C2, IYGB, PAPER H

1\. a) $(1+3x)^8 = 1 + \frac{8}{1}(3x)^1 + \frac{8\times7}{1\times2}(3x)^2 + \frac{8\times7\times6}{1\times2\times3}(3x)^3 + \dots$

$= 1 + 24x + 252x^2 + 1512x^3 + \dots$

b) $\dots + \frac{8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6}(3x)^6 + \dots$

$\uparrow$ $28 \times 729 x^6$ $\therefore 20412$

2\. a) $r = 2a$ $S_\infty = \frac{a}{1-r}$

$1 = \frac{a}{1-2a}$

$1 - 2a = a$

$1 = 3a$

$a = \frac{1}{3}$

b) $\boxed{u_n = ar^{n-1}}$

$u_5 = \frac{1}{3} \times \left(\frac{2}{3}\right)^4$

$u_5 = \frac{16}{243}$

3\. a) $f(x) = x^3 + px^2 + qx + 6$

$f(1) = 0 \Rightarrow 1 + p + q + 6 = 0$
$f(-1) = 8 \Rightarrow -1 + p - q + 6 = 8$
$\Rightarrow$ $p + q = -7$, $p - q = 3$ Add

$\Rightarrow 2p = -4$
$p = -2$

q $p + q = -7$
$-2 + q = -7$
$q = -5$

C2, IYGB, PAPER H

b) FACTORIZE FULLY:

$$\begin{array}{r} x^2 - x - 6 \\ x-1 \overline{\big) x^3 - 2x^2 - 5x + 6} \\ \underline{-x^3 + x^2 \quad\quad\quad} \\ -x^2 - 5x + 6 \\ \underline{+x^2 - x \quad\;} \\ -6x + 6 \\ \underline{+6x - 6} \\ 0 \end{array}$$

THUS

$f(x) = 0$

$x^3 - 2x^2 - 5x + 6 = 0$

$(x-1)(x^2 - x - 6) = 0$

$(x-1)(x+2)(x-3) = 0$

$\therefore x = 1, -2, 3$

4.

x	0	5	10	15	20	25	30
y	0	2.12	2.94	3.03	2.77	1.91	0
	FIRST	←		REST		→	LAST

$$\text{AREA} \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$\approx \frac{5}{2}\left[0 + 0 + 2(2.12 + 2.94 + 3.03 + 2.77 + 1.91)\right]$$

$$\approx 63.85 \text{ m}^2$$

5.

$2\sin\theta = 5\cos\theta$

$\frac{2\sin\theta}{\cos\theta} = \frac{5\cos\theta}{\cancel{\cos\theta}}$

$2\tan\theta = 5$

$\tan\theta = \frac{5}{2}$

$\arctan\left(\frac{5}{2}\right) = 68.198\ldots$

$\theta = 68.2° \pm 180n \qquad n = 0, 1, 2, 3, \ldots$

$\theta_1 = 68.2°$

$\theta_2 = 248.2°$

C2, IYGB, PAPER H

6. $\log_a x + \log_a(x-3) = \log_a 10$

$\Rightarrow \log_a[x(x-3)] = \log_a 10$

$\Rightarrow \log_a[x^2-3x] = \log_a(10)$

$\Rightarrow x^2-3x = 10$

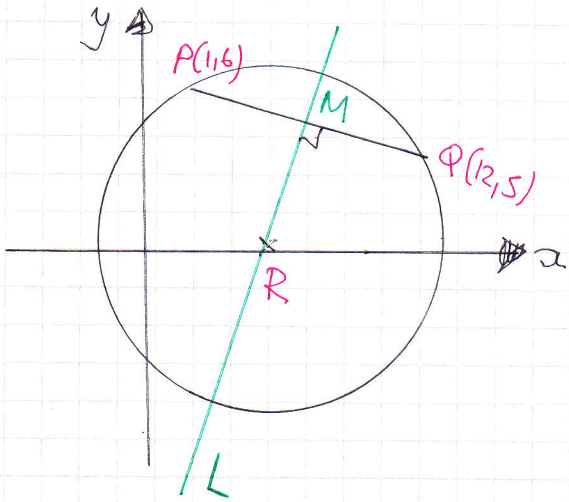
$\Rightarrow x^2-3x-10=0$

$\Rightarrow (x-5)(x+2)=0$

$x = 5$ or $\cancel{-2}$

7. a) GRAD $PQ = \frac{y_2-y_1}{x_2-x_1} = \frac{5-6}{12-1} = \frac{-1}{11} = -\frac{1}{11}$

b)



- MIDPOINT OF $PQ = \left(\frac{1+12}{2}, \frac{6+5}{2}\right) = \left(\frac{13}{2}, \frac{11}{2}\right)$
- GRADIENT OF L IS 11
- EQUATION OF L IS

$\boxed{y-\frac{11}{2} = 11\left(x-\frac{13}{2}\right)}$

- WHEN $y=0$

$-\frac{11}{2} = 11\left(x-\frac{13}{2}\right)$

$-\frac{11}{2} = 11x - \frac{143}{2}$

$66 = 11x$

$x = 6$

$\therefore R(6,0)$

AS REQUIRED

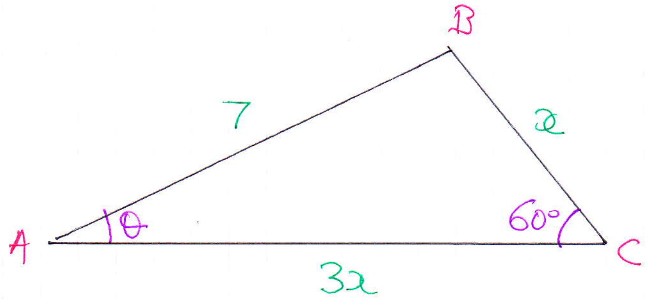
c) RADIUS $= |PR| = \sqrt{(1-6)^2+(6-0)^2} = \sqrt{25+36} = \sqrt{61}$

$\therefore (x-6)^2+(y-0)^2 = (\sqrt{61})^2$

$(x-6)^2+y^2 = 61$

C2, IYGB, PAPER H

8.



- BY THE COSINE RULE

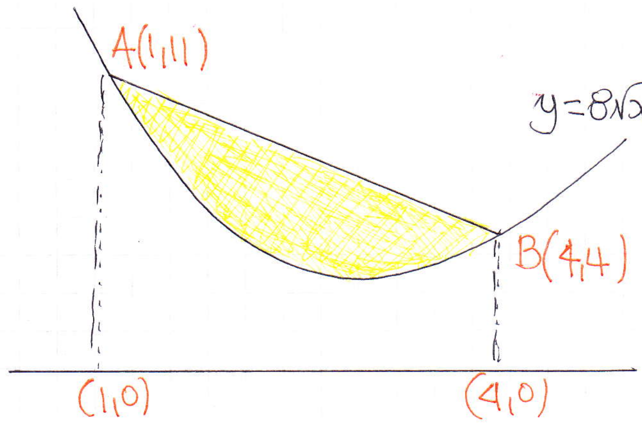
$$\Rightarrow |BA|^2 = |BC|^2 + |AC|^2 - 2|BC||AC|\cos 60$$
$$\Rightarrow 7^2 = x^2 + 9x^2 - 2x(3x) \times \tfrac{1}{2}$$
$$\Rightarrow 49 = x^2 + 9x^2 - 3x^2$$
$$\Rightarrow 49 = 7x^2$$
$$\Rightarrow 7 = x^2$$
$$\Rightarrow \boxed{x = +\sqrt{7}}$$

- BY THE SINE RULE

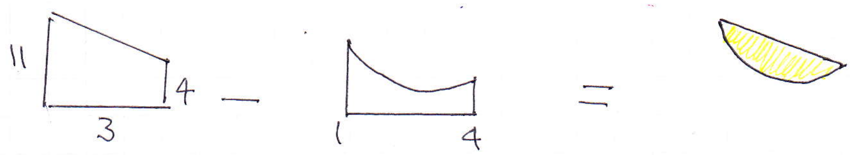
$$\frac{\sin\theta}{x} = \frac{\sin 60°}{7}$$
$$\Rightarrow \frac{\sin\theta}{\sqrt{7}} = \frac{\sqrt{3}/2}{7}$$
$$\Rightarrow 7\sin\theta = \frac{\sqrt{21}}{2}$$
$$\Rightarrow \sin\theta = \frac{\sqrt{21}}{14}$$

// AS REQUIRED

9.



- $y_1 = 8 + 16 - 13 = 11$
- $y_4 = 16 + 1 - 13 = 4$



$$\frac{11+4}{2} \times 3 = \frac{45}{2}$$

$$\int_1^4 8\sqrt{x} + \frac{16}{x^2} - 13 \, dx =$$

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$$= \int_1^4 8x^{\frac{1}{2}} + 16x^{-2} - 13 \, dx = \left[\frac{8}{3/2} x^{\frac{3}{2}} - 16x^{-1} - 13x \right]_1^4$$

$$= \left[\frac{16}{3} x^{\frac{3}{2}} - \frac{16}{x} - 13x \right]_1^4 = \left(\frac{128}{3} - 4 - 52 \right) - \left(\frac{16}{3} - 16 - 13 \right)$$

$$= -\frac{40}{3} - \left(-\frac{71}{3} \right) = \frac{31}{3}$$

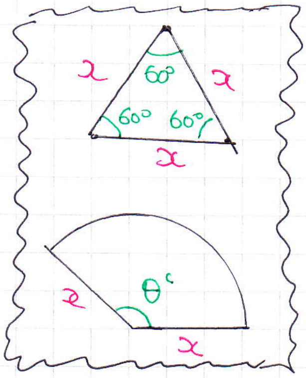
$\therefore$ REQUIRED AREA $= \frac{45}{2} - \frac{31}{3} = \frac{73}{6}$

10. a) TOTAL LENGTH $= 60$

$(x+x+x) + (x + x + \underbrace{x\theta}_{\text{ARC}}) = 60$

$5x + x\theta = 60$

$x\theta = 60 - 5x$



b) TOTAL AREA $= \underbrace{\frac{1}{2}x^2 \sin 60}_{\text{TRIANGLE}} + \underbrace{\frac{1}{2}x^2\theta}_{\text{SECTOR}}$

$$\Rightarrow A = \frac{1}{2}x^2 \times \frac{\sqrt{3}}{2} + \frac{1}{2}x^2\theta$$

$$\Rightarrow A = \frac{1}{4}\sqrt{3}x^2 + \frac{1}{2}x^2\theta$$

$$\Rightarrow A = \frac{1}{4}\sqrt{3}x^2 + \frac{1}{2}x(x\theta)$$

$$\Rightarrow A = \frac{1}{4}\sqrt{3}x^2 + \frac{1}{2}x(60 - 5x)$$

$$\Rightarrow A = \frac{1}{4}\sqrt{3}x^2 + 30x - \frac{5}{2}x^2$$

$$\Rightarrow A = \left(\frac{1}{4}\sqrt{3} - \frac{5}{2}\right)x^2 + 30x$$

$$\Rightarrow A = \frac{1}{4}(\sqrt{3} - 10)x^2 + 30x$$

AS REQUIRED

C2, IYGB, PAPER H

c) $A = \frac{1}{4}(\sqrt{3}-10)x^2 + 30x$

$\frac{dA}{dx} = \frac{1}{2}(\sqrt{3}-10)x + 30$

SOLVE FOR ZERO

$\Rightarrow \frac{1}{2}(\sqrt{3}-10)x + 30 = 0$

$\Rightarrow (\sqrt{3}-10)x = -60$

$\Rightarrow (10-\sqrt{3})x = 60$

$\Rightarrow x = \frac{60}{10-\sqrt{3}}$

$\Rightarrow x \simeq 7.2569\ldots$

$\Rightarrow x \simeq 7.26$

d) $\frac{d^2A}{dx^2} = \frac{1}{2}(\sqrt{3}-10) < 0$

WHICH IS INDEPENDENT OF x

SO MAXIMUM

$V_{MAX} = 108.854\ldots$

$V_{MAX} \simeq 109$

11. • $y = \sin x$ INTERCEPT THE x AXIS EVERY 180° (π) BUT THIS GRAPH INTERCEPTS EVERY $\frac{\pi}{3}$ (60°), BY LOOKING AT THE DIFFERENCES A, B, C.

$\therefore$ THERE IS A HORIZONTAL SCALE FACTOR OF $\frac{1}{3}$

$\therefore n = 3$

• NEXT DEAL WITH THE TRANSLATION, WHICH TAKE PLACE BEFORE THE "STRETCH"

CONSIDER $x=0$ ON $y = \sin x$

$\frac{0+\phi}{3} \longmapsto \frac{\pi}{9}$

$\phi = \frac{\pi}{3}$

ALTERNATIVE

$y = \sin(nx - \phi)$

$\begin{cases} n\left(\frac{\pi}{9}\right) - \phi = 0 \leftarrow \text{"FIRST" } x \text{ INTERCEPT OF } \sin x \\ n\left(\frac{4\pi}{9}\right) - \phi = \pi \leftarrow \text{2ND } x \text{ INTERCEPT OF } \sin x \end{cases}$

$\begin{cases} \phi = \frac{n\pi}{9} \\ \phi = \frac{4n\pi}{9} - \pi \end{cases}$

$\frac{n\pi}{9} = \frac{4n\pi}{9} - \pi$

$n\pi = 4n\pi - 9\pi$

$n = 4n - 9$

$9 = 3n$

$n = 3$

& $\phi = \frac{n\pi}{9}$ $\quad \phi = \frac{\pi}{3}$