1. $\int_1^9 6\sqrt{x} - \frac{6}{\sqrt{x}}\,dx = \int_1^9 6x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}\,dx = \left[4x^{\frac{3}{2}} - 12x^{\frac{1}{2}}\right]_1^9$

$= (108-36) - (4-12) = 72 + 8 = 80$

2. a) $f(x) = 2x^3 - 7x^2 - 5x + 4$

$f(-2) = 2(-2)^3 - 7(-2)^2 - 5(-2) + 4$

$= -16 - 28 + 10 + 4$

$= -30$

b) $f(4) = 2\times4^3 - 7\times4^2 - 5\times4 + 4$

$= 128 - 112 - 20 + 4$

$= 132 - 132 = 0$

$\therefore (x-4)$ IS A FACTOR OF $f(x)$

c) BY LONG DIVISION OR INSPECTION

$$
\begin{array}{r|l}
 & 2x^2 + x - 1 \\
x-4 & 2x^3 - 7x^2 - 5x + 4 \\
 & -2x^3 + 8x^2 \\
\hline
 & x^2 - 5x + 4 \\
 & -x^2 + 4x \\
\hline
 & -x + 4 \\
 & x - 4 \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x-4)(2x^2+x-1)$

$f(x) = (x-4)(2x-1)(x+1)$

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3. a) $r = \frac{u_2}{u_1} = \frac{15}{90} = \frac{1}{6}$

b) $S_\infty = \frac{a}{1-r} = \frac{90}{1-\frac{1}{6}} = \frac{90}{\frac{5}{6}} = 108$

4. $f(x) = x^3 - 5x^2 + 3x + 1$

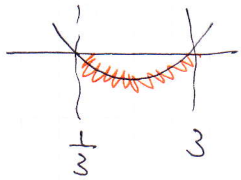
$f'(x) = 3x^2 - 10x + 3$

DECREASING $\Rightarrow f'(x) < 0$

$\Rightarrow 3x^2 - 10x + 3 < 0$

$\Rightarrow (3x-1)(x-3) < 0$

C.V. $= \begin{cases} 1/3 \\ 3 \end{cases}$



$\frac{1}{3} < x < 3$

5. $3\sin^2 3x - 7\cos 3x = 5$

$3(1 - \cos^2 3x) - 7\cos 3x = 5$

$3 - 3\cos^2 3x - 7\cos 3x = 5$

$0 = 3\cos^2 3x + 7\cos 3x + 2$

$0 = (3\cos 3x + 1)(\cos 3x + 2)$

$\cos 3x = \begin{cases} \cancel{-2} \\ -\frac{1}{3} \end{cases}$

$\arccos\left(-\frac{1}{3}\right) = 109.47°$

$\begin{cases} 3x = 109.47 \pm 360n \\ 3x = 250.53 \pm 360n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\begin{cases} x = 36.5 \pm 120n \\ x = 83.5 \pm 120n \end{cases}$

$\therefore x = 36.5°, 156.5°, 83.5°$

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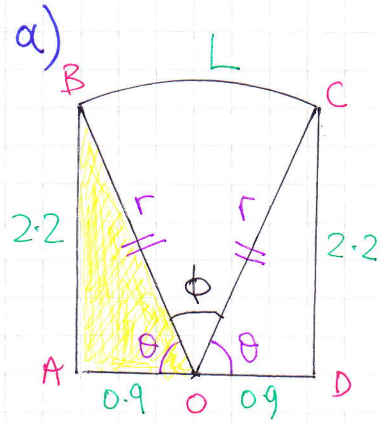
a)



• LOOKING AT THE RIGHT ANGLED TRIANGLE $A\hat{B}O$

$$\tan\theta = \frac{2.2}{0.9}$$

$$\theta = \arctan\left(\frac{2.2}{0.9}\right)$$

$$\theta \simeq 1.1824776\ldots$$

$$\therefore \phi = \pi - 2(1.1824776\ldots)$$

$$\phi \simeq 0.7766^c$$

CORRECT TO 4. d.p.

b) PYTHAGORAS YIELDS $r^2 = 2.2^2 + 0.9^2$

$$r \approx 2.37697\ldots$$

$$\therefore L = r\phi = 2.37697 \times 0.7766 \simeq 1.846 \text{ m}$$

$$\therefore \text{PERIMETER} = 2.2 + 1.8 + 2.2 + 1.846 \simeq 8.05 \text{ m}$$

c) AREA of SECTOR $= \frac{1}{2}r^2\phi^c = \frac{1}{2} \times (2.376\ldots)^2 \times 0.7766^c \simeq 2.194\ldots$

AREA OF TRIANGLE $= \frac{1}{2} \times 0.9 \times 2.2 = 0.99$

$$\therefore \text{AREA} \simeq 2.194 + 0.99 \times 2 \simeq 4.17 \text{ m}^2$$

(ACCEPT 4.18)

(P.T.O)

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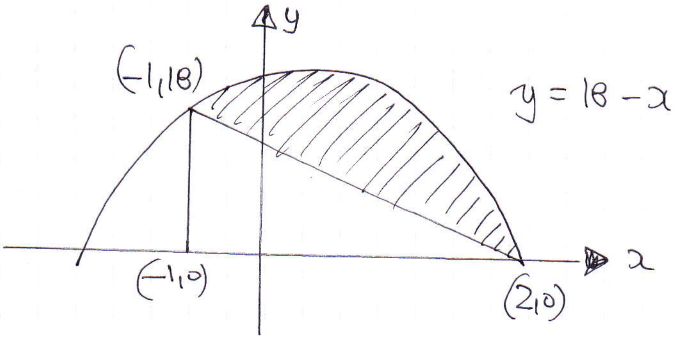
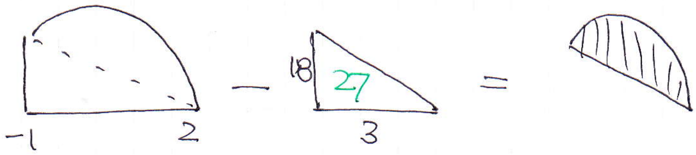
7.





$$\int_{-1}^{2} 18 - x - x^4 \, dx = \left[18x - \tfrac{1}{2}x^2 - \tfrac{1}{5}x^5\right]_{-1}^{2}$$

$$= \left(36 - 2 - \tfrac{32}{5}\right) - \left(-18 - \tfrac{1}{2} + \tfrac{1}{5}\right) = \tfrac{138}{5} - \left(-\tfrac{183}{10}\right) = \tfrac{459}{10}$$

$\therefore$ REQUIRED AREA $= \frac{459}{10} - 27 = \frac{189}{10} = 18.9$

8.

$$(1+kx)^n = 1 + \frac{n}{1}(kx)^1 + \frac{n(n-1)}{1\times 2}(kx)^2 + \cdots$$

$$= 1 + \underbrace{\boxed{nk}}_{40}x + \underbrace{\boxed{\tfrac{1}{2}n(n-1)}}_{120}k^2x^2 + \cdots$$

THUS $\frac{1}{2}n(n-1) = 120$

$n(n-1) = 240$

BY INSPECTION $15 \times 16 = 240$

IE $16(15-1) = 240$

$\therefore n = 16$

OR
$n^2 - n = 240$
$n^2 - n - 240 = 0$
$(n+15)(n-16) = 0$
$n = 16$ or $\cancel{-15}$

$nk = 40$

$16k = 40$

$k = 5/2$

9. $\log_4 x - \log_{16}(x-4) = 1$

$\Rightarrow \log_4 x - \dfrac{\log_4(x-4)}{\log_4 16} = 1$

$\Rightarrow \log_4 x - \dfrac{\log_4(x-4)}{\log_4 4^2} = 1$

$\Rightarrow \log_4 x - \dfrac{\log_4(x-4)}{2\log_4 4} = 1$

$\Rightarrow \log_4 x - \dfrac{\log_4(x-4)}{2} = 1$

$\Rightarrow 2\log_4 x - \log_4(x-4) = 2$

$\Rightarrow \log_4 x^2 - \log_4(x-4) = 2\log_4 4$

$\Rightarrow \log_4\left(\dfrac{x^2}{x-4}\right) = \log_4 16$

$\Rightarrow \dfrac{x^2}{x-4} = 16$

$\Rightarrow x^2 = 16x - 64$

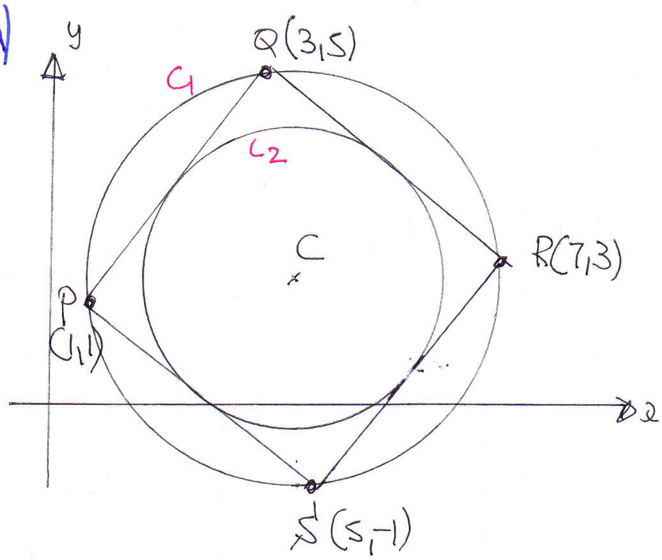
$\Rightarrow x^2 - 16x + 64 = 0$

$\Rightarrow (x-8)^2 = 0$

$\Rightarrow x = 8$

(P.T.O)

10. a)



- FIRSTLY FIND THE MIDPOINT

$C\left(\frac{1+7}{2}, \frac{1+3}{2}\right) = (4,2)$

RADIUS IS THE DISTANCE PC
P(1,1) & C(4,2)

$|PC| = \sqrt{3^2+1} = \sqrt{10}$

$\therefore$ CENTRE AT $(4,2)$
RADIUS $= \sqrt{10}$

b) WORK OUT THE DISTANCE $|SR| = \sqrt{(7-5)^2+(3+1)^2} = \sqrt{4+16}$
$= \sqrt{20}$

$\therefore$ RADIUS OF $C_2 = \frac{1}{2}|SR| = \frac{1}{2}\sqrt{20} = \frac{1}{2}\times 2\sqrt{5} = \sqrt{5}$

$\therefore (x-4)^2+(y-2)^2 = 5$

11.

$$y = x^3+ax^2+bx-10$$
$$\frac{dy}{dx} = 3x^2+2ax+b$$

- $(-1,-5) \Rightarrow -5 = (-1)^3+a(-1)^2+b(-1)-10$
$\Rightarrow -5 = -1+a-b-10$
$\Rightarrow \boxed{6 = a-b}$

- $\left.\frac{dy}{dx}\right|_{x=-1} = 0 \Rightarrow 0 = 3(-1)^2+2a(-1)+b$
$\Rightarrow 0 = 3-2a+b$
$\Rightarrow \boxed{2a-b=3}$

THUS $a-b=6$, $2a-b=3$ SUBTRACT $-a=3$

$\boxed{a=-3}$

& $-3-b=6$

$\boxed{b=-9}$

THUS $\frac{dy}{dx}=3x^2-6x-9$

$0=3x^2-6x-9$

$0=x^2-2x-3$

$0=(x+1)(x-3)$

$x = -1 \leftarrow P$, $x = 3 \leftarrow Q$

$\therefore Q(3,-37) \longleftarrow y=x^3-3x^2-9x-10$

$\frac{d^2y}{dx^2}=6x-6$

$\left.\frac{d^2y}{dx^2}\right|_{x=3}=6\times3-6=12>0$

$\therefore$ LOCAL MIN AT $(3,-37)$