C2, IYGB, PAPER D

1. a) $y = (x+1)(x-2)(x-4) = (x+1)(x^2-6x+8)$

$$= x^3 - 6x^2 + 8x$$
$$\quad\;\; x^2 - 6x + 8$$
$$= x^3 - 5x^2 + 2x + 8$$

b) $\therefore \int_2^4 x^3 - 5x^2 + 2x + 8 \; dx = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_2^4$

$$= \left(64 - \frac{320}{3} + 16 + 32\right) - \left(4 - \frac{40}{3} + 4 + 16\right)$$

$$= \frac{16}{3} - \frac{32}{3} = -\frac{16}{3}$$

$\therefore$ AREA $= \frac{16}{3}$

2. $f(x) = ax^3 - x^2 - 5x + b$

$\left.\begin{array}{l} f(2) = 36 \\ f(-2) = 40 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8a - 4 - 10 + b = 36 \\ -8a - 4 + 10 + b = 40 \end{array}\right)$ add equations

$$-8 + 2b = 76$$
$$2b = 84$$
$$b = 42$$

& $8a - 14 + b = 36$
$$8a + b = 50$$
$$8a = 8$$
$$a = 1$$

3. a) $x^2 + y^2 - 20x + 8y + 16 = 0$

$\Rightarrow x^2 - 20x + y^2 + 8y + 16 = 0$

$\Rightarrow (x-10)^2 - 100 + (y+4)^2 \cancel{-16} \cancel{+16} = 0$

$\Rightarrow (x-10)^2 + (y+4)^2 = 100$

$\therefore$ CENTRE IS AT $C(10, -4)$

RADIUS $= 10$

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b) $C(10,-4)$ $P(4,4)$ GRADIENT $CP = \frac{y_2-y_1}{x_2-x_1} = \frac{4+4}{4-10} = \frac{8}{-6} = -\frac{4}{3}$

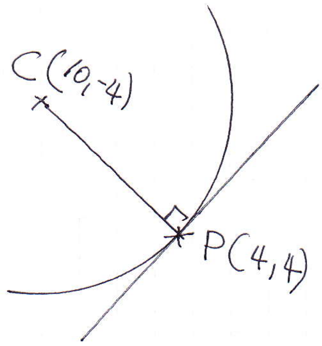
c) GRAD OF TANGENT $= \frac{3}{4}$

$y - y_0 = m(x - x_0)$

$y - 4 = \frac{3}{4}(x-4)$

$4y - 16 = 3x - 12$

$4y = 3x + 4$



4. $(1+kx)^6 = 1 + \frac{6}{1}(kx) + \frac{6\times5}{1\times2}(kx)^2 + \frac{6\times5\times4}{1\times2\times3}(kx)^3 + \dots$

$= 1 + 6kx + \boxed{15k^2}x^2 + \boxed{20k^3}x^3$

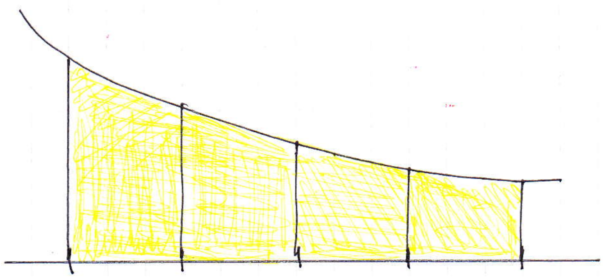
NOW $20k^3 = 2 \times 15k^2$

$20k^3 = 30k^2$

$20k = 30 \quad (k \neq 0)$

$k = \frac{3}{2}$

5. a)



x	4	6	8	10	12
y	0.7071	0.5	0.4082	0.3536	0.3162
	FIRST	←	REST	→	LAST

AREA $\simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$

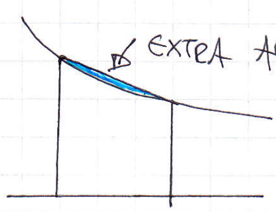
$\simeq \frac{2}{2}\left[0.7071 + 0.3162 + 2(0.5 + 0.4082 + 0.3536)\right]$

$\simeq 3.5467$

$\simeq 3.55$ (3 s.f.)

b) INCREASE THE NUMBER OF STRIPS (USE MORE TRAPEZIUMS)

c) OVERESTIMATE, AS THE TRAPEZIUMS "GO OVER THE CURVE" PRODUCING A LITTLE BIT EXTRA



6. $\frac{1}{\tan^2\phi} = 3$

$\Rightarrow \tan^2\phi = \frac{1}{3}$

$\Rightarrow \tan\phi = \pm\sqrt{\frac{1}{3}}$

- $\tan\phi = \sqrt{\frac{1}{3}}$

  $\arctan\left(\sqrt{\frac{1}{3}}\right) = \frac{\pi}{6}$

  $\phi = \frac{\pi}{6} \pm n\pi$

- $\tan\phi = -\sqrt{\frac{1}{3}}$

  $\arctan\left(-\sqrt{\frac{1}{3}}\right) = -\frac{\pi}{6}$

  $\phi = -\frac{\pi}{6} \pm n\pi$

$n = 0, 1, 2, 3 \ldots$

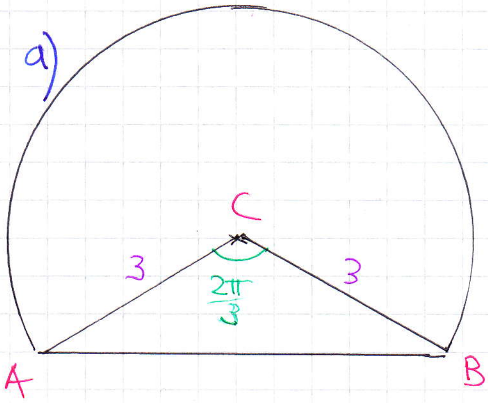
$\phi_1 = \frac{\pi}{6}$

$\phi_2 = \frac{7\pi}{6}$

$\phi_3 = \frac{5\pi}{6}$

$\phi_4 = \frac{11\pi}{6}$

7. a)



- BY THE COSINE RULE

$|AB|^2 = |AC|^2 + |CB|^2 - 2|AC||CB|\cos\frac{2\pi}{3}$

$|AB|^2 = 3^2 + 3^2 - 2\times3\times3\times\left(-\frac{1}{2}\right)$

$|AB|^2 = 27$

$|AB| = \sqrt{27} = 3\sqrt{3}$

(ALTERNATIVE SPLIT IN THE MIDDLE & USE TRIGONOMETRY ON A RIGHT ANGLE TRIANGLE)

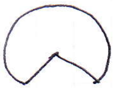
b) AREA OF TRIANGLE $= \frac{1}{2}|AC||CB|\sin\frac{2\pi}{3}$

$= \frac{1}{2}\times3\times3\times\frac{\sqrt{3}}{2} = \frac{9}{4}\sqrt{3} \approx 3.897$

(OR USE RIGHT ANGLE TRIANGLES )

c) THE REFLEX ANGLE $A\hat{C}B = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

AREA OF MAJOR SECTOR $= \frac{1}{2}r^2\theta^c = \frac{1}{2} \times 3^2 \times \frac{4\pi}{3} = 6\pi$

$\therefore$  =  + 

$= 6\pi + \frac{9}{4}\sqrt{3}$

AS REQUIRED

8. a)

- TOTAL SURFACE AREA = 3600

$\frac{1}{2}(15x)(20x) \times 2 + 20xy + 15xy + 25xy = 3600$

$300x^2 + 60xy = 3600$

$\boxed{5x^2 + xy = 60}$

- VOLUME = X-SECTIONAL AREA × LENGTH

$V = \frac{1}{2}(15x)(20x)y$

$V = 150x^2y$

BUT $xy = 60 - 5x^2$

$150xy = 9000 - 750x^2$

$\therefore V = 150x^2y = 9000x - 750x^3$

AS REQUIRED

b) $V = 9000 - 750x^3$

$\frac{dV}{dx} = 9000 - 2250x^2$

SOLVE FOR ZERO

$9000 - 2250x^2 = 0$

$9000 = 2250x^2$

$x^2 = 4$

$x = 2 \quad (x > 0)$

c) $\frac{d^2V}{dx^2} = -4500x$

$\left.\frac{d^2V}{dx^2}\right|_{x=2} = -9000 < 0$

$\therefore$ INDEED A MAX

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d) $5x^2 + xy = 60$

$5 \times 2^2 + 2y = 60$

$20 + 2y = 60$

$2y = 40$

$\therefore y = 20$

9. a) $x^2 \quad x+12 \quad 2x-3$

$$\Rightarrow \frac{x+12}{x^2} = \frac{2x-3}{x+12}$$

$$\Rightarrow (x+12)^2 = 2x^3 - 3x^2$$

$$\Rightarrow x^2 + 24x + 144 = 2x^3 - 3x^2$$

$$\Rightarrow 0 = 2x^3 - 4x^2 - 24x - 144$$

$$\Rightarrow 0 = x^3 - 2x^2 - 12x - 72$$

AS REQUIRED

b) FACTORIZE BY INSPECTION OR LONG DIVISION

$$x^3 - 2x^2 - 12x - 72 = 0$$

$$(x-6)(x^2 + 4x + 12) = 0$$

$b^2 - 4ac = 4^2 - 4 \times 1 \times 12 = -32 < 0$

$\therefore$ ONLY SOLUTION $x = 6$

c) IF $x = 6$

$u_1 = 36$

$u_2 = 18$

$u_3 = 9$

$$\left.\begin{array}{l} \therefore a = 36 \\ r = \frac{1}{2} \end{array}\right\} \Rightarrow S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{36}{1-\frac{1}{2}}$$

$$S_\infty = 72$$

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10. a) $6\times\left(\frac{1}{2}\right)^{\frac{x-4}{3}}=1.89$

$\Longrightarrow \left(\frac{1}{2}\right)^{\frac{x-4}{3}}=0.315$

$\Longrightarrow \log\left(\frac{1}{2}\right)^{\frac{x-4}{3}}=\log(0.315)$

$\Longrightarrow \frac{x-4}{3}=\frac{\log(0.315)}{\log(0.5)}$

$\Longrightarrow \frac{x-4}{3}\simeq 1.66657\ldots$

$\Longrightarrow x\simeq 9.00$

b) $\log_2(8y-1)-2\log_2(y+1)=3-\log_2(y+4)$

$\Longrightarrow \log_2(8y-1)-\log_2(y+1)^2=3\log_2 2-\log_2(y+4)$

$\Longrightarrow \log_2\left(\frac{8y-1}{(y+1)^2}\right)=\log_2 8-\log(y+4)$

$\Longrightarrow \log_2\left(\frac{8y-1}{y^2+2y+1}\right)=\log_2\left(\frac{8}{y+4}\right)$

$\Longrightarrow \frac{8y-1}{y^2+2y+1}=\frac{8}{y+4}$

$\Longrightarrow (8y-1)(y+4)=8(y^2+2y+1)$

$\Longrightarrow \cancel{8y^2}+31y-4=\cancel{8y^2}+16y+8$

$\Longrightarrow 15y=12$

$\Longrightarrow y=\frac{4}{5}$