C2, IYGB, PAPER F

1. a) $f(x) = x^3 - 2x^2 + kx + 6$

$$f(3) = 0 \Rightarrow 3^3 - 2 \times 3^2 + k \times 3 + 6 = 0$$
$$\Rightarrow 27 - 18 + 3k + 6 = 0$$
$$\Rightarrow 3k = -15$$
$$k = -5$$

b)
$$\begin{array}{r} x^2 + x - 2 \\ x-3 \overline{\big) x^3 - 2x^2 - 5x + 6} \\ \underline{-x^3 + 3x^2 } \\ x^2 - 5x + 6 \\ \underline{-x^2 + 3x } \\ -2x + 6 \\ \underline{2x - 6} \\ 0 \end{array}$$

$\therefore f(x) = (x-3)(x^2 + x - 2)$

$f(x) = (x-3)(x-1)(x+2)$

c) $f(x) = (x-3)(x-1)(x+2)$

$f(-3) = (-6)(-4)(-1) = -24$

2. a) FIRSTLY THE "GAP" IS 1

x	0	1	2	3	4
y	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{8}{5}$	$\frac{8}{3}$

$y = \frac{2^x}{x+2}$

$$\int_0^4 \frac{2^x}{x+2}\,dx \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$
$$\approx \frac{1}{2}\left[\frac{1}{2} + \frac{8}{3} + 2\left(\frac{2}{3} + 1 + \frac{8}{5}\right)\right]$$
$$\approx 4.85$$

b) USE MORE STRIPS FOR BETTER ACCURACY

C2, IYGB, PAPER F

3. a) $(1-2x)^{10} = 1 + \frac{10}{1}(-2x) + \frac{10\times9}{1\times2}(-2x)^2 + \frac{10\times9\times8}{1\times2\times3}(-2x)^3 + \dots$

$= 1 - 20x + 180x^2 - 960x^3 + \dots$

b) $1 - 2x = 0.98$
$1 - 0.98 = 2x$
$0.02 = 2x$
$x = 0.01$

$(1-2x)^{10} = 1 - 20x + 180x^2 - 960x^3 + \dots$

$\Rightarrow (1-2\times0.01)^{10} = 1 - 20(0.01) + 180(0.01)^2 - 960(0.01)^3$

$\Rightarrow 0.98^{10} = 1 - 0.2 + 0.018 - 0.00096$

$\Rightarrow 0.98^{10} \approx 0.81704$

$\Rightarrow 0.98^{10} \simeq 0.817$ 3 d.p.

4. a) $a = 22000$
$r = 1.05$
$n = 30$

$\boxed{u_n = a \times r^{n-1}}$

$u_{30} = 22000 \times 1.05^{29}$

$u_{30} = 90554.9831\dots$

$\therefore$ APPROX £90555

b) $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

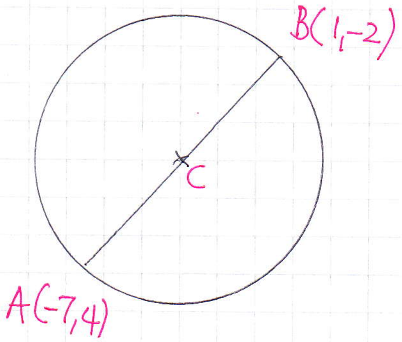
$S_{30} = \frac{22000(1-1.05^{30})}{1-1.05}$

$S_{30} \simeq 1461654.645\dots$

$\therefore$ APPROX £1,461,655

C2, IYGB, PAPER F

5. a)



- C IS THE MIDPOINT OF AB

$C\left(\frac{-7+1}{2}, \frac{4-2}{2}\right) \Rightarrow C(-3,1)$

- $C(-3,1)$ $B(1,-2)$

$r = |BC| = \sqrt{(-3-1)^2+(1+2)^2}$

$r = \sqrt{16+9} = 5$

$\therefore (x+3)^2+(y-1)^2 = 25$

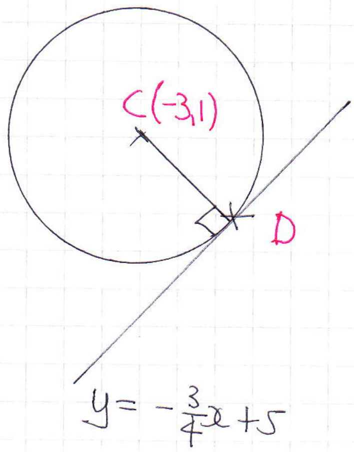
b) $4y+3x=20$ IS A TANGENT AT D

$4y = -3x+20$

$y = -\frac{3}{4}x+5$

$\therefore$ GRADIENT CD MUST BE $\frac{4}{3}$

$\therefore$ USING $C(-3,1)$ & $m=\frac{4}{3}$



$y-y_0 = m(x-x_0)$

$y-1 = \frac{4}{3}(x+3)$

$3y-3 = 4x+12$

$3y = 4x+15$

c) SOLVING SIMULTANEOUSLY

$\left.\begin{aligned} 4y &= -3x+20 \quad (\times 3) \\ 3y &= 4x+15 \quad (\times 4) \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 12y &= -9x+60 \\ 12y &= 16x+60 \end{aligned}\right\} \Rightarrow$

$-9x+60 = 16x+60$

$0 = 25x$

$x = 0$

$y = -\frac{3}{4}\times 0+5 = 5$ $\quad\therefore D(0,5)$

C2, IYGB, PAPER F

6. $2\log_3 x - \log_3(x-2) = 2$

$\Rightarrow \log_3 x^2 - \log_3(x-2) = 2\log_3 3$

$\Rightarrow \log_3\left(\frac{x^2}{x-2}\right) = \log_3 9$

$\Rightarrow \frac{x^2}{x-2} = 9$

$\Rightarrow x^2 = 9x - 18$

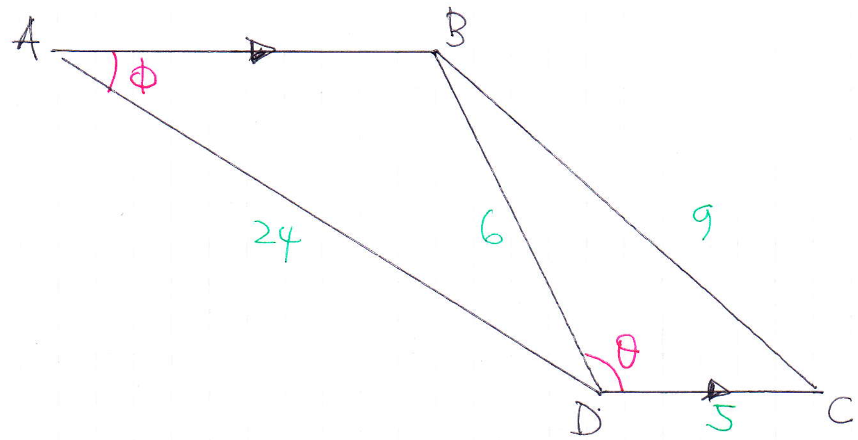
$\Rightarrow x^2 - 9x + 18 = 0$

$\Rightarrow (x-3)(x-6) = 0$

$\Rightarrow x = \begin{cases} 3 \\ 6 \end{cases}$

BOTH O.K.

7. a)



BY THE COSINE RULE ON BCD

$\Rightarrow |BC|^2 = |BD|^2 + |DC|^2 - 2|BD||DC|\cos\theta$

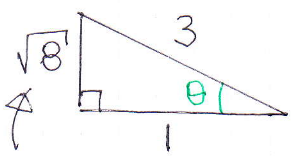
$\Rightarrow 9^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos\theta$

$\Rightarrow 81 = 36 + 25 - 60\cos\theta$

$\Rightarrow 60\cos\theta = -20$

$\Rightarrow \cos\theta = -\frac{1}{3}$ AS REQUIRED

b) METHOD A



BY PYTHAGORAS

$\cos\theta = \cancel{-}\frac{1}{3}$

$\therefore \sin\theta = \frac{\sqrt{8}}{3}$

$\sin\theta = \frac{2}{3}\sqrt{2}$

METHOD B

$\cos^2\theta + \sin^2\theta = 1$

$\left(-\frac{1}{3}\right)^2 + \sin^2\theta = 1$

$\frac{1}{9} + \sin^2\theta = 1$

$\sin^2\theta = \frac{8}{9}$

$\sin\theta = +\sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$

c) ANGLE $\hat{ABD} = B\hat{D}C = \theta$ (ALTERNATE ANGLES)

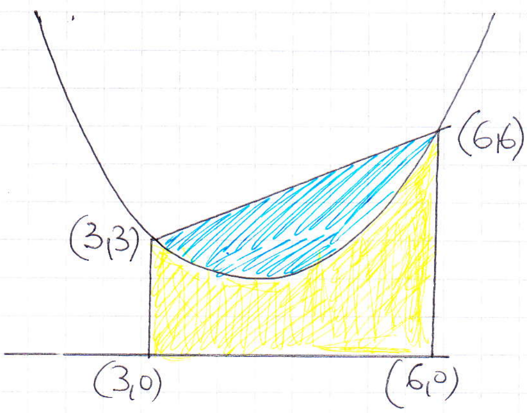
BY THE SINE RULE IN $\hat{ABD}$: $\dfrac{\sin\phi}{6} = \dfrac{\sin\theta}{24}$ ← $\theta = A\hat{B}D$

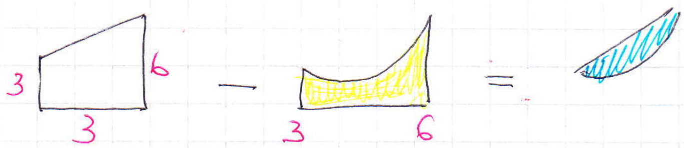
$\Rightarrow \dfrac{\sin\phi}{6} = \dfrac{\frac{2}{3}\sqrt{2}}{24}$

$\Rightarrow 24\sin\phi = 4\sqrt{2}$

$\Rightarrow \sin\phi = \frac{1}{6}\sqrt{2}$

8.





$\therefore$ AREA $= \dfrac{3+6}{2} \times 3$

$= \dfrac{27}{2}$

$\displaystyle\int_3^6 x^2 - 8x + 18 \, dx = \left[\tfrac{1}{3}x^3 - 4x^2 + 18x\right]_3^6$

$= (72 - 144 + 108) - (9 - 36 + 54)$

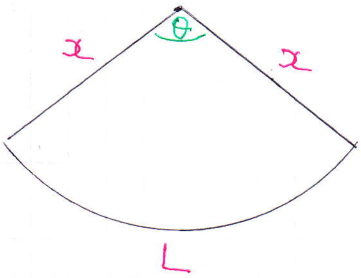
$= 36 - 27$

$= 9$

$\therefore$ REQUIRED AREA $= \dfrac{27}{2} - 9 = \dfrac{9}{2}$

C2, IYGB, PAPER F

9. a)



- $A = 36$

$\frac{1}{2}x^2\theta = 36$

$x^2\theta = 72$

NOW $P = 2x + L$ "$L = r\theta$"

$\Rightarrow P = 2x + x\theta$

$\Rightarrow P = 2x + x\left(\frac{72}{x^2}\right) \longleftarrow$ $x^2\theta = 72$, $\theta = \frac{72}{x^2}$

$\Rightarrow P = 2x + \frac{72}{x}$

AS REQUIRED

b) $P = 2x + 72x^{-1}$

$\Rightarrow \frac{dP}{dx} = 2 - 72x^{-2}$

$\Rightarrow \frac{dP}{dx} = 2 - \frac{72}{x^2}$

- FOR MIN|MAX $\frac{dP}{dx} = 0$

$\Rightarrow 2 - \frac{72}{x^2} = 0$

$\Rightarrow 2 = \frac{72}{x^2}$

$\Rightarrow 2x^2 = 72$

$\Rightarrow x^2 = 36$

$\Rightarrow x = 6 \quad (x > 0)$

c) $P = 2x + \frac{72}{x}$

$P_{MIN} = 2 \times 6 + \frac{72}{6}$

$P_{MIN} = 24$

$\frac{d^2P}{dx^2} = 144x^{-3} = \frac{144}{x^3}$

$\left.\frac{d^2P}{dx^2}\right|_{x=6} = \frac{144}{6^3} = \frac{2}{3} > 0$

$\therefore$ INDEED A MINIMUM

d) $\theta = \frac{72}{x^2}$

$\theta_{MIN} = \frac{72}{6^2} = 2$

$\therefore \theta = 2^c$

10. $3\tan\theta\sin\theta = \cos\theta + 1$

$\Rightarrow 3\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta = \cos\theta + 1$

$\Rightarrow \frac{3\sin^2\theta}{\cos\theta} = \cos\theta + 1$

$\Rightarrow 3\sin^2\theta = \cos\theta(\cos\theta + 1)$

$\Rightarrow 3\sin^2\theta = \cos^2\theta + \cos\theta$

$\Rightarrow 3(1-\cos^2\theta) = \cos^2\theta + \cos\theta$

$\Rightarrow 3 - 3\cos^2\theta = \cos^2\theta + \cos\theta$

$\Rightarrow 0 = 4\cos^2\theta + \cos\theta - 3$

$\Rightarrow 0 = (4\cos\theta - 3)(\cos\theta + 1)$

$\Rightarrow \cos\theta = \begin{cases} 3/4 \\ -1 \end{cases}$

- $\arccos\left(\frac{3}{4}\right) = 0.723^c$

$\left(\begin{array}{l} \theta = 0.723^c \pm 2n\pi \\ \theta = 5.56^c \pm 2n\pi \end{array}\right.$

- $\arccos(-1) = \pi$

$\left(\begin{array}{l} \theta = \pi \pm 2n\pi \\ \theta = \pi \pm 2n\pi \end{array}\right.$ $n = 0, 1, 2, 3, \ldots$

$0.72^c,\ 5.56^c,\ 3.14^c \leftarrow \pi$