C2, IYGB, PAPER A

1. a) $f(x) = 3x^3 - 2x^2 - 12x + 8$

$f(-2) = 3(-2)^3 - 2(-2)^2 - 12(-2) + 8 = -24 - 8 + 24 - 8 = 0$

$\therefore\ x+2$ IS A FACTOR OF $f(x)$

b) BY INSPECTION OR LONG DIVISION

$$\begin{array}{r|l} & 3x^2 - 8x + 4 \\ x+2 & 3x^3 - 2x^2 - 12x + 8 \\ & -3x^3 - 6x^2 \\ \hline & \quad -8x^2 - 12x + 8 \\ & \quad 8x^2 + 16x \\ \hline & \qquad 4x + 8 \\ & \qquad -4x - 8 \\ \hline & \qquad 0 \end{array}$$

$\therefore\ f(x) = (x+2)(3x^2 - 8x + 4)$

$f(x) = (x+2)(3x-2)(x-2)$

2. $(3-2x)^5 = \binom{5}{0}(3)^5(-2x)^0 + \binom{5}{1}(3)^4(-2x)^1 + \binom{5}{2}(3)^3(-2x)^2 + \binom{5}{3}(3)^2(-2x)^3 + \dots$

$= (1 \times 243 \times 1) + [5 \times 81 \times (-2x)] + [10 \times 27 \times 4x^2] + [10 \times 9 \times (-8x^3)] + \dots$

$= 243 - 810x + 1080x^2 - 720x^3 + \dots$

3. a) $r = \frac{u_4}{u_3} = \frac{108}{144} = \frac{3}{4}$

b) $u_n = ar^{n-1}$

$144 = a \times 0.75^2$

$a = \frac{144}{0.75^2}$

$a = 256$

$\therefore\ u_5 = 256 \times 0.75^4$

$u_5 = 81$

ALTERNATIVE

IF $u_4 = 108$ & $r = \frac{3}{4}$

THEN $u_5 = 108 \times \frac{3}{4} = 81$

c) $S_\infty = \frac{a}{1-r} = \frac{256}{1-0.75} = \frac{256}{0.25} = 1024$

4. $\boxed{\begin{aligned} y &= 2x + 8x^{-2} \\ \frac{dy}{dx} &= 2 - 16x^{-3} \\ \frac{d^2y}{dx^2} &= 48x^{-4} \end{aligned}}$

FOR STATIONARY POINTS $\frac{dy}{dx} = 0$

$$2 - 16x^{-3} = 0$$
$$2 = \frac{16}{x^3}$$
$$2x^3 = 16$$
$$x^3 = 8$$
$$x = 2 \quad \& \quad y = 2 \times 2 + \frac{8}{2^2}$$
$$y = 4 + 2 = 6$$

SO $(2,6)$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = \frac{48}{2^4} = 3 > 0$$

$\therefore$ $(2,6)$ IS A LOCAL MINIMUM

5. a) WHEN $x=3$ $\quad y = \sqrt{3^3 - 3} = \sqrt{24} \simeq 4.899$

WHEN $x = 3.5$ $\quad y = \sqrt{3.5^3 - 3.5} \simeq 6.275$

b)

	FIRST	REST					LAST
Y VALUES:	0	1.369	2.449	3.623	4.899	6.275	7.746

$$\int_1^4 \sqrt{x^3 - x} \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$\simeq \frac{0.5}{2}\left[0 + 7.746 + 2(1.369 + 2.449 + 3.623 + 4.899 + 6.275)\right]$$

$$\simeq 11.244$$

$$\simeq 11.24$$

CORRECT TO 2 d.p.

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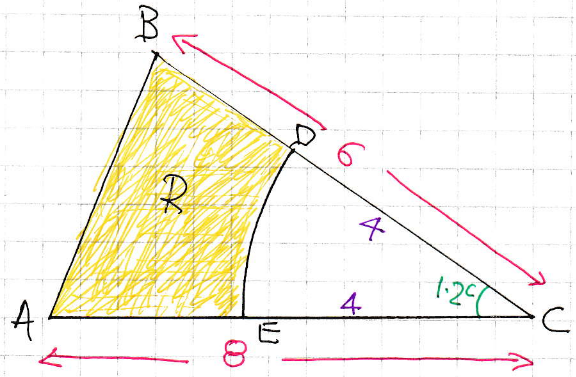
6.

$$\log_5(4-w) - 2\log_5 w = 1$$
$$\Rightarrow \log_5(4-w) - \log_5 w^2 = \log_5 5$$
$$\Rightarrow \log_5\left(\frac{4-w}{w^2}\right) = \log_5 5$$
$$\Rightarrow \frac{4-w}{w^2} = 5$$
$$\Rightarrow 4 - w = 5w^2$$
$$\Rightarrow 0 = 5w^2 + w - 4$$
$$\Rightarrow 0 = (5w-4)(w+1)$$
$$\Rightarrow w = \begin{cases} \cancel{-1} \\ \frac{4}{5} \end{cases}$$

MAKES ONE OF THE LOG ARGUMENTS NEGATIVE

7.



a) BY THE COSINE RULE

$$|AB|^2 = |AC|^2 + |BC|^2 - 2|AC||BC|\cos 1.2^c$$
$$|AB|^2 = 8^2 + 6^2 - 2\times 8\times 6\cos 1.2^c$$
$$|AB|^2 = 65.2136\ldots$$
$$|AB| \simeq 8.08 \text{ cm}$$

b) AREA OF THE TRIANGLE

$$= \tfrac{1}{2}\times|AC||BC|\sin(1.2^c)$$
$$= \tfrac{1}{2}\times 8\times 6\times\sin(1.2^c)$$
$$= 22.36893\ldots$$
$$\simeq 22.4 \text{ cm}^2$$

c) AREA OF SECTOR $\overset{\frown}{CDE} = \frac{1}{2}r^2\theta^c = \frac{1}{2}\times 4^2\times 1.2 = 9.6$

AREA OF R $= 22.368\ldots - 9.6 = 12.7689 \approx 12.8 \text{ cm}^2$

d) LENGTH OF ARC DE $= r\theta^c = 4\times 1.2 = 4.8$

PERIMETER OF R $= |AB| + |BD| + |\overset{\frown}{DE}| + |AE|$

$$\approx 8.08 + 2 + 4.8 + 4$$
$$\approx 18.9 \text{ cm}$$

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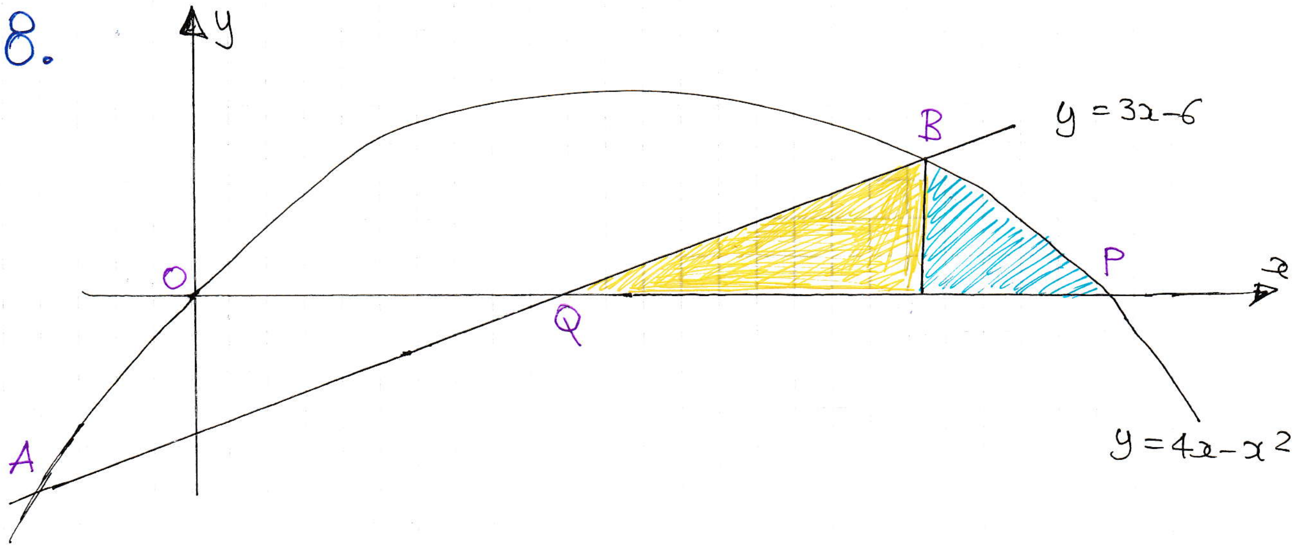
8.



• FIRSTLY FIND P

$4x - x^2 = 0$

$x(4-x) = 0$

$x = \begin{cases} 4 \leftarrow \boxed{P(4,0)} \\ 0 \end{cases}$

• NEXT FIND B

$\left.\begin{array}{l} y = 3x-6 \\ y = 4x - x^2 \end{array}\right\} \Rightarrow 3x - 6 = 4x - x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = \begin{cases} -2 \\ 3 \end{cases} \quad y = \begin{cases} -10 \\ 3 \end{cases}$

$\therefore \boxed{B(3,3)} \quad A(-2,-10)$

NOT NEEDED

• FINALLY FIND Q

$y = 3x - 6$

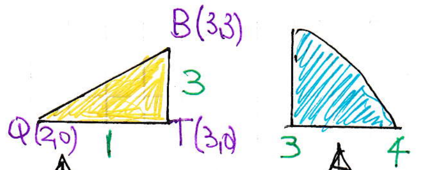
$0 = 3x - 6$

$6 = 3x$

$x = 2$

$\therefore \boxed{Q(2,0)}$

THUS



$\frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

$\int_3^4 4x - x^2 \, dx = \left[2x^2 - \frac{1}{3}x^3\right]_3^4 = \left(32 - \frac{64}{3}\right) - (18 - 9) = \frac{5}{3}$

$\therefore$ REQUIRED AREA $= \frac{3}{2} + \frac{5}{3} = \frac{9}{6} + \frac{10}{6} = \frac{19}{6}$ // AS REQUIRED

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9\. a) $x^2+y^2-10x-8y+21=0$

$\Rightarrow x^2-10x+y^2-8y+21=0$

$\Rightarrow (x-5)^2-25+(y-4)^2-16+21=0$

$\Rightarrow (x-5)^2+(y-4)^2=20$

$\therefore$ CENTRE AT $(5,4)$ RADIUS $=\sqrt{20}$

b) $\sqrt{20}>4$ SO IT CROSSES THE x-AXIS

$\sqrt{20}<5$ SO IT DOES <u>NOT</u> CROSS THE y-AXIS

c) $\left.\begin{array}{l} y=2x+4 \\ (x-5)^2+(y-4)^2=20 \end{array}\right\} \Rightarrow$ SOLVE SIMULTANEOUSLY

$\Rightarrow (x-5)^2+(2x+\cancel{4}-\cancel{4})^2=20$

$\Rightarrow x^2-10x+25+4x^2=20$

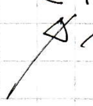
$\Rightarrow 5x^2-10x+5=0$

$\Rightarrow x^2-2x+1=0$

$(x-1)^2=0$

$\therefore$ REPEATED ROOT, SO TANGENT

$\therefore$ POINT OF TANGENCY IS $(1,6)$



$y=2x+4$

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10. $\sqrt{3} + 2\sin\left(3x + \frac{\pi}{4}\right) = 0$

$\Rightarrow 2\sin\left(3x + \frac{\pi}{4}\right) = -\sqrt{3}$

$\Rightarrow \sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

$$\begin{cases} 3x + \frac{\pi}{4} = -\frac{\pi}{3} \pm 2n\pi \\ 3x + \frac{\pi}{4} = \frac{4\pi}{3} \pm 2n\pi \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\begin{cases} 3x = -\frac{7\pi}{12} \pm 2n\pi \\ 3x = \frac{13\pi}{12} \pm 2n\pi \end{cases}$$

$$\begin{cases} x = -\frac{7\pi}{36} \pm \frac{2}{3}n\pi \\ x = \frac{13\pi}{36} \pm \frac{2}{3}n\pi \end{cases}$$

$\therefore x_1 = \frac{17\pi}{36}$ //

$x_2 = \frac{13\pi}{36}$ //