C2, IYGB, PAPER O

1. a) $p(x) = 2x^3 - 11x^2 + 20x - 12$

$p(2) = 2\times2^3 - 11\times2^2 + 20\times2 - 12 = 16 - 44 + 40 - 12 = 56 - 56 = 0$

$\therefore (x-2)$ IS A FACTOR OF $p(x)$

b) BY LONG DIVISION OR INSPECTION

$p(x) = (x-2)(2x^2 - 7x + 6)$

$p(x) = (x-2)(2x-3)(x-2)$

c) $p(-2) = (-2-2)(-4-3)(-2-2) = (-4)(-7)(-4) = -112$

d)

$$\begin{array}{r|l} & 2x^2 - 15x + 50 \\ x+2 & 2x^3 - 11x^2 + 20x - 12 \\ & -2x^3 - 4x^2 \\ \hline & -15x^2 + 20x - 12 \\ & 15x^2 + 30x \\ \hline & 50x - 12 \\ & -50x - 100 \\ \hline & -112 \end{array}$$

$\therefore a = -15$

$b = 50$

$c = -112$

2. a) $x^2 + y^2 = 8x + 4y$

$x^2 - 8x + y^2 - 4y = 0$

$(x-4)^2 - 16 + (y-2)^2 - 4 = 0$

$(x-4)^2 - (y-2)^2 = 20$

$\therefore$ CENTRE AT $(4,2)$

RADIUS $= \sqrt{20} = 2\sqrt{5}$

b) $x=0 \Rightarrow y^2 = 4y$

$\Rightarrow y^2 - 4y = 0$

$\Rightarrow y(y-4) = 0$

$\therefore A(0,4)$

$y=0 \Rightarrow x^2 = 8x$

$\Rightarrow x^2 - 8x = 0$

$\Rightarrow x(x-8) = 0$

$\therefore B(8,0)$

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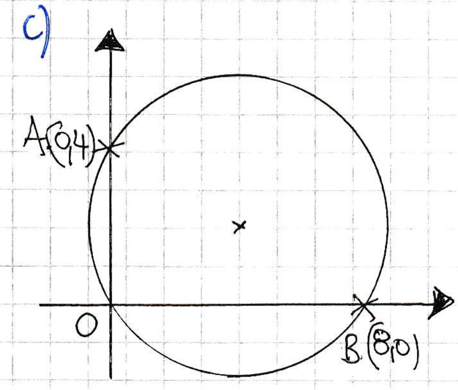
c)



d) AB IS A DIAMETER BECAUSE $A\hat{O}B = 90°$

$\therefore |AB| = 2 \times 2\sqrt{5}$

$|AB| = 4\sqrt{5}$

3. $\log_2(2z+1) = 2 + \log_2 z$

$\Rightarrow \log_2(2z+1) - \log_2 z = 2\log_2 2$

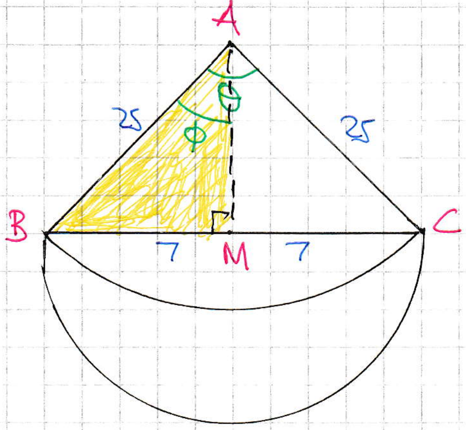
$\Rightarrow \log\left(\frac{2z+1}{z}\right) = \log_2 4$

$\Rightarrow \frac{2z+1}{z} = 4$

$\Rightarrow 1 = 2z$

$\Rightarrow z = \frac{1}{2}$

4. a)



- BY PYTHAGORAS ON $A\hat{M}B$

$|BM|^2 + |AM|^2 = |BA|^2$

$7^2 + |AM|^2 = 25^2$

$|AM|^2 = 576$

$|AM| = 24$

$\therefore \text{AREA} = \left(\frac{1}{2} \times 7 \times 24\right) \times 2$

$= 168 \text{ cm}^2$

b) $\sin\phi = \frac{7}{25}$

$\phi = 0.28379...$

$\theta = 2\phi \simeq 0.567588..$

$\theta = 0.568^c$

C2, IYGB, PAPER O

AREA OF SEMICIRCLE $= \frac{1}{2} \times \pi \times 7^2 = \frac{49}{2}\pi$

AREA OF SECTOR $= \frac{1}{2} \times 25^2 \times 0.568 \approx 177.5$

REQUIRED AREA = △ + ◡ − ◠

$= 168 + \frac{49}{2}\pi - 177.5 \simeq 67.469$

$\simeq 67.5 \text{ cm}^2$

5. a) $2k-5 \xrightarrow{\times r} k \xrightarrow{\times r} k-6$

$\Rightarrow \frac{k}{2k-5} = \frac{k-6}{k}$

$\Rightarrow k^2 = (k-6)(2k-5)$

$\Rightarrow k^2 = 2k^2 - 17k + 30$

$\Rightarrow 0 = k^2 - 17k + 30$

$\Rightarrow k^2 - 17k + 30 = 0$ AS REQUIRED

b) $(k-2)(k-15) = 0$

$k = \begin{cases} 2 \\ 15 \end{cases}$

SO EITHER $-1, 2, -4, \ldots$ $r = -2$ DIVERGES

OR $25, 15, 9, \ldots$ $r = \frac{3}{5}$ CONVERGES

SO $S_\infty = \frac{a}{1-r} = \frac{25}{1-\frac{3}{5}} = \frac{125}{2}$

c) $a = -1$, $r = -2$, $n = 10$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_{10} = \frac{-1((-2)^{10}-1)}{-2-1} = \frac{-(1023)}{-3} = 341$

6. a) $(k+x)^n = \binom{n}{0}k^{n}x^{0} + \binom{n}{1}k^{n-1}x^{1} + \binom{n}{2}k^{n-2}x^{2} + \binom{n}{3}k^{n-3}x^{3} + \dots$

$$= k^n + \frac{n}{1}k^{n-1}x + \frac{n(n-1)}{1\times 2}k^{n-2}x^2 + \frac{n(n-1)(n-2)}{1\times2\times3}k^{n-3}x^3 + \dots$$

$$= k^n + nk^{n-1}x + \boxed{\tfrac{1}{2}n(n-1)k^{n-2}}x^2 + \boxed{\tfrac{1}{6}n(n-1)(n-2)k^{n-3}}x^3 + \dots$$

THUS $\frac{1}{\cancel{2}}\cancel{n}\cancel{(n-1)}k^{n-2} = \frac{1}{6}\cancel{n}\cancel{(n-1)}(n-2)k^{n-3}$

$\Rightarrow \frac{1}{2}k^{n-2} = \frac{1}{6}(n-2)k^{n-3}$

$\Rightarrow 3k^{n-2} = (n-2)k^{n-3}$

DIVIDE BOTH SIDES BY k^{n-3}

$\Rightarrow \frac{3k^{n-2}}{k^{n-3}} = (n-2)\frac{\cancel{k^{n-3}}}{\cancel{k^{n-3}}}$

$\Rightarrow 3k = n-2$

$\Rightarrow n = 3k+2$

AS REQUIRED

b) IF $k=2$ $n=8$

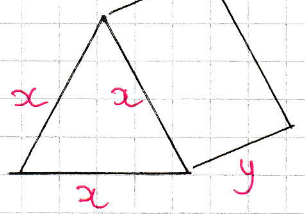
HENCE $f(x) = \dots + \binom{8}{4}2^4x^4 + \dots$

$\binom{n}{4}$

k

$\therefore 70\times16 = 1120$

7. a)



- AREA OF TRIANGLE $= \frac{1}{2}x^2\sin60°$

$= \frac{1}{2}x^2\frac{\sqrt{3}}{2}$

$= \frac{1}{4}x^2\sqrt{3}$

- SURFACE AREA $= 54\sqrt{3}$

$3xy + \left(\frac{1}{4}x^2\sqrt{3}\right)\times2 = 54\sqrt{3}$

$$\boxed{3xy + \frac{1}{2}x^2\sqrt{3} = 54\sqrt{3}}$$

C2, IYGB, PAPER O

- $V =$ CROSS SECTIONAL AREA × "LENGTH"

$\Rightarrow V = \frac{1}{4}x^2\sqrt{3} \times y$

$\Rightarrow V = \frac{1}{4}x^2y\sqrt{3}$

$V = \frac{1}{4}(xy)x\sqrt{3}$

$V = \frac{1}{4}\left[18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}\right]x\sqrt{3}$

$V = \frac{1}{4}\left[54x - \frac{1}{2}x^3\right]$

$V = \frac{27}{2}x - \frac{1}{8}x^3$ AS REQUIRED

SINCE

$3xy = 54\sqrt{3} - \frac{1}{2}x^2\sqrt{3}$

$\boxed{xy = 18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}}$

b) $\frac{dV}{dx} = \frac{27}{2} - \frac{3}{8}x^2$

SOLVE FOR ZERO

$\frac{27}{2} - \frac{3}{8}x^2 = 0$

$\frac{27}{2} = \frac{3}{8}x^2$

$x^2 = 36$

$x = 6$

$\frac{d^2V}{dx^2} = -\frac{3}{4}x$

$\left.\frac{d^2V}{dx^2}\right|_{x=6} = -\frac{9}{2} < 0$

IE MAX

$V_{MAX} = \frac{27}{2} \times 6 - \frac{1}{8} \times 6^3$

$V_{MAX} = 54$

c) USING $xy = 18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}$

$6y = 18\sqrt{3} - \frac{1}{6} \times 6^2\sqrt{3}$

$6y = 12\sqrt{3}$

$y = 2\sqrt{3}$

$(\simeq 3.46)$

C2, IYGB, PAPER O

8. a) $y = P + Q\cos 2x$

$$\left.\begin{array}{l}(0,-3) \Rightarrow -3 = P + Q \\ (\tfrac{\pi}{2},5) \Rightarrow 5 = P - Q\end{array}\right\} \Rightarrow \begin{array}{l}2P = 2 \\ P = 1 \\ \\ Q = -4\end{array}$$

ALTERNATIVE

$y = \cos 2x$ LIES BETWEEN -1 & 1 IE A "GAP" OF 2

OUR GRAPH HAS A "GAP" OF 8 (FROM -3 TO 5), IE A VERTICAL STRETCH OF 4, BUT THE COSINE GRAPH IS "UPSIDE DOWN" ALSO $\therefore Q = 4$

BUT IT DOES NOT LIE BETWEEN -4 & 4; IT LIES BETWEEN -3 & 5 SO IT HAS BEEN TRANSLATED UP BY 1

$\therefore P = 1$

b) $y = 0 \Rightarrow 0 = 1 - 4\cos 2x$

$\Rightarrow \cos 2x = \frac{1}{4}$

$\arccos\left(\frac{1}{4}\right) = 1.3181^c\ldots$

$$\Rightarrow \begin{cases} 2x = 1.3181^c \pm 2n\pi \\ 2x = 4.9651^c \pm 2n\pi \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \begin{cases} x = 0.659^c \pm n\pi \\ x = 2.483^c \pm n\pi \end{cases}$$

$\therefore x = 0.66^c, 2.48^c, 3.80^c, 5.62^c, 6.94^c, 8.27^c,$

A B C D E F

C2, IYGB, PAPER 7

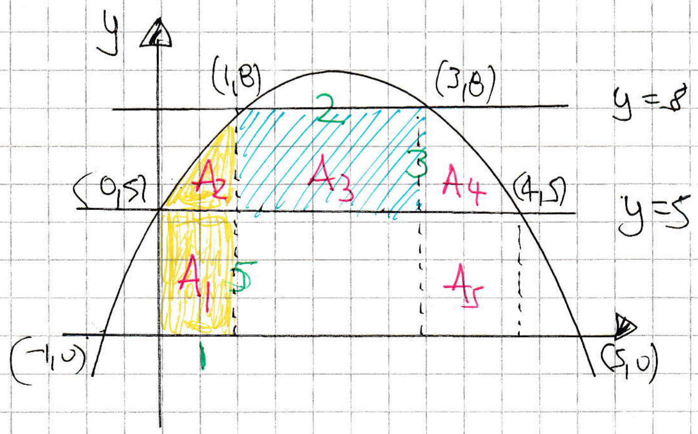
9.

- $5+4x-x^2=8$
  $0=x^2-4x+3$
  $0=(x-1)(x-3)$
  $x=\begin{cases}1\\3\end{cases}$

- $5+4x-x^2=5$
  $0=x^2-4x$
  $0=x(x-4)$
  $x=\begin{cases}0\\4\end{cases}$

- $5+4x-x^2=0$
  $0=x^2-4x-5$
  $0=(x+1)(x-5)$
  $x=\begin{cases}-1\\5\end{cases}$

WE MAY NOT USE ALL THESE CO-ORDINATES



- $A_3 = 2\times 3 = 6$
- $A_1 = A_5 = 1\times 5 = 5$

- $A_1+A_2 = A_4+A_5 = \int_0^1 5+4x-x^2\,dx = \left[5x+2x^2-\frac{1}{3}x^3\right]_0^1$
  $= \left(5+2-\frac{1}{3}\right)-0 = \frac{20}{3}$

$\therefore A_2 = \frac{20}{3}-5 = \frac{5}{3}$

REQUIRED AREA $= A_2+A_3+A_4 = \frac{5}{3}+6+\frac{5}{3} = \frac{28}{3}$