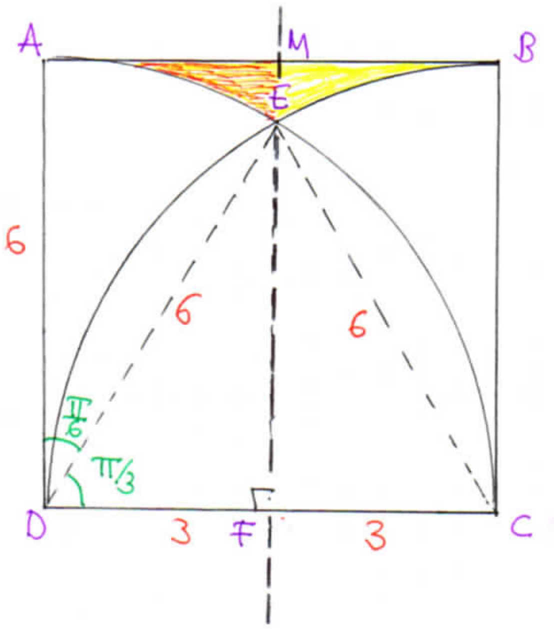
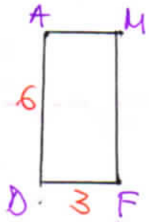
C2, IYGB, PAPER T

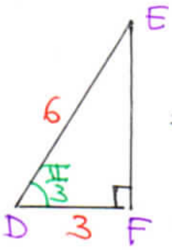
1.



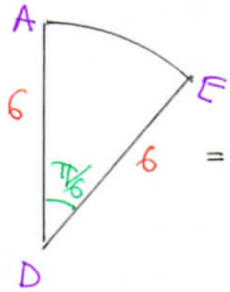
- $\overset{\Delta}{DEC}$ IS EQUILATERAL WITH SIDE LENGTH 6
  $\Rightarrow E\hat{D}C = \frac{\pi}{3} = 60°$
  $\Rightarrow A\hat{D}E = \frac{\pi}{6} = 30°$

- 

  $= 18$

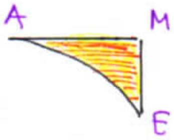
- 

  $= \frac{1}{2} \times 6 \times 3 \times \sin\frac{\pi}{3} = \frac{9}{2}\sqrt{3}$



$= \frac{1}{2} \times 6^2 \times \frac{\pi}{6} = 3\pi$

HENCE THE "ORANGE AREA"



$= \boxed{18} - \boxed{\frac{9}{2}\sqrt{3}} - \boxed{3\pi} = 18 - \frac{9}{2}\sqrt{3} - 3\pi$

HENCE THE REQUIRED AREA $= 2\left(18 - \frac{9}{2}\sqrt{3} - 3\pi\right)$

$= 36 - 9\sqrt{3} - 6\pi$

$= 3\left[12 - 3\sqrt{3} - 2\pi\right]$

AS REQUIRED

C2, IYGB, PAPER T

2. a) $\boxed{f(x) = \sum_{r=0}^{n} \binom{n}{r} x^r (1+x+x^2)^{n-r}}$

RECOGNISING THE BINOMIAL EXPANSION ...

$$f(x) = \left[x + (1+x+x^2)\right]^n = (x^2+2x+1)^n = \left[(x+1)^2\right]^n$$

$$\boxed{f(x) = (x+1)^{2n}}$$

$$\therefore f(-1) = 0^{2n} = 0$$

b) NON CALCULATOR APPROACH

$$\Rightarrow f(3)f(2) = 1728^{1728}$$

$$\Rightarrow 4^{2n} \times 3^{2n} = (12^3)^{1728}$$

$$\Rightarrow 12^{2n} = 12^{3 \times 1728}$$

$$\Rightarrow 2n = 3 \times 1728$$

$$\Rightarrow n = 3 \times 864$$

$$\Rightarrow n = 2592.$$

CALCULATOR APPROACH

$$\Rightarrow f(3)f(2) = 1728^{1728}$$

$$\Rightarrow 4^{2n} \times 3^{2n} = 1728^{1728}$$

$$\Rightarrow 12^{2n} = 1728^{1728}$$

$$\Rightarrow \log_{10} 12^{2n} = \log_{10} 1728^{1728}$$

$$\Rightarrow 2n(\log_{10} 12) = 1728 \log_{10} 1728$$

$$\Rightarrow n = \frac{1728 \log_{10} 1728}{2 \log_{10} 12}$$

$$\Rightarrow n = 2592$$

3. LET $f(x) = ax^3 + ax^2 + ax + b$

$f(-b) = 0 \implies -ab^3 + ab^2 - ab + b = 0$

$\implies -ab^2 + ab - a + 1 = 0 \quad (a \neq 0)$

$\implies ab^2 - ab + a - 1 = 0$

THIS IS A QUADRATIC IN b

FOR <u>REAL</u> ROOTS IN b

$B^2 - 4AC \geqslant 0$.

$\implies (-a)^2 - 4 \times a(a-1) \geqslant 0$

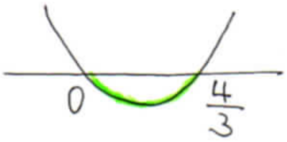
$\implies a^2 - 4a(a-1) \geqslant 0$

$\implies a[a - 4(a-1)] \geqslant 0$

$\implies a[a - 4a + 4] \geqslant 0$

$\implies a(4 - 3a) \geqslant 0$

$\implies a(3a - 4) \leqslant 0$



$0 \leqslant a \leqslant \frac{4}{3}$

$0 < a \leqslant \frac{4}{3} \quad (a \neq 0)$

C2, IYGB, PAPER T

4.
$$4\sin x - \frac{\cos x}{2} = \frac{4}{\sin x} - \frac{1}{2\cos x}$$

MULTIPLY BY 2

$$\Rightarrow 8\sin x - \cos x = \frac{8}{\sin x} - \frac{1}{\cos x}$$

MULTIPLY BY $\sin x$

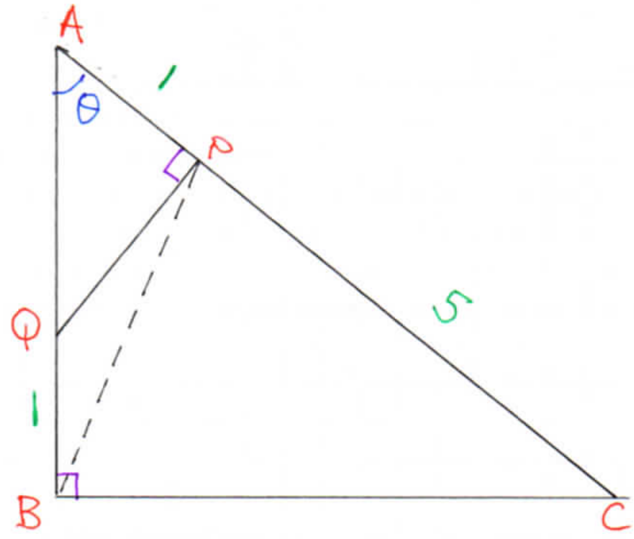
$$\Rightarrow 8\sin^2 x - \cos x\sin x = 8 - \frac{\sin x}{\cos x}$$

MULTIPLY BY $\cos x$

$$\Rightarrow 8\sin^2 x\cos x - \cos^2 x\sin x = 8\cos x - \sin x$$
$$\Rightarrow 8\sin^2 x\cos x - 8\cos x = \cos^2 x\sin x - \sin x$$
$$\Rightarrow 8\cos x(\sin^2 x - 1) = \sin x(\cos^2 x - 1)$$
$$\Rightarrow 8\cos x(1 - \sin^2 x) = \sin x(1 - \cos^2 x)$$
$$\Rightarrow 8\cos x\cos^2 x = \sin x\sin^2 x$$
$$\Rightarrow 8\cos^3 x = \sin^3 x$$
$$\Rightarrow 8 = \frac{\sin^3 x}{\cos^3 x}$$
$$\Rightarrow \tan^3 x = 8$$
$$\Rightarrow \tan x = 2$$

AS REQUIRED

5.



- LOOKING AT $\hat{ABC}$

$$\frac{|AB|}{|AC|} = \cos\theta$$

$$|AB| = |AC|\cos\theta$$

$$|AB| = 6\cos\theta$$

- LOOKING AT $\hat{APQ}$

$$\frac{|AP|}{|AQ|} = \cos\theta$$

$$\frac{1}{|AQ|} = \cos\theta$$

$$|AQ| = \frac{1}{\cos\theta}$$

COMBINE

$$|AQ| + |QB| = |AB|$$

$$\frac{1}{\cos\theta} + 1 = 6\cos\theta$$

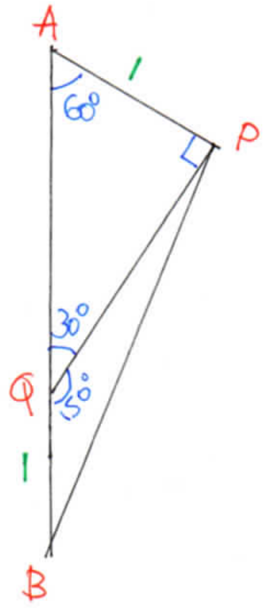
$$1 + \cos\theta = 6\cos^2\theta$$

$$6\cos^2\theta - \cos\theta - 1 = 0$$

$$(3\cos\theta + 1)(2\cos\theta - 1) = 0$$

$$\cos\theta = \begin{cases} -\frac{1}{3} \\ \frac{1}{2} \end{cases}$$

ONLY PHYSICAL SOLUTION $\theta = 60$
($\cos\theta = -\frac{1}{3}$ YIELDS AN OBTUSE ANGLE)



- NOW LOOKING AT $\triangle APQ$

$$\frac{|QP|}{|AP|} = \tan 60°$$

$$|QP| = |AP| \tan 60$$

$$|QP| = 1 \tan 60°$$

$$|QP| = \sqrt{3}$$

- BY THE COSINE RULE ON $\triangle PQB$

$$|PB|^2 = |QB|^2 + |PQ|^2 - 2|QB||PQ|\cos 150°$$

$$|PB|^2 = 1^2 + (\sqrt{3})^2 - 2 \times 1 \times \sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right)$$

$$|PB|^2 = 1 + 3 + 3$$

$$|PB|^2 = 7$$

$$|BP| = \sqrt{7}$$

AS REQUIRED

6. $u_r = ak^{r-1}$

- $u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad \ldots \quad u_n$

$a \quad ak \quad ak^2 \quad ak^3 \quad ak^4 \quad \ldots \quad ak^{n-1}$

IE A STANDARD GEOMETRIC PROGRESSION LAYOUT

NOW

$$\sum_{r=1}^{n}(u_r u_{r+1}) = u_1u_2 + u_2u_3 + u_3u_4 + \dots + u_n u_{n+1}$$

$$= a(ak) + ak(ak^2) + ak^2(ak^3) + \dots ak^{n-1}(ak^n)$$

$$= a^2k + a^2k^3 + a^2k^5 + \dots + a^2k^{2n-1}$$

$$= a^2k\underbrace{\left[1 + k^2 + k^4 + \dots + k^{2n-2}\right]}$$

THIS IS A G.P.
"a" = 1
"r" = k^2
n TERMS

$$= a^2k\left[\frac{1\left[1-(k^2)^n\right]}{1-k^2}\right]$$

$$= \frac{a^2k(1-k^{2n})}{1-k^2}$$

AS REQUIRED

7. LET P BE THE WEEKLY PROFIT

$$P = 500 \times (100-35)$$
$$P = 520 \times (99-35)$$
$$P = 540 \times (98-35)$$

SO IN GENERAL

$$P = (500+20x)\left[(100-x)-35\right]$$
$$P = (500+20x)(65-x)$$
$$P = 20(x+25)(65-x)$$
$$P = -20(x+25)(x-65)$$
$$\boxed{P = -20(x^2-40x-1625)}$$

$$\begin{array}{r} 65 \\ \times\ 25 \\ \hline 325 \\ 130 \\ \hline 1625 \end{array}$$

- NOW

$$P = -20(x^2-40x-1625)$$

$$\frac{dP}{dx} = -20(2x-40)$$

$$\frac{dP}{dx} = -40(x-20)$$

- SOLVING FOR ZERO YIELDS $\boxed{x=20}$

$$\frac{d^2P}{dx^2} = -40<0 \text{ FOR ALL } x$$

$\therefore$ LOCAL MAX

- $P = 20(x+25)(65-x)$ (EARLIER)

- WHEN $x=20$

$$P_{MAX} = 20(20+25)(65-20)$$

$$P = 20 \times 45 \times 45$$

$$\boxed{P = 40500}$$

IE A MAX PROFIT OF £40500 IF PHONES ARE SOLD AT £80

$\underbrace{(100-20)}_{x}$

P.T.O

ALTERNATIVE BY COMPLETING THE SQUARE

$$P = -20(x^2-40x-1625)$$

$$P = -20\left[(x-20)^2-400-1625\right]$$

$$P = -20\left((x-20)^2-2025\right)$$

$$P = -20(x-20)^2+40500$$

$$P = 40500-20(x-20)^2$$

$\therefore$ MAX VALUE OF P IS 40500 (OCCURING WHEN $x=20$)

8. a) $C_1: x^2+y^2-18x+ky+90=0$

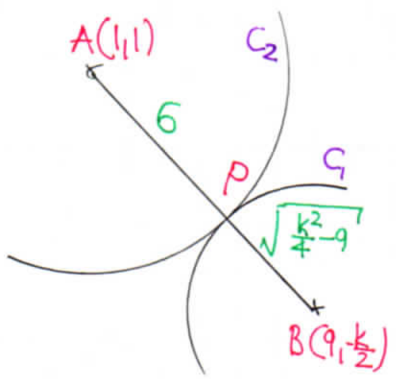
$\Rightarrow x^2-18x+y^2+ky+90=0$

$\Rightarrow (x-9)^2-81+\left(y+\frac{k}{2}\right)^2-\frac{k^2}{4}+90=0$

$\Rightarrow (x-9)^2+\left(y+\frac{k}{2}\right)^2=\frac{k^2}{4}-9$

$\therefore$ CENTRE $\left(9,-\frac{k}{2}\right)$, RADIUS $\sqrt{\frac{k^2}{4}-9}$

b)



$C_2: x^2+y^2-2x-2y=34$

$\Rightarrow x^2-2x+y^2-2y=34$

$\Rightarrow (x-1)^2-1+(y-1)^2-1=34$

$\Rightarrow (x-1)^2+(y-1)^2=36$

CENTRE $(1,1)$, RADIUS 6

- LOOKING AT THE ABOVE PICTURE

$|AB|=\sqrt{\left(-\frac{k}{2}-1\right)^2+(9-1)^2}=\sqrt{\frac{k^2}{4}+k+1+64}=\sqrt{\frac{k^2}{4}+k+65}$

- BUT $|AP|+|PB|=|AB|$

$\Rightarrow 6+\sqrt{\frac{k^2}{4}-9}=\sqrt{\frac{k^2}{4}+k+65}$

SQUARE BOTH SIDES

$\Rightarrow 6^2+2\times6\times\sqrt{\frac{k^2}{4}-9}+\left[\sqrt{\frac{k^2}{4}-9}\right]^2=\left[\sqrt{\frac{k^2}{4}+k+65}\right]^2$

$\Rightarrow 36+12\sqrt{\frac{k^2}{4}-9}+\cancel{\frac{k^2}{4}}-9=\cancel{\frac{k^2}{4}}+k+65$

$\Rightarrow 12\sqrt{\frac{k^2}{4}-9}=k+38$

SQUARE BOTH SIDES AGAIN

$\Rightarrow 144\left[\sqrt{\frac{k^2}{4}-9}\right]^2=(k+38)^2$

$\Rightarrow 144\left(\frac{k^2}{4}-9\right)=k^2+76k+1444$

$$\Rightarrow 35k^2 - 76k - 2740 = 0$$

BY THE <u>QUADRATIC</u> FORMULA

$$k = \frac{-(-76) \pm \sqrt{(-76)^2 - 4 \times 35 \times (-2740)}}{2 \times 35} = \frac{76 \pm 624}{70} = \begin{cases} 10 \\ \cancel{-\frac{274}{35}} \end{cases}$$

$$\therefore k = 10$$

FINALLY

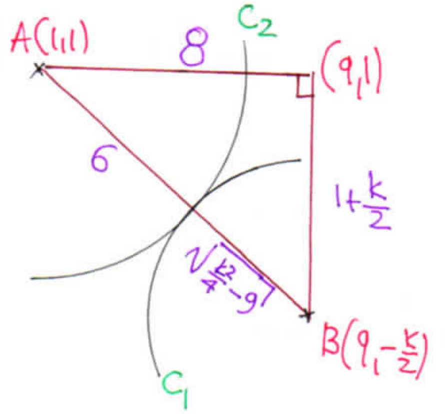
A(1,1) 6 P(x,y) 4 B(9,−5)

$$\boxed{\begin{aligned} \text{RADIUS } |PB| &= \sqrt{\frac{k^2}{4} - 9} \\ &= \sqrt{25 - 9} \\ &= 4 \end{aligned}}$$

- $x = 1 + \frac{6}{10}(9 - 1) = 1 + \frac{3}{5} \times 8 = \frac{29}{5}$
- $y = 1 + \frac{6}{10}(-5 - 1) = 1 + \frac{3}{5} \times (-6) = -\frac{13}{5}$

$$\therefore P\left(\frac{29}{5}, -\frac{13}{5}\right)$$

ALTERNATIVE DIAGRAM ONCE THE PARTICULARS OF C_2 HAVE BEEN ESTABLISHED



$$8^2 + \left(1 + \frac{k}{2}\right)^2 = \left[6 + \sqrt{\frac{k^2}{4} - 9}\right]^2$$

$$64 + 1 + k + \frac{k^2}{4} = 36 + 12\sqrt{\frac{k^2}{4} - 9} + \left(\frac{k^2}{4} - 9\right)$$

$$65 + k + \cancel{\frac{k^2}{4}} = 27 + \cancel{\frac{k^2}{4}} + 12\sqrt{\frac{k^2}{4} - 9}$$

$$k + 38 = 12\sqrt{\frac{k^2}{4} - 9}$$

$$(k+38)^2 = 144\left(\frac{k^2}{4} - 9\right)$$

$$k^2 + 76k + 1444 = 36k^2 - 1296$$

$$0 = 35k^2 - 76k - 2740 \quad \text{ETC}$$

WHICH MERGES WITH PREVIOUS SOLUTION

9. $$\left[\log_{\sin x\cos x}(\sin x)\right]\left[\log_{\sin x\cos x}(\cos x)\right] = \frac{1}{4}$$

- $\log_{\sin x\cos x}(\sin x) = A \implies (\sin x\cos x)^{A} = \sin x \quad \text{(I)}$
- $\log_{\sin x\cos x}(\cos x) = B \implies (\sin x\cos x)^{B} = \cos x \quad \text{(II)}$

THUS THE ORIGINAL EQUATION TRANSFORMS TO

$$\boxed{AB = \frac{1}{4}}$$

AND MULTIPLYING (I) & (II)

$$(\sin x\cos x)^{A}(\sin x\cos x)^{B} = \sin x\cos x$$

$$(\sin x\cos x)^{A+B} = (\sin x\cos x)^{1}$$

$$\boxed{A + B = 1}$$

SOLVING SIMULTANEOUSLY BY SUBSTITUTION

$$\boxed{A = 1 - B} \implies (1-B)B = \frac{1}{4}$$

$$\implies B - B^2 = \frac{1}{4}$$

$$\implies B^2 - B = -\frac{1}{4}$$

$$\implies 4B^2 - 4B = -1$$

$$\implies 4B - 4B + 1 = 0$$

$$\implies (2B-1)^2 = 0$$

$$B = \frac{1}{2} \quad \& \quad A = \frac{1}{2}$$

RETURNING TO ONE OF THE ORIGINAL EQUATIONS

$$\Rightarrow \log_{\sin x \cos x} \sin x = \frac{1}{2}$$

$$\Rightarrow (\sin x \cos x)^{\frac{1}{2}} = \sin x$$

$$\Rightarrow \sin x \cos x = \sin^2 x$$

$$\Rightarrow \cos x = \sin x \qquad (\sin x \neq 0)$$

$$\Rightarrow \tan x = 1$$

$$x = \frac{\pi}{4} \pm n\pi \qquad n = 0, 1, 2, 3, \ldots$$

BUT x MUST BE IN THE "FIRST QUADRANT" FOR THE LOGARITHM TO BE DEFINED

$$\therefore x = \frac{\pi}{4} + n\pi \;, \quad n = 0, 1, 2, 3, \ldots$$

$$x = -\frac{7\pi}{4} + 2n\pi \;, \quad n \in \mathbb{N}$$

$$x = \frac{\pi}{4}(8n - 7) \quad n \in \mathbb{N}$$