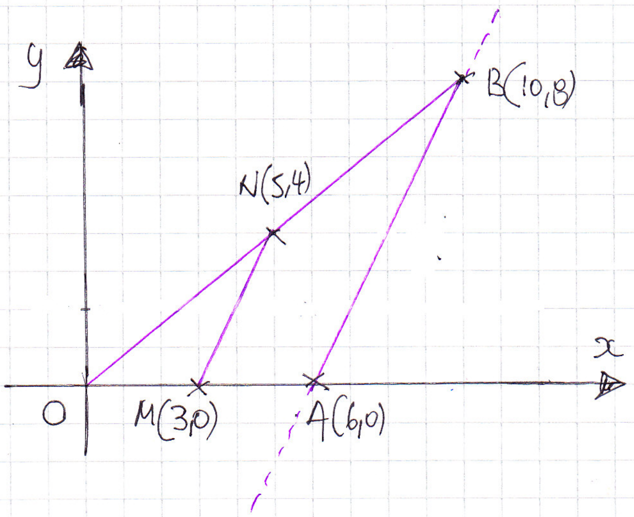
$$y = 2x - 12$$

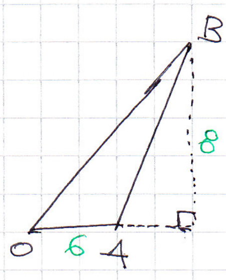
**b)**

$$O(0,0) \quad A(6,0) \Longrightarrow M(3,0)$$

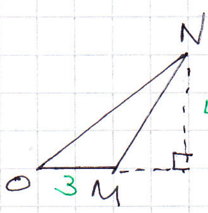
$$O(0,0) \quad B(10,8) \Longrightarrow N(5,4)$$

**c)**



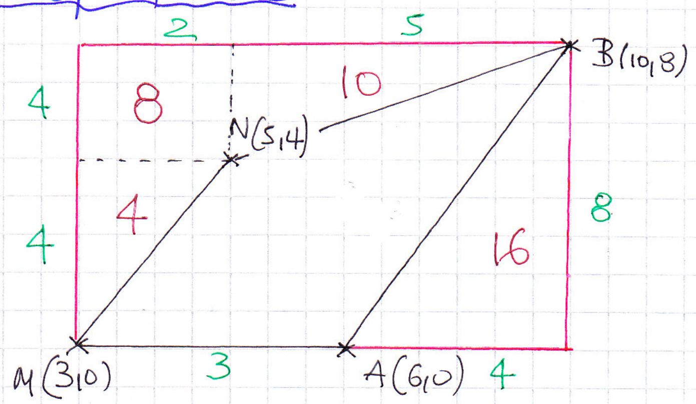




$$= \tfrac{1}{2} \times 6 \times 8 = 24$$




$$= \tfrac{1}{2} \times 3 \times 4 = 6$$

$\therefore$ AREA OF TRAPEZIUM $= 24 - 6 = 18$




ALTERNATIVE

TOTAL $\underset{7}{\boxed{\phantom{xx}}}\ 8 = 56$

REQUIRED AREA IS

$= 56 - (8+4+10+16)$

$= 56 - 38$

$= 18$

3.

- $p+q+r = \frac{3}{2} + \frac{9-\sqrt{17}}{4} + \frac{9+\sqrt{17}}{4} = \frac{3}{2} + \frac{9\cancel{-\sqrt{17}}+9\cancel{+\sqrt{17}}}{4}$

$= \frac{3}{2} + \frac{18}{4} = \frac{3}{2} + \frac{9}{2} = 6$

- $pqr = \frac{3}{2} \times \frac{9-\sqrt{17}}{4} \times \frac{9+\sqrt{17}}{4} = \frac{3(81+\cancel{9\sqrt{17}}-\cancel{9\sqrt{17}}-17)}{32} = \frac{3\times \cancel{64}^{\,2}}{\cancel{32}}$

$= 3\times 2 = 6$

$\therefore p+q+r = pqr$

4. a)

- $100^x = (10^2)^x = (10^x)^2 = y^2$ $\quad \therefore 100^x = y^2$
- $10^{x-1} = 10^x \times 10^{-1} = \frac{1}{10}\times 10^x = \frac{1}{10}y$ $\quad \therefore 10^{x-1} = \frac{1}{10}y$

$\therefore 100^x - 10001(10^{x-1}) + 100 = 0$

$y^2 - 10001\times\frac{1}{10}y + 100 = 0$

$10y^2 - 10001y + 1000 = 0$ $\quad \times 10$

AS REQUIRED

b)

$10y^2 - 10001y + 1000 = 0$

$(10y - 1)(y - 1000) = 0$

$y = \frac{1}{10}$ or $1000$ $\qquad 10^x = \frac{1}{10}$ or $1000$ $\qquad x = -1$ or $3$

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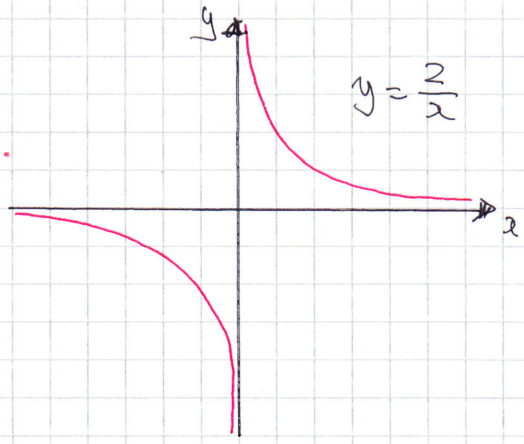
5. a) $f(x) = \frac{1}{x}$ $\quad 2f(x) = 2 \times \frac{1}{x} = \frac{2}{x}$

SO VERTICAL STRETCH, BY SCALE FACTOR of 2

OR $f(x) = \frac{1}{x}$ $\quad f\left(\frac{1}{2}x\right) = \frac{1}{\frac{1}{2}x} = \frac{2}{x}$

SO HORIZONTAL STRETCH, BY SCALE FACTOR 2

b)




ASYMPTOTES $x = 0$
$y = 0$

c) $\left.\begin{array}{l} y = k - 2x \\ y = \frac{2}{x} \end{array}\right\} \Rightarrow$ $k - 2x = \frac{2}{x}$

$kx - 2x^2 = 2$

$0 = 2x^2 - kx + 2$

IF TANGENT $b^2 - 4ac = 0$

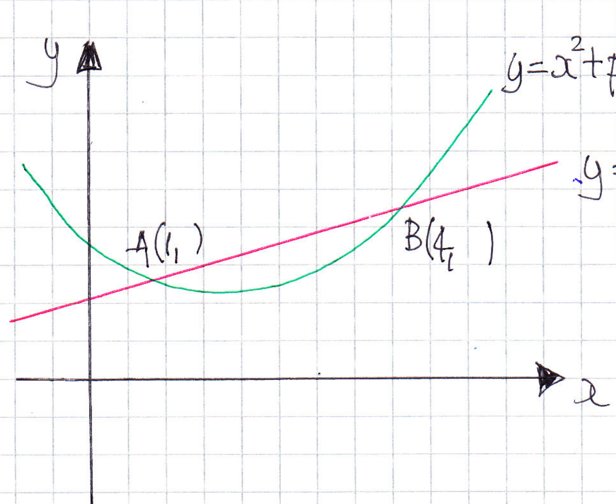
$(-k)^2 - 4 \times 2 \times 2 = 0$

$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm 4$

6.




- USING $x = 1$ WE OBTAIN THE SAME $y$ IN BOTH OBJECTS

$1 + q = 1 + p + 10$

$\boxed{q = p + 10}$

- SIMILARLY USING $x = 4$

$4 + q = 16 + 4p + 10$

$\boxed{q = 4p + 22}$

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$$p+10 = 4p+22$$
$$-12 = 3p$$
$$p = -4$$

& $q = p+10$

$q = 6$

SINCE $y = x+6$

$A(1,7) \quad B(4,10)$

7. $u_n = 2^n + 4n$

- FIRSTLY $u_1 = 2^1 + 4 \times 1 = 2+4 = 6$

- $u_{n+1} = 2^{n+1} + 4(n+1)$

$u_{n+1} = 2^n \times 2^1 + 4n + 4$

$u_{n+1} = 2 \times 2^n + 4n + 4$ BUT $2^n = u_n - 4n$

$u_{n+1} = 2\left[u_n - 4n\right] + 4n + 4$

$u_{n+1} = 2u_n - 4n + 4$

$\therefore$ $u_{n+1} = 2u_n - 4n + 4$ STARTING WITH $u_1 = 6$

8. $V = (p-qt)^2$

$V = p^2 - 2pqt + q^2t^2$

$\frac{dV}{dt} = 0 - 2pq + 2q^2t$

- $t=1, V=9 \implies (p-q\times1)^2 = 9$

$(p-q)^2 = 9$

$p - q = \begin{cases} 3 \\ -3 \end{cases}$

$t=1 \quad \frac{dV}{dt} = -6 \implies -6 = -2pq + 2q^2$

$-3 = -pq + q^2$

$-3 = q(-p+q)$

$\boxed{3 = (p-q)q}$

- IF $p-q=3$ $\quad 3q = 3$

  $q = 1 \qquad p = 4$

- IF $p-q=-3$ $\quad 3 = -3q$

  $\cancel{q=-1}$

9. $t + 2t + 3t + \dots + 50$

$a = t$

$d = t$

$l = 50$

$u_n = a + (n-1)d$

$50 = t + (n-1)t$

$50 - t = (n-1)t$ $\quad$ OR $\quad 50 - t = nt - t$

$\frac{50-t}{t} = n-1$ $\qquad nt = 50$

$n = \frac{50-t}{t} + 1$ $\qquad \boxed{n = \frac{50}{t}}$

$n = \frac{50-t+t}{t}$

$\boxed{n = \frac{50}{t}}$

Thus $S_n = \frac{n}{2}[a+l]$

$S_n = \frac{50/t}{2}[t+50]$

$S_n = \frac{50}{2t}[t+50]$

$S_n = \frac{25}{t}[t+50]$

$S_n = 25 + \frac{1250}{t}$

AS REQUIRED

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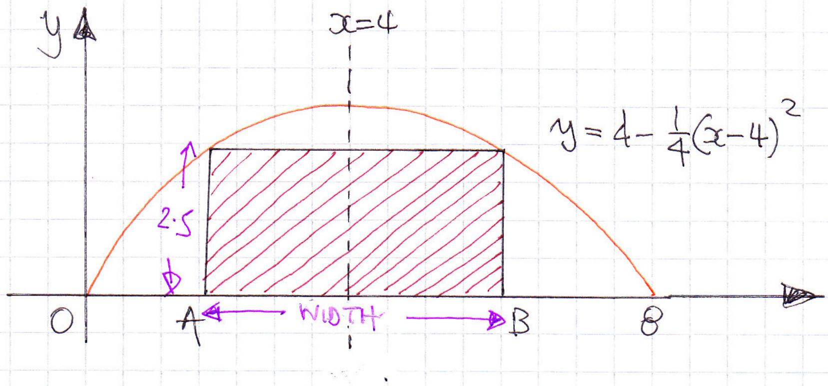
10.




WHEN $y = 2.5$

$$2.5 = 4 - \frac{1}{4}(x-4)^2$$

$$10 = 16 - (x-4)^2$$

$$(x-4)^2 = 6$$

$$x - 4 = \pm\sqrt{6}$$

$$x = 4 \pm \sqrt{6}$$

$\therefore$ $A(4-\sqrt{6}, 0)$

$B(4+\sqrt{6}, 0)$

$$|AB| = 2\sqrt{6}$$