C2, IYGB, PAPER B

1. $(1-5x)^4 = 1 + \binom{4}{1}(-5x)^1 + \frac{4\times3}{1\times2}(-5x)^2 + \frac{4\times3\times2}{1\times2\times3}(-5x)^3 + \frac{4\times3\times2\times1}{1\times2\times3\times4}(-5x)^4$

$$= 1 - 20x + 150x^2 - 500x^3 + 625x^4$$

2. a) $f(x) = x^3 + 4x^2 + 7x + k$

- $f(-2) = 0$

$(-2)^3 + 4(-2)^2 + 7(-2) + k = 0$

$-8 + 16 - 14 + k = 0$

$-6 + k = 0$

$k = 6$

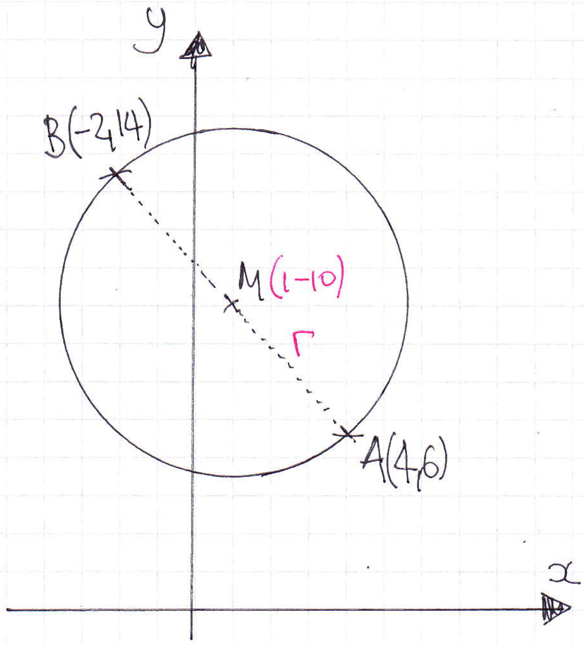
AS REQUIRED

b)

$$\begin{array}{r} x^2 + 2x + 3 \\ x+2 \;\overline{)\; x^3 + 4x^2 + 7x + 6} \\ -x^3 - 2x^2 \qquad\qquad \\ \hline 2x^2 + 7x + 6 \\ -2x^2 - 4x \qquad \\ \hline 3x + 6 \\ -3x - 6 \\ \hline 0 \end{array}$$

$\therefore f(x) = (x+2)(x^2+2x+3)$

3.



- MIDPOINT OF AB IS $\left(\frac{-2+4}{2}, \frac{14+6}{2}\right)$

$\therefore M(1,10)$

- RADIUS IS THE DISTANCE $|MA|$

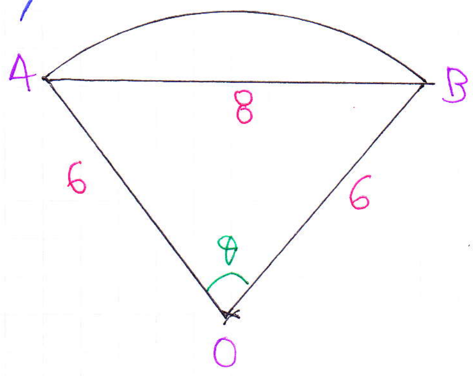
$r = \sqrt{(6-10)^2 + (4-1)^2}$

$r = \sqrt{16+9} = 5$

$\therefore (x-1)^2 + (y-10)^2 = 25$

C2, IYGB, PAPER B

4. a)



- BY THE COSINE RULE

$$|AB|^2 = |AO|^2 + |OB|^2 - 2|AO||OB|\cos\theta$$

$$8^2 = 6^2 + 6^2 - 2\times6\times6\times\cos\theta$$

$$64 = 36 + 36 - 72\cos\theta$$

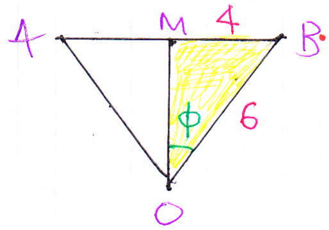
$$72\cos\theta = 8$$

$$\cos\theta = \frac{1}{9}$$

$$\theta = 1.459455\ldots$$

$$\theta \simeq 1.46^c \quad (2\text{ d.p})$$

ALTERNATIVE



$$\sin\phi = \frac{4}{6}$$

$$\phi \simeq 0.72972\ldots$$

$$\therefore \theta = 2\phi \simeq 1.46^c$$

b) AREA OF SECTOR $= \frac{1}{2}r^2\theta^c = \frac{1}{2}\times6^2\times1.46^c \simeq 26.27$

AREA OF TRIANGLE $A\hat{O}B = \frac{1}{2}|AO||OB|\sin\theta = \frac{1}{2}\times6\times6\times\sin(1.46^c)$
$\simeq 17.89$

$\therefore$ REQUIRED AREA (SEGMENT) $= 26.27 - 17.89 \approx 8.38\text{ cm}^2$

(ACCEPT 8.39)

5. a) $u_n = ar^{n-1}$

$$\left.\begin{array}{l} u_3 = 54 \\ u_6 = 1458 \end{array}\right\} \Longrightarrow \left.\begin{array}{l} ar^2 = 54 \\ ar^5 = 1458 \end{array}\right\} \Rightarrow \text{DIVIDE}$$

$$\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{1458}{54} \Longrightarrow r^3 = 27$$

$$\therefore r = 3$$

C2, IYGB, PAPER B

THUS $ar^2 = 54$

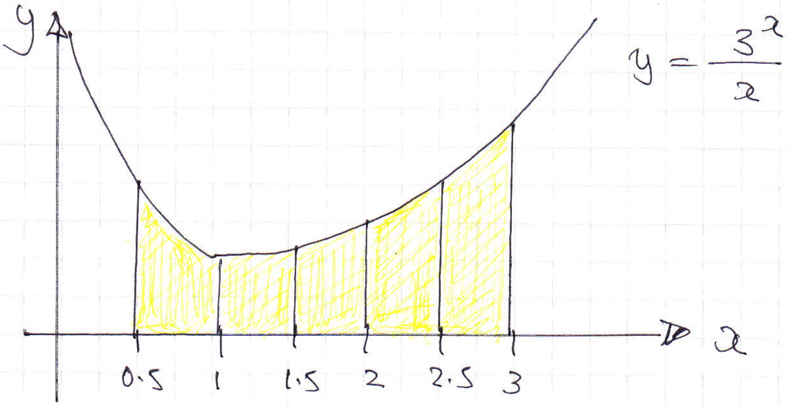
$9a = 54$

$a = 6$

b) $S_n = \dfrac{a(1-r^n)}{1-r} \implies S_{10} = \dfrac{6(1-3^{10})}{1-3}$

$S_{10} = 177144$

6. a)



x	0.5	1	1.5	2	2.5	3
y	3.4641	3	3.4641	4.5	6.2354	9
	FIRST	←	R E S T		→	LAST

AREA $\simeq \dfrac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$

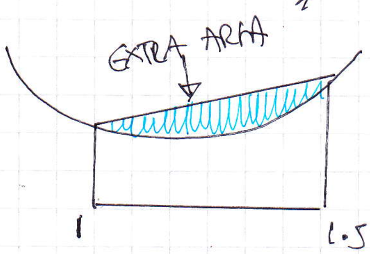
$\simeq \dfrac{0.5}{2}\left[3.4641 + 9 + 2(3 + 3.4641 + 4.5 + 6.2354)\right]$

$\simeq 11.71577\ldots$

$\simeq 11.7$ 3 s.f.

b) INCREASE THE NUMBER OF STRIPS I.E TAKE MORE TRAPEZIUMS

c) OVERESTIMATE AS THE TRAPEZIUMS IN THIS CURVE GO OVER, PRODUCING EXTRA AREA

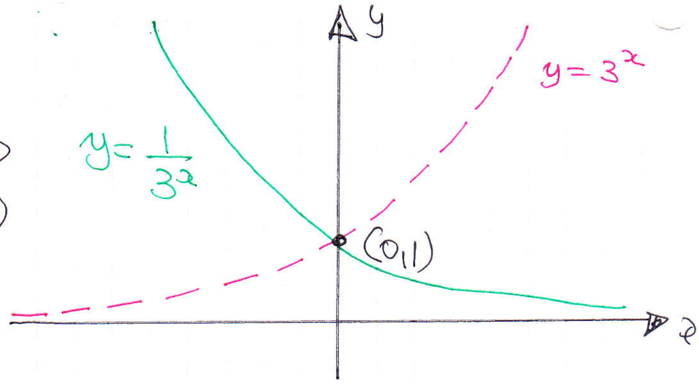


7. a) $y = \frac{1}{3^x} = 3^{-x}$

THIS IS A REFLECTION OF $y = 3^x$ ABOUT THE y AXIS

$\therefore f(x) = 3^x$

$f(-x) = 3^{-x}$



b) $y = \frac{2}{3}$

$\Rightarrow \frac{1}{3^x} = \frac{2}{3}$

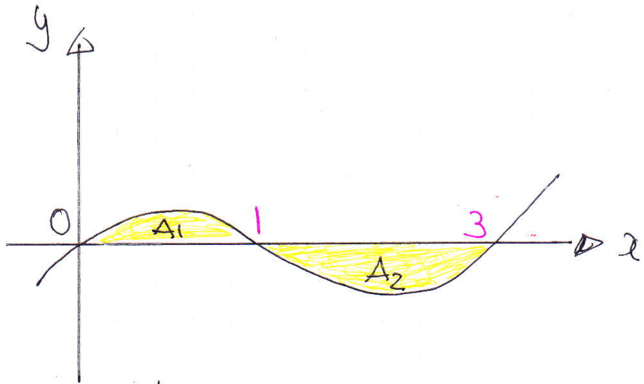
$\Rightarrow 3^x = \frac{3}{2}$

$\Rightarrow \log 3^x = \log \frac{3}{2}$

$\Rightarrow x \log 3 = \log(1.5)$

$\Rightarrow x = \frac{\log(1.5)}{\log 3}$

$\Rightarrow x \simeq 0.369$ (3 s.f.)

8. 

$y = 2x(x-1)(x-3)$

$y = 2x(x^2-4x+3)$

$y = 2x^3-8x^2+6x$

- $A_1 = \int_0^1 2x^3-8x^2+6x \, dx = \left[\frac{1}{2}x^4 - \frac{8}{3}x^3 + 3x^2\right]_0^1$

$= \left(\frac{1}{2} - \frac{8}{3} + 3\right) - (0) = \frac{5}{6}$

- $A_2 = \int_1^3 2x^3-8x^2+6x \, dx = \left[\frac{1}{2}x^4 - \frac{8}{3}x^3 + 3x^2\right]_1^3$

$= \left(\frac{81}{2} - 72 + 27\right) - \left(\frac{1}{2} - \frac{8}{3} + 3\right) = -\frac{16}{3}$

$\therefore$ REQUIRED AREA $= \frac{5}{6} + \frac{16}{3} = \frac{37}{6}$

C2, IYGB, PAPER B

9\. a) $\sin(2\theta + 30) = \frac{\sqrt{3}}{2}$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60°$

$$\begin{cases} 2\theta + 30 = 60 \pm 360n \\ 2\theta + 30 = 120 \pm 360n \end{cases}$$

$n = 0, 1, 2, 3, \ldots$

$$\begin{cases} 2\theta = 30 \pm 360n \\ 2\theta = 90 \pm 360n \end{cases}$$

$$\begin{cases} \theta = 15° \pm 180n \\ \theta = 45° \pm 180n \end{cases}$$

$\therefore \theta = 15°, -165°, 45°, -135°$

b) $\sin x = 2\cos x$

$\frac{\sin x}{\cos x} = \frac{2\cos x}{\cos x}$

$\tan x = 2$

$\arctan(2) = 63.4$

$x = 63.4° \pm 180n$

$n = 0, 1, 2, 3, 4, \ldots$

$x_1 = 63.4°$

$x_2 = 243.4°$

c) $2\sin^2 y - 5\cos y + 1 = 0$

$2(1 - \cos^2 y) - 5\cos y + 1 = 0$

$2 - 2\cos^2 y - 5\cos y + 1 = 0$

$0 = 2\cos^2 y + 5\cos y - 3$

$0 = (2\cos y - 1)(\cos y + 3)$

$\cos y = \begin{cases} 1/2 \\ \cancel{-3} \end{cases}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

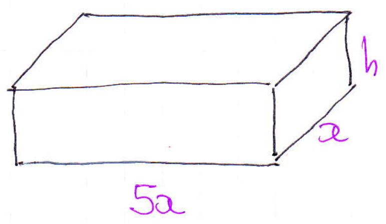
$\therefore y = \frac{\pi}{3} \pm 2n\pi$

$y = \frac{5\pi}{3} \pm 2n\pi$ $\quad n = 0, 1, 2, 3, \ldots$

$\therefore y_1 = \frac{\pi}{3}$

$y_2 = \frac{5\pi}{3}$

10. a)



TOTAL SURFACE AREA = 720

$(5x^2 + 5xh + xh) \times 2 = 720$

$5x^2 + 6xh = 360$ $\quad) \div 6$

$\frac{5}{6}x^2 + xh = 60$ $\quad) \times 5$

$\frac{25}{6}x^2 + 5xh = 300$ $\quad) \times x$

$\frac{25}{6}x^3 + 5x^2h = 300x$

$5x^2h = 300x - \frac{25}{6}x^3$

• $V = 5x^2h$

$\therefore V = 300x - \frac{25}{6}x^3$

AS REQUIRED

OR TAKE $5x^2 + 6xh = 360$

$6xh = 360 - 5x^2$

$h = \frac{360 - 5x^2}{6}$

& SUB INTO

$V = 5x^2h$

b) $\frac{dV}{dx} = 300 - \frac{25}{2}x^2$

• SOLVE FOR ZERO

$300 - \frac{25}{2}x^2 = 0$

$300 = \frac{25}{2}x^2$

$x^2 = 24$

$x = +\sqrt{24}$

$x \simeq 4.898979...$

$x \simeq 4.90$ cm

c) WHEN $x = \sqrt{24} \simeq 4.90$

$V = 300\sqrt{24} - \frac{25}{6}(\sqrt{24})^3$

$V \simeq 979.7958...$

$\therefore V_{MAX} \simeq 980$

$\frac{d^2V}{dx^2} = -25x$

$\left.\frac{d^2V}{dx^2}\right|_{x=\sqrt{24}} = -25\sqrt{24} = -122.47... < 0$

So A MAX VALUE FOR V