C2, IYGB, PAPER J

1. $\tan(5y-35) = -2-\sqrt{3}$

$\arctan(-2-\sqrt{3}) = -75$

$5y - 35° = -75° \pm 180n \qquad n = 0,1,2,3,4 \ldots$

$5y = -40° \pm 180n$

$y = -8° \pm 36n$

$\therefore y = 28°, 64°$

2. $f(2) = p \times 2^3 - 32 \times 2^2 - 10 \times 2 + q$

$R = 8p - 128 - 20 + q$

$R = 8p + q - 148$

$f(-\frac{3}{2}) = p(-\frac{3}{2})^3 - 32(-\frac{3}{2})^2 - 10(-\frac{3}{2}) + q$

$R = -\frac{27}{8}p - 72 + 15 + q$

$R = -\frac{27}{8}p + q - 57$

Now $8p + \cancel{q} - 148 = -\frac{27}{8}p + \cancel{q} - 57$

$\frac{91}{8}p = 91$

$p = 8$

3. a) $x^2 + y^2 - 6x + ay = 15$

$(x-3)^2 + (y + \frac{a}{2})^2 - \frac{a^2}{4} - 9 = 15$

$\frac{a}{2} = 5$

$a = 10$

b) CONTINUE....

$(x-3)^2 + (y+5)^2 - \frac{10^2}{4} - 9 = 15$

$(x-3)^2 + (y+5)^2 - 25 - 9 = 15$

$(x-3)^2 + (y+5)^2 = 49$

$\therefore$ RADIUS $= \sqrt{49} = 7$

C2, IYGB, PAPER J

4. a) $\left(1+\frac{1}{4}x\right)^{10} = 1 + \frac{10}{1}\left(\frac{1}{4}x\right) + \frac{10\times9}{1\times2}\left(\frac{1}{4}x\right)^2 + \frac{10\times9\times8}{1\times2\times3}\left(\frac{1}{4}x\right)^3 + \dots$

$= 1 + \frac{5}{2}x + \frac{45}{16}x^2 + \frac{15}{8}x^3 + \dots$

b) $1+\frac{1}{4}x = \frac{41}{40}$

$4 + x = 4.1$

$x = 0.1$

$\left(1+\frac{1}{4}x\right)^{10} \approx 1 + \frac{5}{2}x + \frac{45}{16}x^2 + \frac{15}{8}x^3$ FOR SMALL x

$\left(1+\frac{1}{4}\times0.1\right)^{10} \approx 1 + \frac{5}{2}(0.1) + \frac{45}{16}(0.1)^2 + \frac{15}{8}(0.1)^3$

$\left(\frac{41}{40}\right)^{10} \simeq 1 + \frac{1}{4} + \frac{9}{320} + \frac{3}{1600}$

$\left(\frac{41}{40}\right)^{10} \simeq \frac{32}{25}$

$\left(\frac{41}{40}\right)^{10} \simeq 1.28$ AS REQUIRED

5. a)

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
✓	✓	✓	✓	✓	✗

$10 + 20 + 40 + 80 + 160 + 0 = 310$

$\frac{1}{10}\times 310 = £31$

b) RAY MUST HAVE ACCUMULATED 10×2097151 WHEN HE GAVE AN INCORRECT QUESTION

$a = 10$

$r = 2$

$S_n = \frac{a(r^n-1)}{r-1}$

$20971510 = \frac{10(2^n-1)}{2-1}$

$20971510 = 10(2^n-1)$

$2097151 = 2^n - 1$

C2, IYGB, PAPER J

$2^n = 2097152$

NOW

EITHER TRY POSITIVE INTEGERS AS THE ANSWER IS EXACT

OR

$\log 2^n = \log(2097152)$

$n \log 2 = \log(2097152)$

$n = \frac{\log(2097152)}{\log 2}$

$n = 21$

6. a) $7^x = 10$

$\log 7^x = \log 10$

$x \log 7 = 1$

$x = \frac{1}{\log 7}$

$x \simeq 1.18$ 3sf

b) $\log_2 y = \frac{9}{\log_2 y}$

$\Rightarrow (\log_2 y)^2 = 9$

$\Rightarrow \log_2 y = 3$ or -3

$\Rightarrow \log_2 y = 3\log_2 2$ or $-3\log_2 2$

$\Rightarrow \log_2 y = \log_2 8$ or $\log_2\left(\frac{1}{8}\right)$

$\Rightarrow y = 8$ or $\frac{1}{8}$

C2, IYGB, PAPER J

7. a) $C = \frac{200}{V} + \frac{2V}{25} = 200V^{-1} + \frac{2}{25}V$

$\frac{dC}{dV} = -200V^{-2} + \frac{2}{25}$

SOLVE FOR ZERO

$-\frac{200}{V^2} + \frac{2}{25} = 0$

$\frac{2}{25} = \frac{200}{V^2}$

$2V^2 = 5000$

$V^2 = 2500$

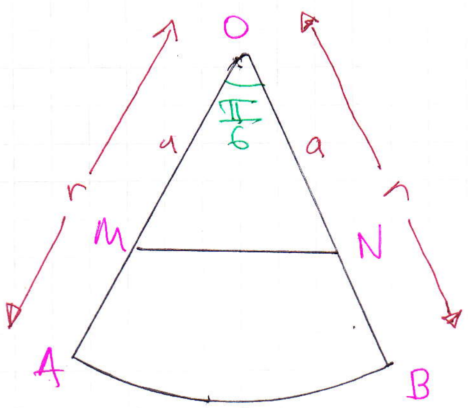
$V = +50$

b) $\frac{d^2C}{dV^2} = 400V^{-3} = \frac{400}{V^3}$

$\left.\frac{d^2C}{dV^2}\right|_{V=50} = \frac{40}{50^3} = \frac{40}{125000} > 0$ INDEED IT MINIMIZES

c) WHEN $V = 50$ $C = \frac{200}{50} + \frac{2 \times 50}{25} = £8$

8.



AREA OF SECTOR $= \frac{1}{2}r^2\theta^c$

$= \frac{1}{2}r^2\frac{\pi}{6} = \frac{\pi r^2}{12}$

AREA OF TRIANGLE $= \frac{1}{2}a^2\sin\frac{\pi}{6}$

$= \frac{1}{4}a^2$

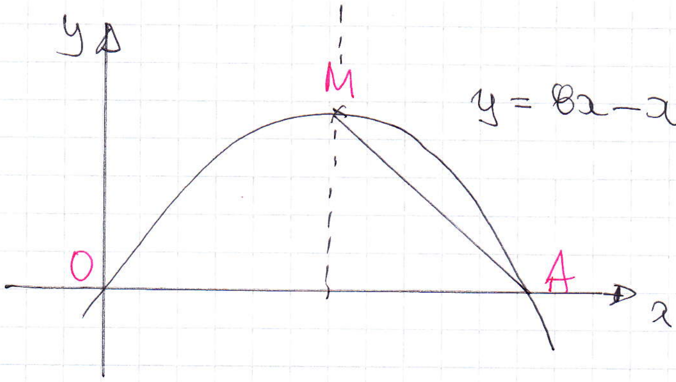
AREA OF TRIANGLE $= \frac{1}{2}$ AREA OF SECTOR

$\frac{1}{4}a^2 = \frac{1}{2} \times \frac{\pi r^2}{12}$

$a^2 = \frac{\pi r^2}{6}$

$a = \sqrt{\frac{\pi r^2}{6}} = \sqrt{\frac{\pi}{6}}\, r$ AS REQUIRED

9.

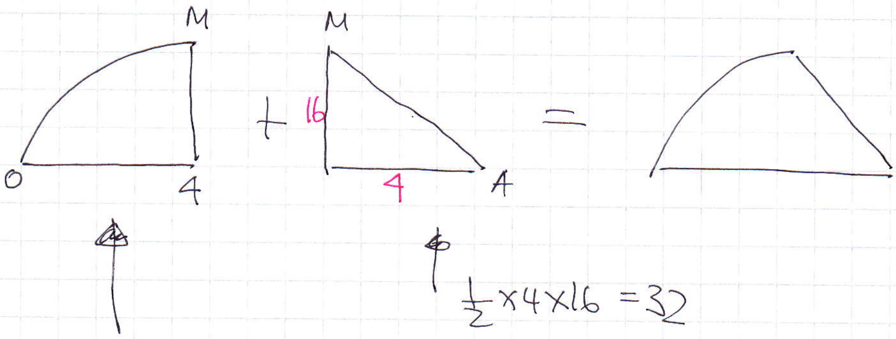


$y = 8x - x^2$
$y = x(8-x)$
$\therefore A(8,0)$

- BY SYMMETRY (OR COMPLETING THE SQUARE OR EVEN CALCULUS!)

$M(4, ?)$ $\quad y = 4\times8 - 4^2 = 32 - 16 = 16$

$\therefore M(4,16)$



$\frac{1}{2}\times4\times16 = 32$

$$\int_0^4 8x - x^2 \, dx = \left[4x^2 - \frac{1}{3}x^3\right]_0^4 = \left(64 - \frac{64}{3}\right) - (0)$$

$$= \frac{128}{3}$$

$\therefore$ REQUIRED AREA $= \frac{128}{3} + 32 = \frac{224}{3}$

10. a) THE PERIOD IS $120°$

IE A FULL CYCLE

b) $C = 3$ (3 full cycles from 0 to 360)

$B = 2$ (BECAUSE THERE IS A "GAP" OF 4, WHICH IS TWICE AS LARGE THAN THE NORMAL GAP BETWEEN -1 & 1)

$A = 1$

C2, IYGB, PAPER J

11\. a)

x	0	0.25	0.5	0.75	1
y	1	1.4142	1.6325	1.8226	2

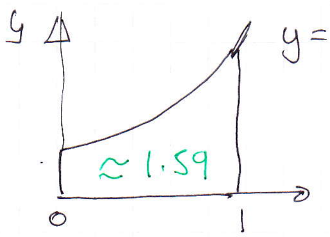
$$\int_0^1 2^{\sqrt{x}} \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times \text{REST}\right]$$

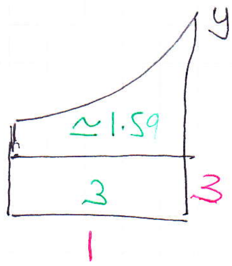
$$\simeq \frac{0.25}{2}\left[1+2+2(1.4142+1.6325+1.8226)\right]$$

$$\simeq 1.59233\ldots$$

$$\simeq 1.59$$

b) I) GRAPHICALLY





$\therefore$ APPROX 4.59

OR ALGEBRAICALLY

$$\int_0^1 2^{\sqrt{x}} + 3\,dx = \int_0^1 2^{\sqrt{x}}\,dx + \int_0^1 3\,dx$$

$$\simeq 1.59 + \left[3x\right]_0^1$$

$$\simeq 1.59 + (3-0) \simeq 4.59$$

(II) $$\int_0^1 2^{\sqrt{x}+3}\,dx = \int_0^1 2^{\sqrt{x}}\times 2^3\,dx = \int_0^1 8\times 2^{\sqrt{x}}\,dx$$

$$= 8\int_0^1 2^{\sqrt{x}}\,dx \simeq 8\times 1.59$$

$$\simeq 12.7$$