C2, IYGB, PAPER V

1. a)

$f(x) = (2x-1)(x+4) - 4(x-3)^2$

$f(x) = 2x^2+8x-x-4-4(x^2-6x+9)$

$f(x) = 2x^2+7x-4-4x^2+24x-36$

$f(x) = -2x^2+31x-40$

b)

$g(x) = (3x-2)(x+4)(x+k)$

$g(x) = (3x^2+10x-8)(x+k)$

$10x^2$

$3kx^2$

Thus

$10x^2 + 3kx^2 = -2x^2$

$10+3k = -2$

$3k = -12$

$k = -4$

2. a)

$\boxed{\begin{array}{l} a = 10 \\ r = 1.2 \\ n = 10 \end{array}}$

$u_n = ar^{n-1}$

$u_{10} = 10 \times 1.2^9$

$u_{10} = 51.5978\ldots$

$\therefore$ Approx £51.60

b)

$\boxed{u_n = a \times r^{n-1}}$

$10 \times 1.2^{n-1} > 1000$

$\Rightarrow \quad 1.2^{n-1} > 100$

$\Rightarrow \quad \log_{10}(1.2^{n-1}) > \log_{10} 100$

$\Rightarrow \quad (n-1)\log_{10}(1.2) > 2$

$\Rightarrow \quad n-1 > \dfrac{2}{\log_{10}(1.2)}$

$\Rightarrow \quad n > \dfrac{2}{\log_{10}(1.2)} + 1$

AS REQUIRED

c)

$n > 26.258\ldots$

$\therefore n = 27$

C2, IYGB, PAPER V

3. $(2x+k)^4 = \dots + \binom{4}{2}(2x)^2(k)^2 + \binom{4}{3}(2x)^3(k)^1 + \dots$

$(2x+k)^4 = \dots + 6 \times 4x^2 \times k^2 + 4 \times 8x^3 \times k + \dots$

$(2x+k)^4 = \dots + 24k^2x^2 + 32kx^3 + \dots$

$\therefore \quad 24k^2 = 12(32k)$

$24k^2 = 384k$

$24k = 384 \quad (k \neq 0)$

$k = 16$

4. a) TRANSLATION, 4 UNITS, TO THE "RIGHT" IE $f(x-4)$

b) $|AB| = 4$ UNITS

c) $\left.\begin{array}{l} y_1 = \log_2 x \\ y_2 = \log_2(x-4) \end{array}\right\} \Rightarrow |y_1 - y_2| = 2$

$\log_2 x - \log_2(x-4) = 2$

$\log_2\left(\frac{x}{x-4}\right) = 2\log_2 2$

$\log_2\left(\frac{x}{x-4}\right) = \log_2 4$

$\frac{x}{x-4} = 4$

$x = 4x - 16$

$16 = 3x$

$x = \frac{16}{3}$

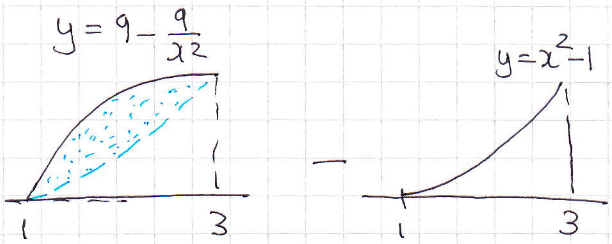
I.E $k = \frac{16}{3}$

C2, IYGB, PAPER V

5.

$$\left.\begin{array}{l} y = x^2 - 1 \\ y = 9\left(1 - \frac{1}{x^2}\right) \end{array}\right\}$$ SOLVING SIMULTANEOUSLY

$$x^2 - 1 = 9\left(1 - \frac{1}{x^2}\right)$$
$$x^2 - 1 = 9 - \frac{9}{x^2}$$
$$x^2 + \frac{9}{x^2} - 10 = 0$$
$$x^4 + 9 - 10x^2 = 0$$
$$x^4 - 10x^2 + 9 = 0$$
$$(x^2 - 1)(x^2 - 9) = 0$$
$$x^2 = \begin{cases} 1 \\ 9 \end{cases}$$
$$x = \begin{cases} 1 \\ 3 \end{cases}$$ $x > 0$ (1ST QUADRANT)



$$\int_1^3 9 - 9x^{-2}\,dx$$
$$= \left[9x + 9x^{-1}\right]_1^3$$
$$= \left[9x + \frac{9}{x}\right]_1^3$$
$$= (27 + 3) - (9 + 9)$$
$$= 12$$

$$\int_1^3 x^2 - 1\,dx$$
$$= \left[\tfrac{1}{3}x^3 - x\right]_1^3$$
$$= (9 - 3) - \left(\tfrac{1}{3} - 1\right)$$
$$= 7 - \frac{1}{3}$$
$$= \frac{20}{3}$$

REQUIRED AREA $= 12 - \frac{20}{3} = \frac{16}{3}$ //

ALTERNATIVE

$$\int_1^3 9 - \frac{9}{x^2}\,dx - \int_1^3 x^2 - 1\,dx$$
$$= \int_1^3 10 - \frac{9}{x^2} - x^2\,dx$$
$$= \left[10x + \frac{9}{x} - \frac{1}{3}x^3\right]_1^3$$
$$= (30 + 3 - 9) - \left(10 + 9 - \frac{1}{3}\right)$$
$$= 24 - \frac{56}{3}$$
$$= \frac{16}{3}$$ // AS BEFORE

6. $D = 12 + 3\sin\left(\frac{\pi t}{6}\right)$

$\Rightarrow 10 = 12 + 3\sin\left(\frac{\pi t}{6}\right)$

$\Rightarrow -2 = 3\sin\left(\frac{\pi t}{6}\right)$

$\Rightarrow \boxed{\sin\frac{\pi t}{6} = -\frac{2}{3}}$

$\arcsin\left(-\frac{2}{3}\right) = -0.7297\ldots$

$\Rightarrow \begin{cases} \frac{\pi t}{6} = -0.7297 \pm 2n\pi \\ \frac{\pi t}{6} = 3.8713.. \pm 2n\pi \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} \pi t = -4.3784.. \pm 12n\pi \\ \pi t = 23.2279 \pm 12n\pi \end{cases}$

$\Rightarrow \begin{cases} t = -1.3937 \pm 12n \\ t = 7.3937 \pm 12n \end{cases}$

$t_1 = 22.606\ldots$

$t_2 = 19.394\ldots$

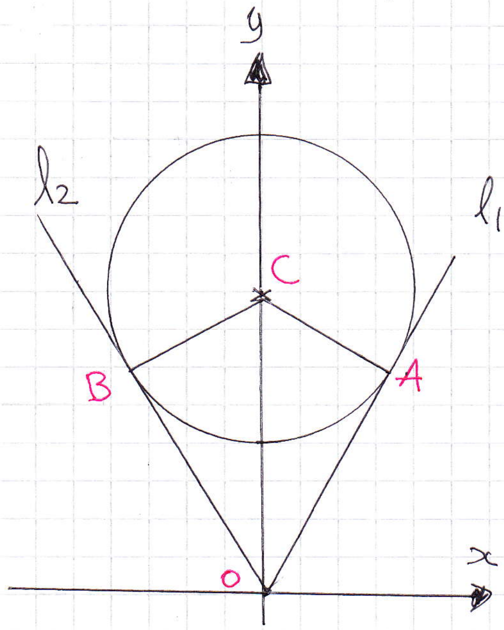
$\therefore$ 19:24

22:36

e.g. $0.606.. \times 60 \approx 36$

$0.394 \times 60 \approx 24$

7.



a) $9x^2 + (3y - 25)^2 = 225$

$9x^2 + 9\left(y - \frac{25}{3}\right)^2 = 225$

$x^2 + \left(y - \frac{25}{3}\right) = 25$

$\therefore$ CENTRE $\left(0, \frac{25}{3}\right)$ RADIUS $= 5$

b) $A\left(4, \frac{16}{3}\right)$ $C\left(0, \frac{25}{3}\right)$

GRAD AC $= \dfrac{\frac{25}{3} - \frac{16}{3}}{0 - 4} = \dfrac{3}{-4} = -\dfrac{3}{4}$

l_1: $y - y_0 = m(x - x_0)$

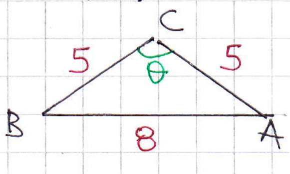
$y - \frac{16}{3} = \frac{4}{3}(x - 4)$

$y - \frac{16}{3} = \frac{4}{3}x - \frac{16}{3}$

$y = \frac{4}{3}x$

IT PASSES THROUGH O AS IT IS OF THE FORM $y = mx$

c) EITHER



$|BA|^2 = |BC|^2 + |CA|^2 - 2|BC||CA|\cos\theta$

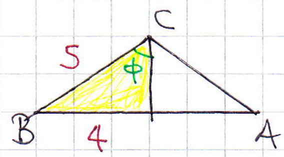
$8^2 = 5^2 + 5^2 - 2 \times 5 \times 5\cos\theta$

$50\cos\theta = -14$

$\cos\theta = -\frac{7}{25}$

$\theta = 1.8546...$

OR



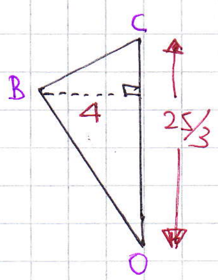
$\sin\phi = \frac{4}{5}$

$\phi \approx 0.9273...$

$\therefore \theta = 2 \times 0.9273$

$\theta \approx 1.8546^c$

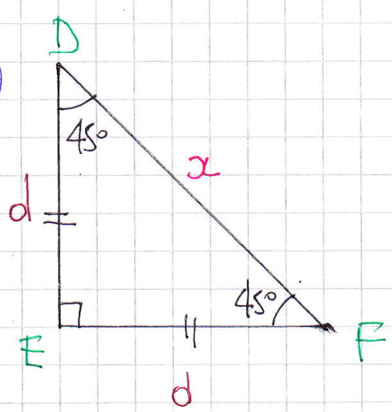
d) AREA OF KITE $= 2 \times$ TRIANGLES $= 2 \times \left(\frac{1}{2} \times \frac{25}{3} \times 4\right) = \frac{100}{3}$



AREA OF SECTOR $=$ "$\frac{1}{2}r^2\theta$" $= \frac{1}{2} \times 5^2 \times 1.8546... \approx 23.182...$

REQUIRED AREA $= \frac{100}{3} - 23.182... \approx 10.15$

8. a)



BY PYTHAGORAS

$d^2 + d^2 = x^2$

$2d^2 = x^2$

$d^2 = \frac{1}{2}x^2$

$\boxed{d = \frac{\sqrt{2}}{2}x}$

OR TRIGONOMETRY

$\frac{d}{x} = \sin 45°$

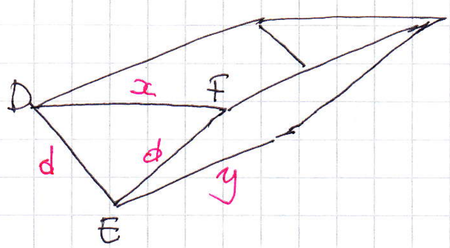
$\frac{d}{x} = \frac{\sqrt{2}}{2}$

$\sqrt{2}x = 2d$

$d = \frac{\sqrt{2}}{2}x$

C2, IYGB, PAPER V — 6 —

CAPACITY = VOLUME = CROSS-SECTIONAL AREA × LENGTH



$\Rightarrow \frac{1}{2}d^2y = 4$

$\Rightarrow \frac{1}{2}\left(\frac{\sqrt{2}}{2}x\right)^2 y = 4$

$\Rightarrow \frac{1}{4}x^2y = 4$

$\Rightarrow \boxed{x^2y = 16}$

SURFACE AREA = 2 TRIANGLES + 2 RECTANGLES

$A = 2 \times \frac{1}{2}d^2 + 2dy$

$A = d^2 + 2dy$

$A = \underbrace{\left(\frac{\sqrt{2}}{2}x\right)^2}_{d^2} + 2\underbrace{\left(\frac{\sqrt{2}}{2}x\right)}_{d}\underbrace{\frac{16}{x^2}}_{y}$

$A = \frac{1}{2}x^2 + \frac{16\sqrt{2}}{x}$ // AS REQUIRED

b) $A = \frac{1}{2}x^2 + 16\sqrt{2}x^{-1}$

$\Rightarrow \frac{dA}{dx} = x - 16\sqrt{2}x^{-2}$

$\Rightarrow \frac{dA}{dx} = x - \frac{16\sqrt{2}}{x^2}$

SOLVE FOR ZERO

$\Rightarrow 0 = x - \frac{16\sqrt{2}}{x^2}$

$\Rightarrow \frac{16\sqrt{2}}{x^2} = x$

$16\sqrt{2} = x^3$

$2^4 \times 2^{\frac{1}{2}} = x^3$

$x^3 = 2^{\frac{9}{2}}$

$x = \left(2^{\frac{9}{2}}\right)^{\frac{1}{3}}$

$x = 2^{\frac{3}{2}}$

$x = \sqrt{2}\sqrt{2}\sqrt{2}$

$x = 2\sqrt{2}$ //

$(k = 2)$