C2, IYGB, PAPER P

1. a) $(1+2x)^{12} = 1 + \frac{12}{1}(2x)^1 + \frac{12\times11}{1\times2}(2x)^2 + \frac{12\times11\times10}{1\times2\times3}(2x)^3 + \frac{12\times11\times10\times9}{1\times2\times3\times4}(2x)^4 + \dots$

$(1+2x)^{12} = 1 + 24x + 264x^2 + 1760x^3 + 7920x^4 + \dots$

b) LET $1+2x = 1.02$
$2x = 0.02$
$x = 0.01$

$[1+2(0.01)]^{12} = 1 + 24(0.01) + 264(0.01)^2 + 1760(0.01)^3 + 7920(0.01)^4 + \dots$

$1.02^{12} \simeq 1 + 0.24 + 0.0264 + 0.00176 + 0.0000792$

$1.02^{12} \simeq 1.2682392$

c) ERROR $= 1.02^{12} - 1.2682392 \simeq 0.0000026$

2. $f(x) \equiv x^2 - 4x + 12$

$f(-k) = 3 \times f(k)$

$\Rightarrow (-k)^2 - 4(-k) + 12 = 3[k^2 - 4k + 12]$

$\Rightarrow k^2 + 4k + 12 = 3k^2 - 12k + 36$

$\Rightarrow 0 = 2k^2 - 16k + 24$

$\Rightarrow k^2 - 8k + 12 = 0$

$\Rightarrow (k-2)(k-6) = 0$

$k = \begin{cases} 2 \\ 6 \end{cases}$

3. a) $2 \times 3^x = 900$

$3^x = 450$

$\log 3^x = \log 450$

$x \log 3 = \log 450$

$x = \frac{\log 450}{\log 3}$

$x \simeq 5.56$

b) $\log_2(7y-1) = 3 + \log_2(y-1)$

$\log_2(7y-1) - \log_2(y-1) = 3\log_2 2$

$\log_2\left(\frac{7y-1}{y-1}\right) = \log_2 8$

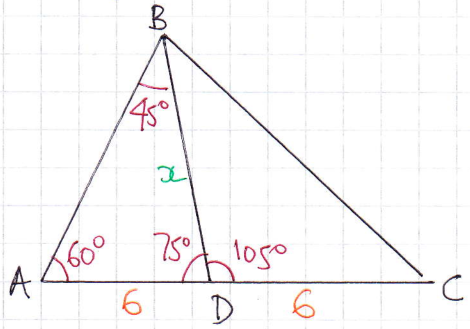
$\frac{7y-1}{y-1} = 8$

$7y - 1 = 8y - 8$

$7 = y$

$\therefore y = 7$

4.



a) BY THE SINE RULE ON $\hat{BAD}$

$$\frac{x}{\sin 60} = \frac{6}{\sin 45}$$

$$x = \frac{6\sin 60}{\sin 45}$$

$$x = \frac{3\sqrt{3}}{\frac{1}{\sqrt{2}}} = 3\sqrt{6}$$

$$x \simeq 7.35 \text{ cm}$$

b) $\text{AREA } \hat{BAD} = \frac{1}{2}|AD||BD|\sin 75 = \frac{1}{2} \times 6 \times 3\sqrt{6} \times \sin 75 = \frac{27+9\sqrt{3}}{2}$

$$\simeq 21.3 \text{ cm}^2$$

c) THIS IS THE HEIGHT OF THE TRIANGLE $\hat{BAD}$

$$\frac{1}{2} \times \text{BASE} \times \text{HEIGHT} = \frac{27+9\sqrt{3}}{2}$$

$$\frac{1}{2} \times 6 \times h = \frac{27+9\sqrt{3}}{2}$$

$$h = \frac{9+3\sqrt{3}}{2} \simeq 7.10 \text{ cm}$$

d) BY THE COSINE RULE ON $\hat{BDC}$: $|BC|^2 = |BD|^2 + |DC|^2 - 2|BD||BC|\cos 105°$

$$|BC|^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos 105$$

$$|BC|^2 = (3\sqrt{6})^2 + 36 - 2 \times 6 \times 3\sqrt{6} \times \cos 105$$

$$|BC|^2 = 54 + 36 - 36\sqrt{6}\cos 105$$

$$|BC|^2 = 112.82\ldots$$

$$|BC| \simeq 10.6 \text{ cm}$$

5. a) $(x-2)^2 + (y-5)^2 = (\sqrt{10})^2$

$$x^2 - 4x + 4 + y^2 - 10y + 25 = 10$$

$$x^2 + y^2 - 4x - 10y + 19 = 0$$

AS REQUIRED

C2, IYGB, PAPER P

b) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = x+5 \\ (x-2)^2+(y-5)^2=10 \end{array}\right\} \Rightarrow (x-2)^2+(x+\cancel{5}-\cancel{5})^2=10$$

$$x^2-4x+4+x^2=10$$

$$2x^2-4x-6=0$$

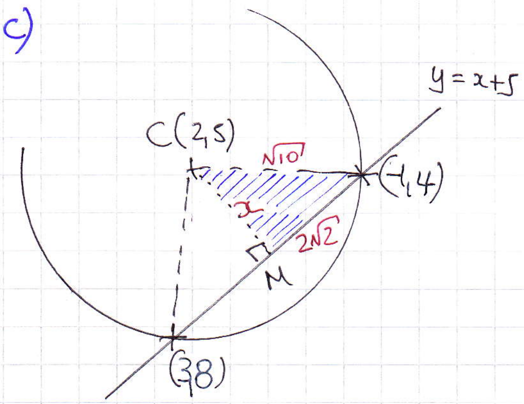
$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$$x = \begin{cases} 3 \\ -1 \end{cases} \qquad y = \begin{cases} 8 \\ 4 \end{cases}$$

$\therefore$ $(3,8)$ & $(-1,4)$

c)



DISTANCE $|PQ| = \sqrt{(4-8)^2+(-1-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

BY PYTHAGORAS

$$x^2+(2\sqrt{2})^2 = (\sqrt{10})^2$$

$$x^2+8=10$$

$$x^2=2$$

$$x=\sqrt{2}$$

AS REQUIRED

ALTERNATIVE

MIDPOINT $M\left(\frac{3-1}{2}, \frac{8+4}{2}\right) = M(1,6)$

$|MC| = \sqrt{(6-5)^2+(1-2)^2} = \sqrt{1+1} = \sqrt{2}$

6. a) $A(0,2)$ (when $x=0$)

$B(\pi,-4)$ ← GRAPH OF $y=\cos x$ HAS MIN AT $(180,-1) = (\pi,-1)$

$y=3\cos x$ HAS MIN AT $(\pi,-3)$

$y=3\cos x-1$ HAS MIN AT $(\pi,-4)$

b) $y = 3\cos x - 1$

$0 = 3\cos x - 1$

$1 = 3\cos x$

$\boxed{\cos x = \frac{1}{3}}$

$\Rightarrow \arccos\left(\frac{1}{3}\right) = 1.23^c$

$\Rightarrow \begin{cases} x = 1.23^c \pm 2\pi n \\ x = 5.05^c \pm 2\pi n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\therefore x_1 = 1.23^c$, $x_2 = 5.05^c$ $\therefore$ $C(1.23, 0)$, $D(5.05, 0)$

7. a) EITHER

250×0.9^2

$= 202.5$

OR

$a = 250$
$r = 0.9$
$n = 3$

$u_n = ar^{n-1}$

$u_3 = 250 \times 0.9^2$

$u_3 = 202.5$

b)

START	END OF 1	END OF 2	....	END OF 12
250 a	u_2	u_3		u_{13}

$u_n = ar^{n-1}$

$u_{13} = 250 \times 0.9^{12} = 60.71$ ltrs

c) AT THE END OF MONTH...

1 $\quad 250 \times 0.9$

2 $\quad 250 \times 0.9^2 + 250 \times 0.9$

3 $\quad 250 \times 0.9^3 + 250 \times 0.9^2 + 250 \times 0.9$

4 $\quad 250 \times 0.9^4 + 250 \times 0.9^3 + 250 \times 0.9^2 + 250 \times 0.9$

$\vdots$

12 $\quad 250 \times 0.9^{12} + 250 \times 0.9^{11} + 250 \times 0.9^{10} + \cdots + 250 \times 0.9^1$

WE REQUIRE $= 250 \times 0.9 + 250 \times 0.9^2 + 250 \times 0.9^3 + \cdots + 250 \times 0.9^{12}$

$$= 250\big[\underbrace{0.9 + 0.9^2 + 0.9^3 + \dots + 0.9^{12}}_{G.P}\big]$$

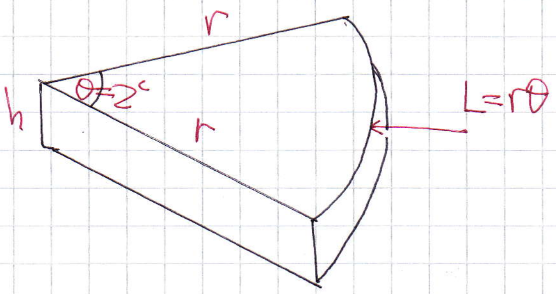
$$\boxed{\begin{aligned} \text{FIRST TERM} &= 0.9 \\ r &= 0.9 \\ n &= 12 \end{aligned}}$$

USING $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_{12} = \frac{0.9(1-0.9^{12})}{1-0.9} = 6.458\dots$$

$\therefore$ TOTAL $= 250 \times 6.458 \simeq 1615$ ltrs

8. a)



$$\boxed{\begin{aligned} V &= 1000 \\ \left(\tfrac{1}{2}r^2\theta^c\right) \times h &= 1000 \\ \tfrac{1}{2}r^2 \times 2 \times h &= 1000 \\ r^2h &= 1000 \\ rh &= \frac{1000}{r} \\ \boxed{4rh} &\boxed{= \frac{4000}{r}} \end{aligned}}$$

$$S = 2\left(\tfrac{1}{2}r^2\theta\right) + 2(rh) + hL$$
$$S = r^2\theta + 2rh + h(r\theta)$$
$$S = r^2\theta + 2rh + hr\theta$$
$$S = 2r^2 + 2rh + 2rh$$
$$S = 2r^2 + 4rh$$
$$S = 2r^2 + \frac{4000}{r}$$

b) $S = 2r^2 + 4000r^{-1}$

$$\frac{dS}{dr} = 4r - 4000r^{-2}$$

SOLVE FOR ZERO

$$\Rightarrow 4r = \frac{4000}{r^2} \qquad \& \quad r^2h = 1000$$

$$\Rightarrow 4r^3 = 4000 \qquad 100h = 1000$$

$$\Rightarrow r^3 = 1000 \qquad h = 10$$

$$r = 10$$

TO JUSTIFY THAT WHEN $r = h = 10$, S IS LEAST

$$\frac{d^2S}{dr^2} = 4 + 8000r^{-3} = 4 + \frac{8000}{r^3}$$

$$\left.\frac{d^2S}{dr^2}\right|_{r=10} = 4 + \frac{8000}{10^3} = 12 > 0$$

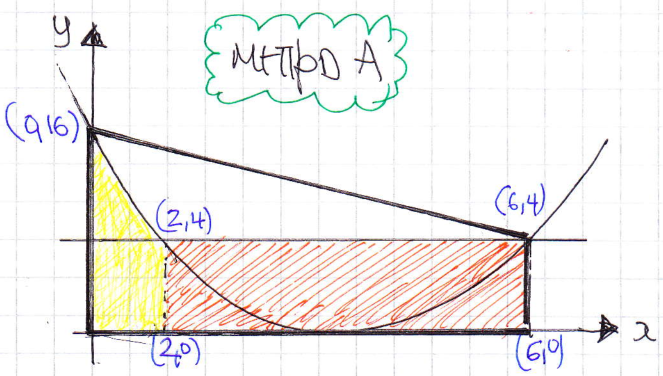
INDEED S IS MINIMISED

9. $y = (x-4)^2$

- BY INSPECTION $C(0,16)$ CURVE TOUCHES THE x AXIS AT $(4,0)$
- $(x-4)^2 = 4$

$$x - 4 = \begin{cases} 2 \\ -2 \end{cases}$$

$$x = \begin{cases} 6 \\ 2 \end{cases} \qquad \therefore \begin{array}{l} A(2,4) \\ B(6,4) \end{array}$$



- AREA OF TRAPEZIUM

$$= \frac{16+4}{2} \times 6$$

$$= 60$$

- AREA OF "ORANGE RECTANGLE"

$$= 4 \times 4$$

$$= 16$$

- "YELLOW AREA" $= \int_0^2 (x-4)^2 \, dx = \int_0^2 x^2 - 8x + 16 \, dx$

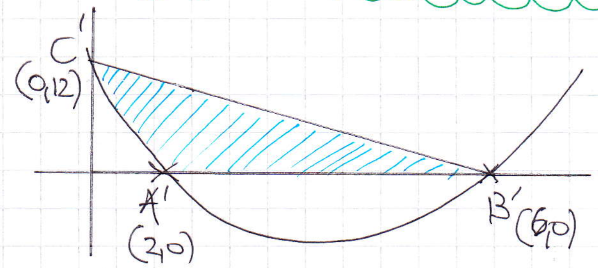
$$= \left[\frac{1}{3}x^3 - 4x^2 + 16x\right]_0^2 = \left(\frac{8}{3} - 16 + 32\right) - 0$$

$$= \frac{56}{3}$$

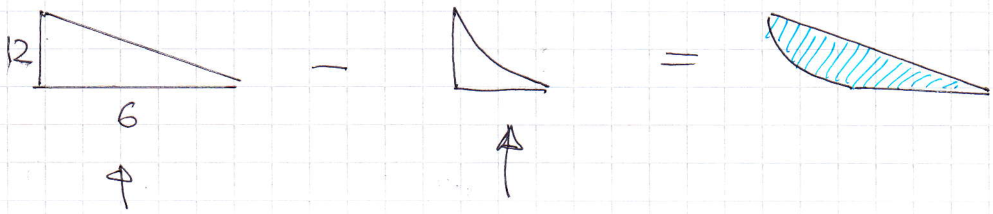
REQUIRED AREA $= 60 - 16 - \frac{56}{3}$

$= \frac{76}{3}$

METHOD B — TRANSLATE "WHOLE PICTURE" 4 UNITS DOWN



$y = (x-4)^2 - 4$



$\frac{1}{2} \times 6 \times 12 = 36$

$$\int_0^2 (x-4)^2 - 4 \, dx = \int_0^2 x^2 - 8x + 12 \, dx$$

$$= \left[\frac{1}{3}x^3 - 4x^2 + 12x\right]_0^2 = \left(\frac{8}{3} - 16 + 24\right) - 0$$

$$= \frac{32}{3}$$

REQUIRED AREA $= 36 - \frac{32}{3} = \frac{76}{3}$