C2, IYGB, PAPER N

1. a) $(2+x)^9 = \binom{9}{0}2^9x^0 + \binom{9}{1}2^8x^1 + \binom{9}{2}2^7x^2 + \binom{9}{3}2^6x^3 + \dots$

$= 512 + 2304x + 4608x^2 + 5376x^3 + \dots$

b) REPLACE x WITH $-\frac{1}{4}x$

$(2-\frac{1}{4}x)^9 = 512 + 2304(-\frac{1}{4}x) + 4608(-\frac{1}{4}x)^2 + 5376(-\frac{1}{4}x)^3 + \dots$

$= 512 - 576x + 288x^2 - 84x^3 + \dots$

2. $f(x) \equiv 6x^2 + x + 7$

$$\left.\begin{array}{l} f(a) = 6a^2 + a + 7 \\ f(-2a) = 6(-2a)^2 + (-2a) + 7 = 24a^2 - 2a + 7 \end{array}\right\} \Rightarrow f(a) = f(-2a)$$

THUS $24a^2 - 2a + \cancel{7} = 6a^2 + a + \cancel{7}$

$18a^2 - 3a = 0$

$3a(6a-1) = 0$

$a = \frac{1}{6}$ $(a \neq 0)$

3. $y = x^3 - 3x^2 - 24x - 1$

$\frac{dy}{dx} = 3x^2 - 6x - 24$

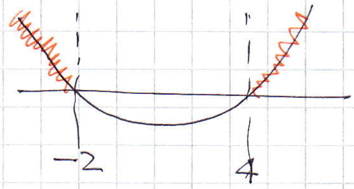
INCREASING $\Rightarrow \frac{dy}{dx} > 0$

$\Rightarrow 3x^2 - 6x - 24 > 0$

$\Rightarrow x^2 - 2x - 8 > 0$

$\Rightarrow (x+2)(x-4) > 0$

C.V = 4, −2



$x < -2$ OR $x > 4$

4. a) $y = x^3 - 8x^2 + 16x$

$y = x(x^2 - 8x + 4)$

$y = x(x-4)^2$

$y = 0 \Longrightarrow x = \begin{cases} 0 & \leftarrow \text{ORIGIN} \\ 4 & \leftarrow \text{POINT } A \end{cases}$

b) $\int_0^4 x^3 - 8x^2 + 16x \, dx = \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 8x^2\right]_0^4 = \left(64 - \frac{512}{3} + 128\right) - (0)$

$= \frac{64}{3}$

5. a) $y = x - 2x^4$

$\frac{dy}{dx} = 1 - 8x^3$

$\frac{d^2y}{dx^2} = -24x^2$

- $\frac{dy}{dx} = 0$

$\Longrightarrow 1 - 8x^3 = 0$

$\Longrightarrow 1 = 8x^3$

$\Longrightarrow x^3 = \frac{1}{8}$

$\Longrightarrow \boxed{x = \frac{1}{2}}$

$y = \frac{1}{2} - 2\left(\frac{1}{2}\right)^4 = \frac{3}{8}$

$\therefore \left(\frac{1}{2}, \frac{3}{8}\right)$

- $\left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{2}} = -24\left(\frac{1}{2}\right)^2 = -6 < 0$

$\therefore \left(\frac{1}{2}, \frac{3}{8}\right)$ IS A (LOCAL) MAX

b) $\frac{d^3y}{dx^3} = -48x$

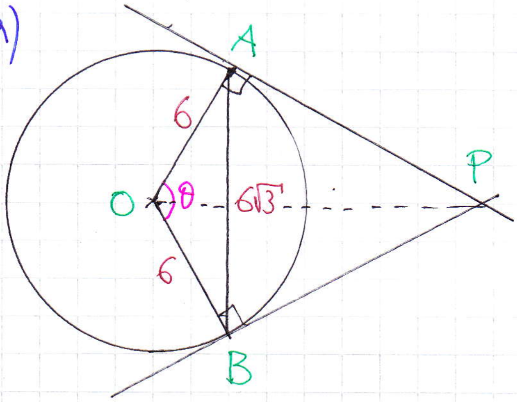
$\frac{d^2y}{dx^2} = 0 \Longrightarrow x = 0$

$\Longrightarrow \left.\frac{d^3y}{dx^3}\right|_{x=0} = 0$

$\therefore$ NO POINTS OF INFLEXION

C2, IYGB, PAPER N

6. a)



- BY COSINE RULE

$|AB|^2 = |OA|^2 + |OB|^2 - 2|OA||OB|\cos\theta$

$(6\sqrt{3})^2 = 6^2 + 6^2 - 2\times6\times6\cos\theta$

$108 = 36 + 36 - 72\cos\theta$

$72\cos\theta = -36$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

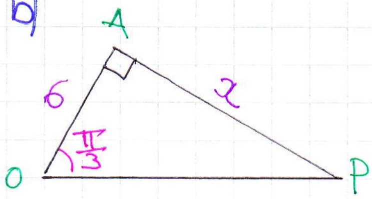
ALTERNATIVE — SPLIT INTO 2 TRIANGLES

$\sin\phi = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\phi = \frac{\pi}{3}$

$\theta = 2\phi = \frac{2\pi}{3}$

b)



- $\tan\frac{\pi}{3} = \frac{x}{6}$

$\sqrt{3} = \frac{x}{6}$

$x = 6\sqrt{3}$

- AREA OF TRIANGLE

$\frac{1}{2}|OA||AP| = \frac{1}{2}\times6\times6\sqrt{3}$

$= 18\sqrt{3}$

- AREA OF KITE OAPB IS $2\times18\sqrt{3} = 36\sqrt{3}$

AS REQUIRED

c) AREA OF SECTOR $= \frac{1}{2}r^2\theta^c$

$= \frac{1}{2}\times6^2\times\frac{2\pi}{3} = 12\pi$

∴ AREA OF SHADED REGION $= 36\sqrt{3} - 12\pi \approx 24.6547... \simeq 24.7$

7. $6\cos\psi = 5\tan\psi$

$\Rightarrow 6\cos\psi = \frac{5\sin\psi}{\cos\psi}$

$\Rightarrow 6\cos^2\psi = 5\sin\psi$

$\Rightarrow 6(1-\sin^2\psi) = 5\sin\psi$

$\Rightarrow 6 - 6\sin^2\psi = 5\sin\psi$

$\Rightarrow 0 = 6\sin^2\psi + 5\sin\psi - 6$

$\Rightarrow 0 = (3\sin\psi - 2)(2\sin\psi + 3)$

$\Rightarrow \sin\psi = \left\langle \begin{array}{l} \cancel{-3/2} \\ 2/3 \end{array}\right.$

$\arcsin\left(\frac{2}{3}\right) \simeq 0.7297...^c$

$\left(\begin{array}{l} \psi = 0.729^c \pm 2n\pi \\ \psi = 2.41^c \pm 2n\pi \end{array}\right.$ $n = 0,1,2,3,...$

$\psi_1 = 0.73^c$

$\psi_2 = 2.41^c$

8.

$(2^2)^y = 2^{2y}$

$(2^y)^2 = 2^{2y}$

$4^y - 3(2^y) - 10 = 0$

$\Rightarrow (2^2)^y - 3(2^y) - 10 = 0$

$\Rightarrow (2^y)^2 - 3(2^y) - 10 = 0$

$\Rightarrow a^2 - 3a - 10 = 0$

$a = 2^y$

$\Rightarrow (a+2)(a-5) = 0$

$\Rightarrow a = \begin{cases} -2 \\ 5 \end{cases}$

$\Rightarrow 2^y = \begin{cases} \cancel{-2} \\ 5 \end{cases}$

$\Rightarrow \log 2^y = \log 5$

$\Rightarrow y \log 2 = \log 5$

$\Rightarrow y = \dfrac{\log 5}{\log 2}$

$\Rightarrow y \simeq 2.32$

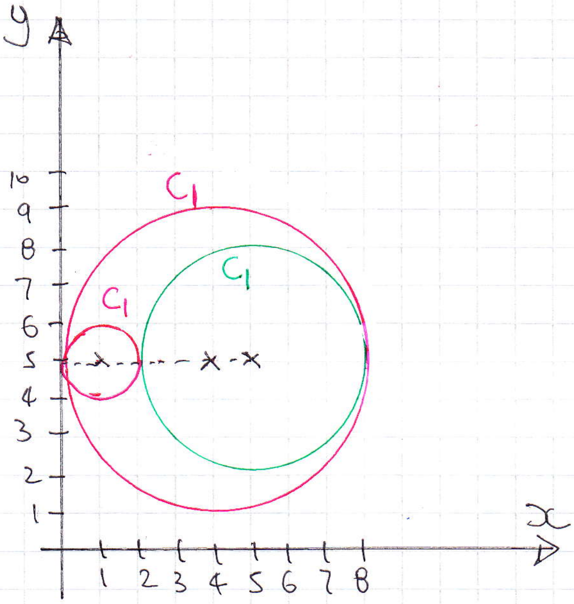
9. a) $x^2 + y^2 - 10x - 10y + 41 = 0$

$(x-5)^2 + (y-5)^2 - 25 - 25 + 41 = 0$

$(x-5)^2 + (y-5)^2 = 9$

CENTRE AT $(5,5)$ RADIUS $= 3$

b)



EITHER CENTRE $(1,5)$, RADIUS 1

OR CENTRE $(4,5)$, RADIUS 4

$\therefore (x-1)^2 + (y-5)^2 = 1$

OR

$(x-4)^2 + (y-5)^2 = 16$

10. a)

$$\left.\begin{array}{l} u_8 = ar^7 \\ u_4 = ar^3 \end{array}\right\} \Rightarrow u_8 = 10u_4$$

$$\Rightarrow \cancel{a}r^7 = 10\cancel{a}r^3$$

$$r^4 = 10$$

$$r = 10^{\frac{1}{4}}$$

$$r = 1.778$$

b)

$$\cancel{S}_8 = 10 \times \cancel{S}_4$$

$$\Rightarrow \frac{\cancel{a}(r^8-1)}{\cancel{r-1}} = \frac{10\cancel{a}(r^4-1)}{\cancel{r-1}}$$

$$\Rightarrow r^8 - 1 = 10r^4 - 10$$

$$\Rightarrow r^8 - 10r^4 + 9 = 0$$

c)

$$(r^4 - 9)(r^4 - 1) = 0$$

$$r^4 = 1, \; 9$$

$$r^2 = 1, \; \cancel{-1}, \; 3, \; \cancel{-3}$$

$$r^2 = 1, \; 3$$

$$r = 1, \; -1, \; \sqrt{3}, \; -\sqrt{3}$$

IF G.P $r \neq 0, 1, -1$

AS TERMS ARE POSITIVE

$$r = +\sqrt{3}$$