C2, IYGB, PAPER U

1. a) WORK IN MINUTES: PAPER 1 $\longrightarrow$ 3h − 20′ = 200 MINUTES

PAPER 2 $\longrightarrow$ 3h − 15′ = 195 MINUTES

COMMON RATIO $= \frac{195}{200} = 0.975$

$u_n = ar^{n-1}$

$\Rightarrow u_6 = 200 \times 0.975^5$

$\Rightarrow u_6 \simeq 176.219\ldots$

IE APPROX. 176 min

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_{12} = \frac{200(1-0.975^{12})}{1-0.975}$

$\Rightarrow S_{12} = 2096.01\ldots \quad) \div 60$

$\Rightarrow S_{12} = 34.933\ldots$

I.E APPROX 35 hours

b) $u_n < 120$

$\Rightarrow ar^{n-1} < 120$

$\Rightarrow 200 \times (0.975)^{n-1} < 120$

$\Rightarrow 0.975^{n-1} < 0.6$

$\Rightarrow \log(0.975)^{n-1} < \log(0.6)$

$\Rightarrow (n-1)\log(0.975) < \log(0.6)$

DIVIDING BY NEGATIVE REVERSES INEQUALITY

$\Rightarrow n-1 > 20.17\ldots$

$\Rightarrow n > 21.17\ldots$

$\therefore n = 22$

2. a) $(2x-4)^5 = \binom{5}{0}(2x)^0(-4)^5 + \binom{5}{1}(2x)^1(-4)^4 + \binom{5}{2}(2x)^2(-4)^3 + \binom{5}{3}(2x)^3(-4)^2 + \binom{5}{4}(2x)^4(-4)^1 + \binom{5}{5}(2x)^5(-4)^0$

$= -1024 + 2560x - 2560x^2 + 1280x^3 - 320x^4 + 32x^5$

$= 32x^5 - 320x^4 + 1280x^3 - 2560x^2 + 2560x - 1024$

b) I) $\left(\frac{y+16}{4}\right)^5 = \left[\frac{1}{4}y + 4\right]^5$

THIS IS THE SAME AS THE EXPANSION OF PART (A) WITH

$2x = \frac{1}{4}y$

$x = \frac{1}{8}y$

$x = \frac{1}{8}y$ AND ALL THE SIGN "PLUS"

$\therefore \ldots 2560\left(\frac{1}{8}y\right)^2 + \ldots \quad \therefore 40y^2 \quad \therefore 40$

II) $(\sqrt{2}z-2)^5(\sqrt{2}z+2)^5 = \left[(\sqrt{2}z-2)(\sqrt{2}z+2)\right]^5$

$= (2z^2-4)^5$

THIS IS THE SAME EXPANSION AS PART (a) WITH $x=z^2$

THUS .... $-320(z^2)^4$ ... $-320z^8$

$\therefore -320$

3. METHOD A

$\begin{cases} \log_2(xy^2)=0 \\ \log_2(x^2y)=3 \end{cases}$

$\begin{cases} \log_2 x + \log_2 y^2 = 0 \\ \log_2 x^2 + \log_2 y = 3 \end{cases}$

$\begin{cases} \log_2 x + 2\log_2 y = 0 \\ 2\log_2 x + \log_2 y = 3 \end{cases}$

$\left.\begin{cases} X+2Y=0 \\ 2X+Y=3 \end{cases}\right\} \Rightarrow \boxed{X=-2Y}$

$-4Y+Y=3$

$-3Y=3$

$\boxed{Y=-1}$ & $\boxed{X=2}$

$\log_2 y = -1$ $\qquad$ $\log_2 x = 2$

$y = 2^{-1}$ $\qquad$ $x = 2^2$

$y = \frac{1}{2}$ $\qquad$ $x = 4$

METHOD B

$\begin{cases} \log_2(xy^2)=0 \\ \log_2(x^2y)=3 \end{cases}$

$\begin{cases} xy^2 = 2^0 \\ x^2y = 2^3 \end{cases}$

$\begin{cases} xy^2=1 \\ x^2y=8 \end{cases}$

$\begin{cases} x^2y^4=1 \\ x^2y=8 \end{cases}$ DIVIDE

$y^3 = \frac{1}{8}$

$y = \frac{1}{2}$

$x^2y=8$

$\frac{1}{2}x^2=8$

$x^2=16$

$x=+4$ $\quad \begin{pmatrix} x>0 \\ y>0 \end{pmatrix}$

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4. a) $f(x) = x^3 + (a+2)x^2 - 2x + b$

$$\left.\begin{array}{l} f(2)=0 \Rightarrow 8+4(a+2)-4+b=0 \\ f(-a)=0 \Rightarrow -a^3+(a+2)(-a)^2-2(-a)+b=0 \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} 8+4a+8-4+b=0 \\ \cancel{-a^3}+\cancel{a^3}+2a^2+2a+b=0 \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} b=-4a-12 \\ b=-2a^2-2a \end{array}\right\} \Rightarrow -4a-12=-2a^2-2a$$

$$2a^2-2a-12=0$$

$$a^2-a-6=0$$

$$(a+2)(a-3)=0$$

$$a = \begin{cases} 3 \\ \cancel{-2} \end{cases}$$

$$\therefore\ b=-4\times3-12$$

$$b=-24$$

b) $f(x) = x^3+5x^2-2x-24$

$f(x) = (x-2)(x+3)(x+4)$

$$\therefore\ x = \begin{cases} 2 \\ -3 \\ -4 \end{cases}$$

5.

$y = x - 2x^4$

$\frac{dy}{dx} = 1 - 8x^3$

SOLVE FOR ZERO.

$1-8x^3=0$

$8x^3=1$

$x^3=\frac{1}{8}$

$x=\frac{1}{2}$ $\quad y=\frac{1}{2}-2\left(\frac{1}{2}\right)^4=\frac{3}{8}$ $\qquad M\left(\frac{1}{2},\frac{3}{8}\right)$

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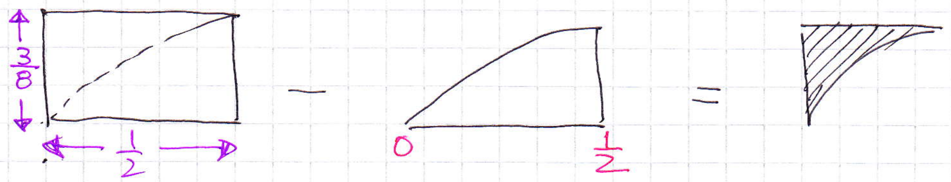
Now



$\frac{1}{2} \times \frac{3}{8} = \frac{3}{16}$

$$\int_0^{\frac{1}{2}} x - 2x^4 \, dx = \left[\tfrac{1}{2}x^2 - \tfrac{2}{5}x^5\right]_0^{\frac{1}{2}}$$

$$= \left(\tfrac{1}{8} - \tfrac{1}{80}\right) - (0) = \tfrac{9}{80}$$

$\therefore$ REQUIRED AREA $= \frac{3}{16} - \frac{9}{80} = \frac{3}{40}$

6. a) $\boxed{P = 16\sqrt{t} + \frac{27}{t}}$

WHEN $t = 2\frac{1}{4} = \frac{9}{4} \implies P = 16\sqrt{\frac{9}{4}} + \frac{27}{\frac{9}{4}} = 36$

$\therefore 36000$

b) $P = 16t^{\frac{1}{2}} + 27t^{-1}$

$$\frac{dP}{dt} = 8t^{-\frac{1}{2}} - 27t^{-2}$$

$\boxed{\text{INCREASING} \implies \frac{dP}{dt} > 0}$

$$8t^{-\frac{1}{2}} - 27t^{-2} > 0$$

$$\frac{8}{t^{\frac{1}{2}}} > \frac{27}{t^2}$$

$$\frac{t^2}{t^{\frac{1}{2}}} > \frac{27}{8} \qquad \boxed{t^2, t^{\frac{1}{2}} > 0}$$

$$t^{\frac{3}{2}} > \frac{27}{8}$$

$$\left(t^{\frac{3}{2}}\right)^{\frac{2}{3}} > \left(\frac{27}{8}\right)^{\frac{2}{3}}$$

$$t > \frac{9}{4}$$

7. a) $f(x) = \sqrt{3} - \tan(2x - \alpha)$

$\Rightarrow -2 = \sqrt{3} - \tan(2 \times 52.5 - \alpha)$

$\Rightarrow -2 = \sqrt{3} - \tan(105 - \alpha)$

$\Rightarrow \tan(105 - \alpha) = 2 + \sqrt{3}$

$\arctan(2 + \sqrt{3}) = 75°$

$\Rightarrow 105° - \alpha = 75° \pm 180n \quad n = 0,1,2,3,\ldots$

$\Rightarrow -\alpha = -30° \pm 180n$

$\Rightarrow \alpha = 30° \pm 180n$

$\therefore \alpha = 30°$

$(0 < \alpha < 90)$

b) $f(x) = 0$

$\Rightarrow 0 = \sqrt{3} - \tan(2x - 30)$

$\Rightarrow \tan(2x - 30) = \sqrt{3}$

$\arctan(\sqrt{3}) = 60°$

$\Rightarrow 2x - 30° = 60 \pm 180n$

$n = 0,1,2,3\ldots$

$\Rightarrow 2x = 90 \pm 180n$

$\Rightarrow x = 45 \pm 90n$

$\therefore B(45, 0)$

$C(135, 0)$

c) WHEN $x = 0$ $f(0) = \sqrt{3} - \tan(-30) = \frac{4}{3}\sqrt{3}$

$x = 180$ $f(180) = \sqrt{3} - \tan(330) = \frac{4}{3}\sqrt{3}$

$\therefore A(0, \frac{4}{3}\sqrt{3}) \quad D(180, \frac{4}{3}\sqrt{3})$

d) PERIOD IS $90°$

$\tan(x + \phi)$ HAS PERIOD 180
$\tan(2x + \phi)$ HAS PERIOD 90

e)
- $\tan x$ HAS ITS FIRST ASYMPTOTE FOR WHICH $x > 0$ AT $x = 90$
- $\tan(x - 30)$ HAS ITS FIRST ASYMPTOTE AT $x = 90 + 30 = 120$
- $\tan(2x - 30)$ HAS ITS FIRST ASYMPTOTE AT $x = \frac{120}{2} = 60$

$\therefore$ ASYMPTOTES ARE $x = 60°$

$x = 150°$ ← $+90$ ONE PERIOD LATER

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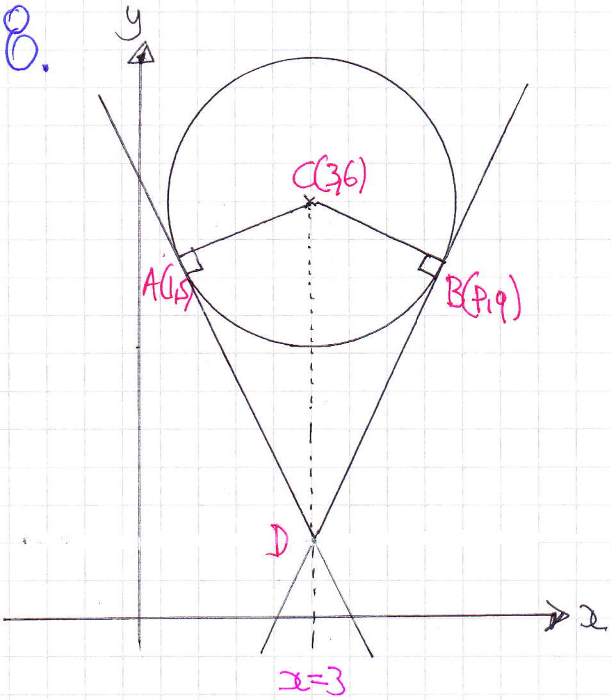
8.



a) RADIUS $= |AC| = \sqrt{(6-5)^2+(3-1)^2}$
$= \sqrt{1+4}$
$= \sqrt{5}$

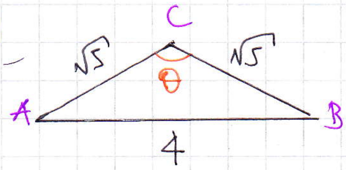
b) $p = 5$ ⟵ SYMMETRICAL ABOUT $x=3$
$q = 5$ ⟵ SAME "HEIGHT" AS A

c) GRAD AC $= \dfrac{6-5}{3-1} = \dfrac{1}{2}$

TANGENT GRADIENT $= -2$ PASSING THROUGH $A(1,5)$

$y-5 = -2(x-1)$
$y-5 = -2x+2$
$y+2x = 7$

d)

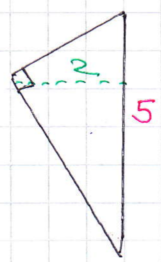


BY THE COSINE RULE $\Longrightarrow 4^2 = \sqrt{5}^2 + \sqrt{5}^2 - 2\sqrt{5}\sqrt{5}\cos\theta$
$\Longrightarrow 16 = 5 + 5 - 10\cos\theta$
$\Longrightarrow 10\cos\theta = -6$
$\cos\theta = -\dfrac{3}{5}$
$\theta \simeq 2.214^c$

ALTERNATIVE SPLIT ABOVE INTO TWO RIGHT ANGLED TRIANGLES

then $\sin\phi = \dfrac{2}{\sqrt{5}}$
$\phi \simeq 1.1071\ldots$
$\theta \simeq 2\phi \simeq 2.214^c$

e)
- $y + 2x = 7$
  $y + 2\times3 = 7$
  $y = 1$
  $\therefore D(3,1)$
- $|CD| = 5$
-



AREA $= \frac{1}{2}\times5\times2 = 5$

- AREA OF KITE
  $2\times5 = 10$
- AREA OF SECTOR
  $\frac{1}{2}r^2\theta = \frac{1}{2}(\sqrt{5})^2\times2.214^c$
  $\simeq 5.5357\ldots$
- REQUIRED AREA
  $10 - 5.5357\ldots$
  $\simeq 4.64$

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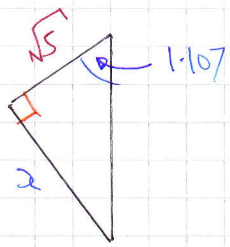
ALTERNATIVE FOR PART OF PART e



$$\frac{x}{\sqrt{5}} = \tan(1.107)$$

$$x = 2\sqrt{5}$$

$$\text{AREA} = \frac{1}{2} \times \sqrt{5} \times 2\sqrt{5} = 5$$

(AS BEFORE)