C2, IYGB, PAPER G

1. $\cos(2\theta+25) = -0.454$

$\arccos(-0.454) = 117.0°$

$$\begin{cases} 2\theta+25 = 117° \pm 360n \\ 2\theta+25 = 243° \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\begin{cases} 2\theta = 92° \pm 360n \\ 2\theta = 218° \pm 360n \end{cases}$$

$\theta = 46° \pm 180n$

$\theta = 109 \pm 180n$

• IF $0 \leq \theta < 360$

$\theta_1 = 46°$

$\theta_2 = 226°$

$\theta_3 = 109°$

$\theta_4 = 289°$

2.

x	1	1.4	1.8	2.2	2.6	3
y	1.000	1.0756	1.1802	1.3015	1.4340	1.5748

$\leftarrow (\sqrt{x} - \log_{10}x)^2$

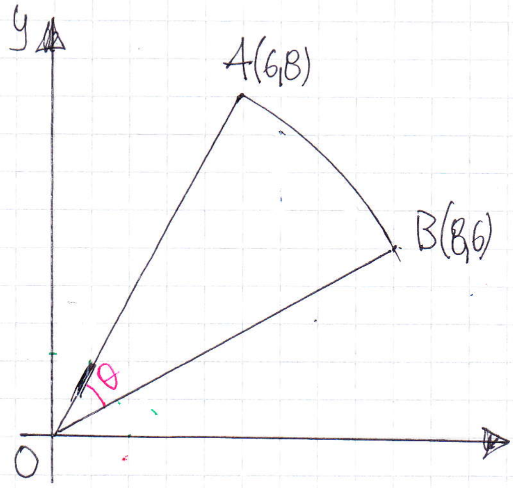
$$\int_1^3 (\sqrt{x} - \log_{10}x)^2 \, dx \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$\approx \frac{0.4}{2}\left[1 + 1.5748 + 2(1.0756 + 1.1802 + 1.3015 + 1.4340)\right]$$

$$\approx 2.511\ldots$$

$$\simeq 2.51 \quad (3\text{ sf})$$

3. a)



• METHOD A

$|OA| = \sqrt{(6-0)^2+(8-0)^2} = \sqrt{36+64} = 10$

$|OB| = \sqrt{(8-0)^2+(6-0)^2} = \sqrt{36+64} = 10$

$|AB| = \sqrt{(8-6)^2+(6-8)^2} = \sqrt{8}$

BY THE COSINE RULE ON $\hat{OAB}$

$(\sqrt{8})^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos\theta$

$8 = 100 + 100 - 200\cos\theta$

$200\cos\theta = 192$

$\cos\theta = 0.96$

$\theta \approx 0.2838^c$

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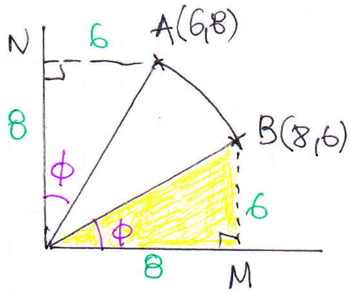
METHOD B



- $\tan\phi = \frac{6}{8}$

  $\phi \simeq 0.64350...$

- $\theta = \frac{\pi}{2} - 2(0.64350...)$

  $\theta \simeq 0.283794...$

  $\theta \simeq 0.2838^c$

b) LENGTH OF $|OB| = \sqrt{6^2+8^2} = 10$

AREA $= \frac{1}{2}r^2\theta^c = \frac{1}{2}\times 10^2 \times 0.2838... \simeq 14.19$

4. a) $\log_a 100 = \log_a(4\times 25) = \log_a 4 + \log_a 25 = \log_a 4 + \log_a 5^2$

$= \log_a 4 + 2\log_a 5 = p + 2q$

b) $\log_a 0.4 = \log_a\left(\frac{2}{5}\right) = \log_a 2 - \log_a 5 = \log_a 4^{\frac{1}{2}} - \log_a 5$

$= \frac{1}{2}\log_a 4 - \log_a 5 = \frac{1}{2}p - q$

5. FIRSTLY $(2+kx)^5 = \binom{5}{0}(2)^5(kx)^0 + \binom{5}{1}(2)^4(kx)^1 + \binom{5}{2}(2)^3(kx)^2 + ...$

$= (1\times 32\times 1) + (5\times 16\times kx) + (10\times 8\times k^2x^2) + ...$

$= 32 + 80kx + 80k^2x^2 + ...$

NEXT $(1-2x)(2+kx)^5 = (1-2x)(32+80kx+80k^2x^2+...)$

$-160kx^2$

$80k^2x^2$

- COEFFICIENT OF $x^2 = 240$

$80k^2 - 160k = 240$

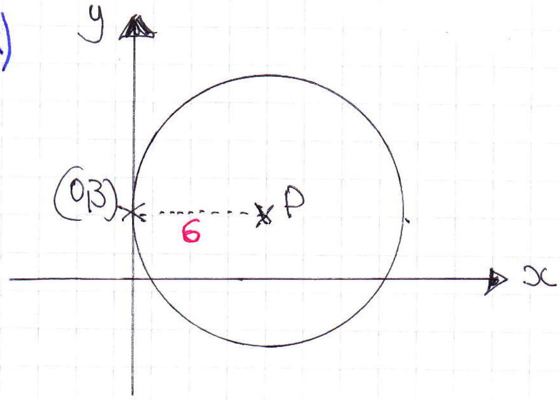
$k^2 - 2k = 3$

$k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

$\therefore k = 3$ or -1

6) a)

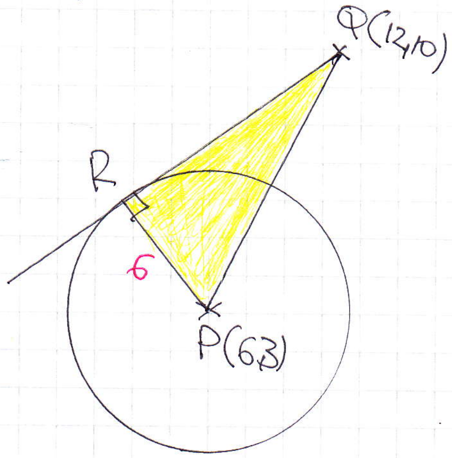


BY INSPECTION THE CENTRE IS AT (6,3)

$\therefore\ (x-6)^2+(y-3)^2=6^2$

$(x-6)^2+(y-3)^2=36$

b)



METHOD A

$|PQ|=\sqrt{(10-3)^2+(12-6)^2}$

$=\sqrt{49+36}=\sqrt{85}$

BY PYTHAGORAS ON $\overset{\triangle}{PQR}$

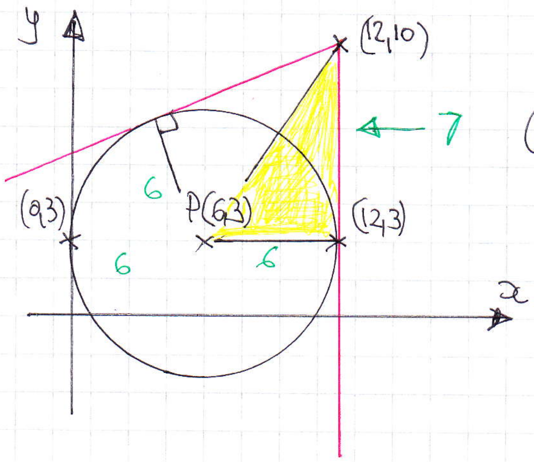
$|PR|^2+|QR|^2=|PQ|^2$

$6^2+|QR|^2=(\sqrt{85})^2$

$|QR|^2=49$

$|QR|=7$

METHOD B NOTICE THAT Q(12,10) IS DIRECTLY ABOVE THE EDGE OF THE CIRCLE!!



(SINCE 10-3)!

(P.T.O)

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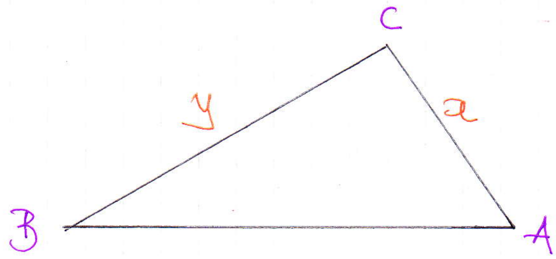
7. a)



$\sin A = \frac{4}{5}$

$\sin B = \frac{8}{17}$

$\sin C = \frac{84}{85}$

- BY THE SINE RULE

$$\frac{y}{\sin A} = \frac{x}{\sin B} \Longrightarrow \frac{y}{\frac{4}{5}} = \frac{x}{\frac{8}{17}}$$

$$\Longrightarrow \frac{8}{17}y = \frac{4}{5}x$$

$$\Longrightarrow 8y = \frac{68}{5}x$$

$$y = \frac{17}{10}x$$

$$y = 1.7x$$

b) AREA = 21

$$\Longrightarrow \frac{1}{2}xy\sin C = 21$$

$$\Longrightarrow \frac{1}{2}xy \times \frac{84}{85} = 21$$

$$\Longrightarrow \frac{42}{85}xy = 21$$

$$\Longrightarrow xy = \frac{85}{2}$$

- BUT $y = 1.7x$

$$\Longrightarrow x(1.7x) = \frac{85}{2}$$

$$\Longrightarrow 1.7x^2 = \frac{85}{2}$$

$$\Longrightarrow x^2 = 25$$

$$\Longrightarrow x = 5 \quad (x>0)$$

& $y = 1.7x$

$y = 1.7 \times 5$

$y = 8.5$ cm

C2, IYGB, PAPER G

8. a) I)

	FLOWERS	
YEAR 1 →	1 →	3^0
YEAR 2 →	3 →	3^1
YEAR 3 →	9 →	3^2
YEAR 4 →	27 →	3^3
⋮	⋮	⋮
YEAR n		3^{n-1}

$\therefore f_n = 3^{n-1}$

OR SIMPLY IT IS A G.P

$a = 1$

$r = 3$

$f_n = a \times r^{n-1}$

$f_n = 1 \times 3^{n-1}$

$f_n = 3^{n-1}$

(II)

YEAR 1	1	$\leftarrow S_1$
YEAR 2	$1+3$	$\leftarrow S_2$
YEAR 3	$1+3+9$	$\leftarrow S_3$
YEAR 4	$1+3+9+27$	$\leftarrow S_4$
⋮		
YEAR n	----------	$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_n = \dfrac{1(1-3^n)}{1-3}$

$S_n = \dfrac{1-3^n}{-2}$

$S_n = \frac{1}{2}(3^n - 1)$

b)

$S_n = 1093$

$\frac{1}{2}(3^n - 1) = 1093$

$3^n - 1 = 2186$

$3^n = 2187$

$n = 7$ BY INSPECTION

$\therefore f_n = 3^{n-1}$

$f_7 = 3^6 = 729$

$\therefore$ 729 flowers

C2, IYGB, PAPER G

c) $f_n = 3^{n-1}$

$f_7 = 729$ found in (b)

$f_8 = 2187$ ← AT LEAST THAT MANY FLOWERS

∴ THE PLANT IS AT LEAST 8 YEARS OLD

$S_8 = \frac{1}{2}(3^8 - 1) = 3280$

9. a) $f(x) = x^3 + x^2 - x + 15$

$f(-3) = (-3)^3 + (-3)^2 - (-3) + 15$

$= -27 + 9 + 3 + 15$

$= 0$

∴ $(x+3)$ IS A FACTOR

b) $f'(x) = 3x^2 + 2x - 1$

SOLVE FOR ZERO

$\Rightarrow 3x^2 + 2x - 1 = 0$

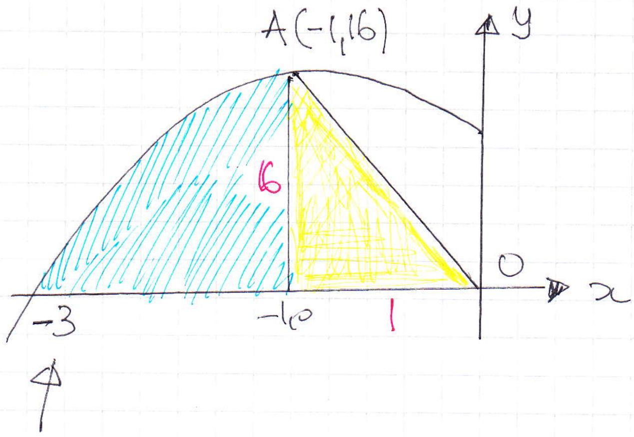
$\Rightarrow (3x-1)(x+1) = 0$

$x = \begin{cases} -1 \\ \frac{1}{3} \end{cases}$ $\quad y = \begin{cases} -1+1+1+15 = 16 \\ \frac{1}{27} + \frac{1}{9} - \frac{1}{3} + 15 = \frac{400}{27} \end{cases}$

∴ $A(-1, 16)$ $\quad B\left(\frac{1}{3}, \frac{400}{27}\right)$

(P.T.O)

c)



SINCE $(x+3)$ IS A FACTOR (PART a)
(ONLY PLACE SINCE NO OTHER STATIONARY POINTS)

- AREA OF "YELLOW TRIANGLE" $= \frac{1}{2}\times 1\times 16 = 8$
- "BLUE AREA" UNDER CURVE $= \displaystyle\int_{-3}^{-1} x^3+x^2-x+15\,dx$

$$= \left[\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{1}{2}x^2+15x\right]_{-3}^{-1}$$

$$= \left(-\frac{187}{12}\right)-\left(-\frac{153}{4}\right)$$

$$= \frac{68}{3}$$

$\therefore$ REQUIRED AREA $= 8+\frac{68}{3} = \frac{92}{3}$