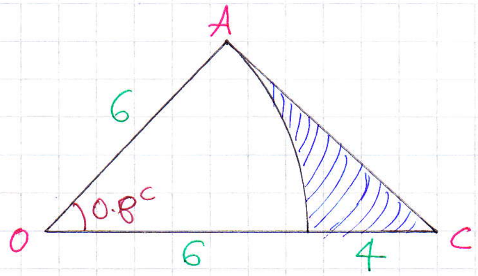
C2, IYGB, PAPER R

1. a)



$\text{AREA} = \frac{1}{2}|OA||OC|\sin 0.8^c$

$\text{AREA} = \frac{1}{2} \times 6 \times 10 \times \sin(0.8^c)$

$\text{AREA} \simeq 21.52 \text{ cm}^2$

b) $\text{AREA OF SECTOR} = \frac{1}{2}r^2\theta^c = \frac{1}{2} \times 6^2 \times 0.8 = 14.40 \text{ cm}^2$

$\text{REQUIRED AREA} = 21.52 - 14.40 \simeq 7.12 \text{ cm}^2$

c) BY THE COSINE RULE

$|AC|^2 = |OA|^2 + |OC|^2 - 2|OA||OC|\cos 0.8^c$

$|AC|^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 0.8^c$

$|AC|^2 = 52.395\ldots$

$|AC| \simeq 7.24 \text{ cm}$

$L = r\theta$

$L = 6 \times 0.8$

$L = 4.8 \text{ cm}$

$\therefore P = 7.24 + 4.8 + 4$

$P = 16.0 \text{ cm}$

2. a) $f(x) = (x+p)(2x^2+5x-4) - 4$

$f(-p) = -4$

b) $f(2) = 10$

$(2+p)(8+10-4) - 4 = 10$

$(2+p) \times 14 = 14$

$2+p = 1$

$p = -1$

c) $f(x) = (x-1)(2x^2+5x-4) - 4$

$$
\begin{aligned}
f(x) = {} & 2x^3 + 5x^2 - 4x \\
& - 2x^2 - 5x + 4 \\
& - 4 \\
\hline
f(x) = {} & 2x^3 + 3x^2 - 9x
\end{aligned}
$$

$f(x) = x(2x^2 + 3x - 9)$

$f(x) = x(2x-3)(x+3)$

3. a) $\left(1+\frac{x}{2}\right)^7 = 1 + \frac{7}{1}\left(\frac{x}{2}\right)^1 + \frac{7\times6}{1\times2}\left(\frac{x}{2}\right)^2 + \frac{7\times6\times5}{1\times2\times3}\left(\frac{x}{2}\right)^3 + \dots$

$= 1 + \frac{7}{2}x + \frac{21}{4}x^2 + \frac{35}{8}x^3 + \dots$

b) $\left(1+\frac{2}{x}\right)^2\left(1+\frac{x}{2}\right)^7 = \left(1+\frac{4}{x}+\frac{4}{x^2}\right)\left(1+\frac{7}{2}x+\frac{21}{4}x^2+\frac{35}{8}x^3+\dots\right)$

$\frac{7}{2}x$ $\quad 21x \quad$ $\frac{35}{2}x$

$\therefore \frac{7}{2}x + 21x + \frac{35}{2}x = 42x$

$\therefore 42$

4. $S_\infty = 675$ $\qquad\qquad$ $u_2 = 27u_5$

$\frac{a}{1-r} = 675$

$\Rightarrow \cancel{a}r = 27\times\cancel{a}r^4$

$\Rightarrow r = 27r^4$

$\Rightarrow 1 = 27r^3 \quad (r\neq0)$

$\Rightarrow r^3 = \frac{1}{27}$

$\boxed{r = \frac{1}{3}}$

$\frac{a}{1-\frac{1}{3}} = 675$

$\frac{3}{2}a = 675$

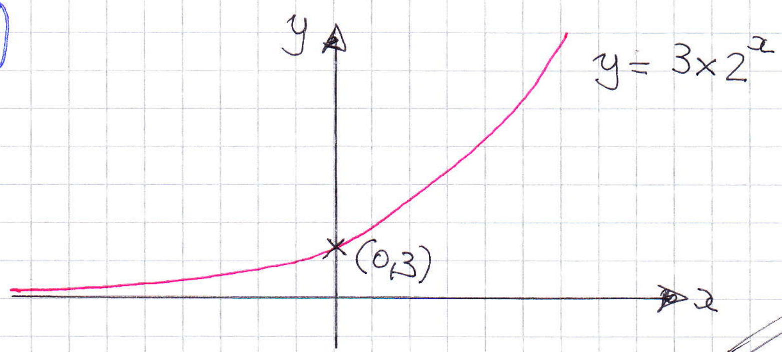
$a = 450$

5. a) $2^x \longmapsto 3\times 2^x$

$f(x) \qquad 3f(x)$

"VERTICAL" STRETCH BY SCALE FACTOR 3

b)



c) $\left.\begin{array}{l} y = 2^{-x} \\ y = 3\times 2^x \end{array}\right\} \Rightarrow 2^{-x} = 3\times 2^x$

$\Rightarrow \dfrac{1}{2^x} = 3\times 2^x$

$\Rightarrow 1 = 3\times 2^x \times 2^x$

$\Rightarrow \dfrac{1}{3} = 2^{2x}$

$\Rightarrow \log\left(\frac{1}{3}\right) = \log 2^{2x}$

$\Rightarrow \log\left(\frac{1}{3}\right) = 2x\log 2$

$\Rightarrow x = \dfrac{\log \frac{1}{3}}{2\log 2}$

$x \simeq -0.792$

ALTERNATIVE

$2^{-x} = 3\times 2^x$

$\log 2^{-x} = \log(3\times 2^x)$

$-x\log 2 = \log 3 + \log 2^x$

$-x\log 2 = \log 3 + x\log 2$

$-\log 3 = 2x\log 2$

$x = \dfrac{-\log 3}{2\log 2} \approx -0.792$

P.T.O

6. a) $f(x) = \dfrac{x+4}{\sqrt{x}} = \dfrac{x}{x^{\frac{1}{2}}} + \dfrac{4}{x^{\frac{1}{2}}} = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$

$$f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$$

SOLVE FOR ZERO

$\Rightarrow \tfrac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}} = 0$

$\Rightarrow \dfrac{1}{2x^{\frac{1}{2}}} = \dfrac{2}{x^{\frac{3}{2}}}$

$\Rightarrow \dfrac{x^{3/2}}{2x^{1/2}} = 2$

$\Rightarrow \boxed{x = 4}$

$\Rightarrow y = \dfrac{4+4}{\sqrt{4}} = 4$

$\therefore M(4,4)$

b)

x	1	1.75	2.5	3.25	4
y	5	4.3466	4.1110	4.0216	4

AREA $\simeq \dfrac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$

AREA $\simeq \dfrac{0.75}{2}\left[5+4+2(4.3466+4.1110+4.0216)\right]$

AREA $\simeq 12.7344\ldots$

AREA $\simeq 12.73$

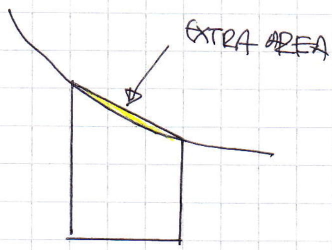
c) AREA $= \displaystyle\int_1^4 \frac{x+4}{\sqrt{x}}\,dx = \int_1^4 x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\,dx = \left[\tfrac{2}{3}x^{\frac{3}{2}} + 8x^{\frac{1}{2}}\right]_1^4$

$= \left(\tfrac{16}{3}+16\right) - \left(\tfrac{2}{3}+8\right) = \tfrac{64}{3} - \tfrac{26}{3} = \tfrac{38}{3}$

d) % ERROR $= \dfrac{12.7344 - \frac{38}{3}}{\frac{38}{3}} \times 100 \simeq 0.5347\ldots$

i.e. 0.53%

e) IT IS AN ~~OVERESTIMATE~~ OVERESTIMATE AS ALL THE STRIPS (TRAPEZIA) GO OVER THE CURVE



7.

$$4\tan^2\theta\cos\theta = 15$$

$$\Rightarrow 4\left(\frac{\sin\theta}{\cos\theta}\right)^2\cos\theta = 15$$

$$\Rightarrow 4\frac{\sin^2\theta}{\cos^{\cancel{2}}\theta}\cancel{\cos\theta} = 15$$

$$\Rightarrow \frac{4\sin^2\theta}{\cos\theta} = 15$$

$$\Rightarrow 4\sin^2\theta = 15\cos\theta$$

$$\Rightarrow 4(1-\cos^2\theta) = 15\cos\theta$$

$$\Rightarrow 4 - 4\cos^2\theta = 15\cos\theta$$

$$\Rightarrow 0 = 4\cos^2\theta + 15\cos\theta - 4$$

$$\Rightarrow (4\cos\theta - 1)(\cos\theta + 4) = 0$$

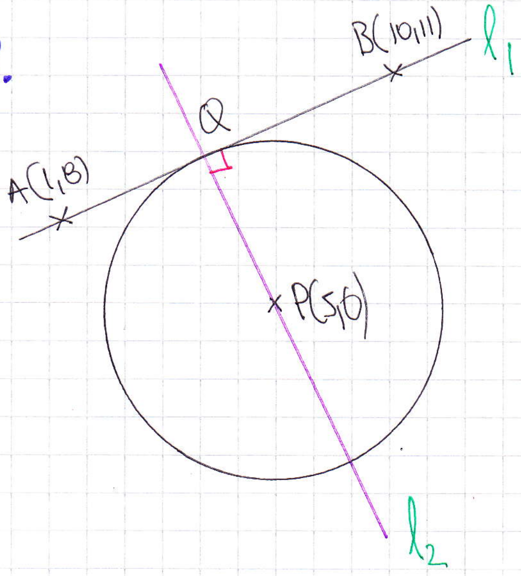
$$\Rightarrow \cos\theta = \begin{cases} 1/4 \\ \cancel{-4} \end{cases}$$

$$\arccos\left(\tfrac{1}{4}\right) = 75.52°$$

$$\begin{pmatrix} \theta = 75.52 \pm 360n \\ \theta = 284.48 \pm 360n \end{pmatrix} \quad n = 0,1,2,3,\ldots$$

$$\theta_1 = 75.5°$$
$$\theta_2 = 284.5°$$

8.



- GRADIENT $l_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-8}{10-1} = \frac{3}{9} = \frac{1}{3}$
- EQUATION l_1: $y - y_0 = m(x - x_0)$

$$y - 8 = \tfrac{1}{3}(x-1)$$
$$3y - 24 = x - 1$$
$$\boxed{3y = x + 23}$$

- EQUATION OF l_2: $y - y_0 = m(x - x_0)$

$$y - 6 = -3(x-5)$$
$$y - 6 = -3x + 15$$
$$\boxed{y = 21 - 3x}$$

- SOLVING SIMULTANEOUSLY TO GET Q

$$\left.\begin{aligned} 3y &= x + 23 \\ y &= 21 - 3x \end{aligned}\right) \Rightarrow 3(21 - 3x) = x + 23$$

$$\Rightarrow 63 - 9x = x + 23$$

$$40 = 10x$$

$$\boxed{x = 4}$$

$$y = 21 - 3 \times 4$$

$$\boxed{y = 9} \qquad \therefore Q(4, 9)$$

THUS RADIUS $= |PQ| = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $\qquad P(5, 6)\ Q(4, 9)$

$$|PQ| = \sqrt{(9 - 6)^2 + (4 - 5)^2}$$

$$|PQ| = \sqrt{9 + 1}$$

$$|PQ| = \sqrt{10}$$

$\therefore$ RADIUS $= \sqrt{10}$