C2, IYGB, PAPER X

1. a) $f(x) = 10x^3 - 21x^2 - x$

$f(2) = 10 \times 2^3 - 21 \times 2^2 - 2 = 80 - 84 - 2 = -6$

b) $$\frac{10x^3 - 21x^2 - x}{x-2} = g(x) - \frac{6}{x-2}$$

$$10x^3 - 21x^2 - x = (x-2)g(x) - 6$$

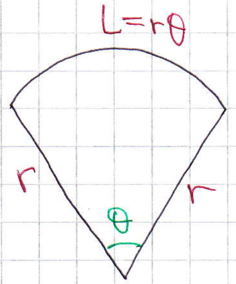
$$10x^3 - 21x^2 - x + 6 = (x-2)g(x)$$

- BY INSPECTION OR LONG DIVISION

$$\begin{array}{r l}
 & 10x^2 - x - 3 \\
x-2 \,\big) & 10x^3 - 21x^2 - x + 6 \\
 & -10x^3 + 20x^2 \\
\hline
 & -x^2 - x + 6 \\
 & x^2 - 2x \\
\hline
 & -3x + 6 \\
 & 3x - 6 \\
\hline
\end{array}$$

$$f(x) = (x-2)(10x^2 - x - 3) = (x-2)(5x-3)(2x+1)$$

2.



- $A = \frac{1}{2}r^2\theta^c$

$15 = \frac{1}{2}r^2\theta$

$\boxed{r^2\theta = 30}$

- $P = 23$

$2r + r\theta = 23$

$\boxed{r\theta = 23 - 2r}$

Thus $r^2\theta = 30$

$r(r\theta) = 30$

$r(23 - 2r) = 30$

$23r - 2r^2 = 30$

$0 = 2r^2 - 23r + 30$

$(2r - 3)(r - 10)$

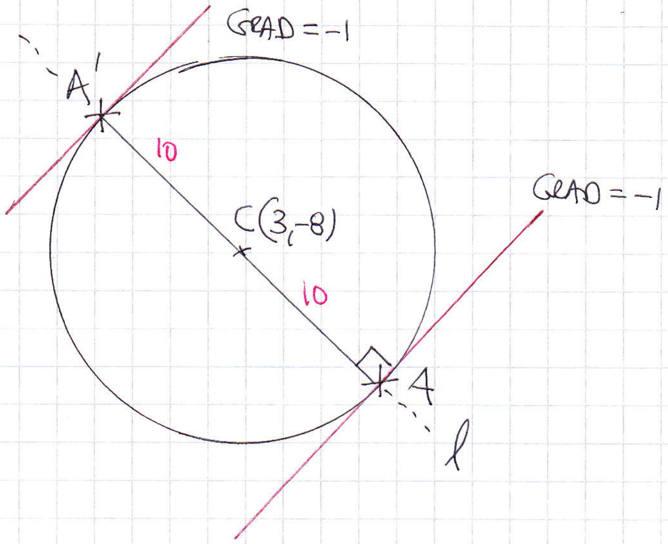
$r = \begin{cases} 10 \\ 3/2 \end{cases}$

BUT $\theta = \dfrac{23 - 2r}{r}$ $\therefore \theta = \begin{cases} 0.3^c \\ \cancel{\frac{40}{3}} \end{cases}$ MORE THAN 2π!

$\therefore r = 10$ & $\theta = 0.3^c$

C2, IYGB, PAPER X

3.



- GRAD OF AC = 1
- EQUATION OF l

$$y+8 = 1(x-3)$$
$$y+8 = x-3$$
$$\boxed{y = x-11}$$

- EQUATION OF CIRCLE

$$(x-3)^2+(y+8)^2=100$$
$$(x-3)^2+(x-11+8)^2=100$$
$$(x-3)^2+(x-3)^2=100$$
$$2(x-3)^2 = 100$$
$$(x-3)^2 = 50$$
$$x-3 = \pm\sqrt{50}$$
$$x = 3 \pm \sqrt{50}$$
$$\left(\text{OR } x = 3 \pm 5\sqrt{2}\right)$$

4. a) $y = 1 + 2\sin(ax+p)$

- $A(0, 1+\sqrt{3}) \Longrightarrow$ $1+\sqrt{3} = 1+2\sin p$

$$\sqrt{3} = 2\sin p$$
$$\sin p = \frac{\sqrt{3}}{2}$$
$$p = 60 \qquad (0 < p < 90)$$

- $C(50,0) \Longrightarrow$ $0 = 1 + 2\sin(a \times 50 + 60)$

$$-1 = 2\sin(50a + 60)$$
$$-\frac{1}{2} = \sin(50a+60)$$

$$\left(\begin{array}{l} 50a + 60 = -30 \pm 360n \\ 50a + 60 = 210 \pm 360n \end{array}\right. \qquad n = 0,1,2,3,\ldots$$

$$\left(\begin{array}{l} 50a = -90 \pm 360n \\ 50a = 150 \pm 360n \end{array}\right.$$

C2, IYGB, PAPER X

$$\begin{cases} d = -\frac{9}{5} \pm 7.2n \\ d = 3 \pm 7.2n \end{cases}$$

$0 < d < 5$

$\therefore d = 3$

b)

$$y = 1 + 2\sin(3x + 60)$$

$$\Rightarrow 0 = 1 + 2\sin(3x + 60)$$

$$\Rightarrow \sin(3x + 60) = -\frac{1}{2}$$

$$\arcsin\left(-\frac{1}{2}\right) = -30$$

$$\Rightarrow \begin{cases} 3x + 60 = -30 \pm 360n \\ 3x + 60 = 210 \pm 360n \end{cases}$$

$n = 0, 1, 2, 3, \ldots$

$$\Rightarrow \begin{cases} 3x = -90 \pm 360n \\ 3x = 150 \pm 360n \end{cases}$$

$$\Rightarrow \begin{cases} x = -30 \pm 120n \\ x = 50 \pm 120n \end{cases}$$

$\therefore x = -30°, 90°, 210°, \ldots$

$x = \ldots 50°, 170°, \ldots$

$D(90, 0)$

$B(10, 3)$

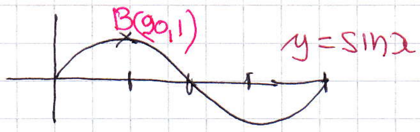
STRETCHED VERTICALLY BY 2

TRANSLATED UP BY 1

- MIDPOINT OF C & D IS $\frac{90 + 50}{2} = 70$
- GRAPH HAS PERIOD $\frac{360}{3} = 120°$
- FROM MIN TO MAX IS $60°$

$\therefore 70 - 60 = 10$

ALTERNATIVE



	$y = \sin x$	$B(90, 1)$
"$f(x+60)$":	$y = \sin(x + 60)$	$B(30, 1)$
"$f(3x)$":	$y = \sin(3x + 60)$	$B(10, 1)$
"$2f(x)$":	$y = 2\sin(3x + 60)$	$B(10, 2)$
"$f(x)+1$":	$y = 1 + 2\sin(3x + 60)$	$B(10, 3)$

C2, IYGB, PAPER X

5. $(1+x-x^2)^6 = (1+(x-x^2))^6 = (1+y)^6$

WHERE $y = x - x^2$

$= 1 + \frac{6}{1}(y) + \frac{6\times5}{1\times2}(y)^2 + \frac{6\times5\times4}{1\times2\times3}(y)^3 + \dots$

$= 1 + 6y + 15y^2 + 20y^3 + \dots$

$= 1 + 6(x-x^2) + 15(x-x^2)^2 + 20(x-x^2)^3$

$= 1 + 6x - 6x^2 + 15(x^2 - 2x^3 \cancel{+ x^4}) + 20(x^3 + \dots)$

$(x-x^2)(x-x^2)(x-x^2)$

EVERYTHING ELSE IS HIGHER THAN x^3

$$
\begin{aligned}
= 1 + 6x &- 6x^2 \\
&+ 15x^2 - 30x^3 \\
&\qquad\quad + 20x^3 \\
\hline
1 + 6x &+ 9x^2 - 10x^3 + \dots
\end{aligned}
$$

$A = 6$
$B = 9$
$C = -10$

6. a) $\boxed{m = m_0 \times 2^{-0.2t}}$

$m = 20 \times 2^{-0.2\times10}$

$m = 5$

b) $\boxed{m = m_0 \times 2^{-0.2t}}$

• WHEN $t = T$ $\quad m = \frac{m_0}{64}$

$\Rightarrow \frac{\cancel{m_0}}{64} = \cancel{m_0} \times 2^{-0.2T}$

$\Rightarrow \frac{1}{64} = 2^{-0.2T}$

$\Rightarrow \log\left(\frac{1}{64}\right) = \log\left(2^{-0.2T}\right)$

C2, IYGB, PAPER X

$\Rightarrow -0.2T\log 2 = \log\left(\frac{1}{64}\right)$

$\Rightarrow T = \dfrac{\log\left(\frac{1}{64}\right)}{-0.2\log 2}$

$\Rightarrow T = 30$

ALTERNATIVE

$\frac{1}{64} = 2^{-0.2T}$

$\frac{1}{2^6} = 2^{-0.2T}$

$2^{-6} = 2^{-0.2T}$

$-6 = -0.2T$

$T = 30$

c) $1\% = \frac{1}{100}$

$m = m_0 \times 2^{-0.2t}$

$\Rightarrow \cancel{m_0} \times 2^{-0.2t} < \frac{1}{100}\cancel{m_0}$

$\Rightarrow 2^{-0.2t} < \frac{1}{100}$

$\Rightarrow \log 2^{-0.2t} < \log\left(\frac{1}{100}\right)$

$\Rightarrow -0.2t\log 2 < -2$

$\Rightarrow t > \dfrac{-2}{-0.2\log 2}$

$\Rightarrow t > \dfrac{10}{\log 2}$

$\Rightarrow t > 33.219\ldots$

$\therefore N = 34$

7.

$S_2 = 40$

$\dfrac{a(1-r^2)}{1-r} = 40$

$a = \dfrac{40(1-r)}{1-r^2}$

$S_4 = 130$

$\dfrac{a(1-r^4)}{1-r} = 130$

$a = \dfrac{130(1-r)}{1-r^4}$

$\Rightarrow \dfrac{40\cancel{(1-r)}}{1-r^2} = \dfrac{130\cancel{(1-r)}}{1-r^4}$

$\Rightarrow \dfrac{4}{1-r^2} = \dfrac{13}{1-r^4}$

C2, IYGB, PAPER X

$\Rightarrow 13 - 13r^2 = 4 - 4r^4$

$\Rightarrow 4r^4 - 13r^2 + 9 = 0$

$(4r^2 - 9)(r^2 - 1)$

$r^2 = \frac{9}{4}$ or 1

$r = \frac{3}{2}$, $-\frac{3}{2}$, $\cancel{1}$, $\cancel{-1}$

- $a = \frac{40(1-r)}{1-r^2}$

$a = \frac{40\cancel{(1-r)}}{\cancel{(1-r)}(1+r)}$

$\boxed{a = \frac{40}{1+r}}$

$a = \frac{40}{1+\frac{3}{2}} = 16$ or $\frac{40}{1-\frac{3}{2}} = -80$

Thus

EITHER $S_5 = \frac{16\left(1-\left(\frac{3}{2}\right)^5\right)}{1-\frac{3}{2}}$

$S_5 = 211$

OR $S_5 = \frac{-80\left(1-\left(-\frac{3}{2}\right)^5\right)}{1-\left(-\frac{3}{2}\right)}$

$S_5 = -275$

ALTERNATIVE

- $S_2 = 40$

$a + ar = 40$

$a(1+r) = 40$

- $S_4 = 130$

$\frac{a(1-r^4)}{1-r} = 130$

$\frac{a(1-r^2)(1+r^2)}{1-r} = 130$

$\frac{a\cancel{(1-r)}(1+r)(1+r^2)}{\cancel{1-r}} = 130$

$40(1+r^2) = 130$

$1 + r^2 = \frac{13}{4}$

$r^2 = \frac{9}{4}$

$r = \pm\frac{3}{2}$

OR

- $S_2 = 40$

$a + ar = 40$

$a(1+r) = 40$

- $S_4 = 130$

$S_2 + ar^2 + ar^3 = 130$

$40 + ar^2 + ar^3 = 130$

$ar^2 + ar^3 = 90$

$ar^2(r+1) = 90$

$r^2 \times 40 = 90$

$r^2 = \frac{9}{4}$

$r = \pm\frac{3}{2}$

C2, IYGB, PAPER X

8. IF $\frac{dy}{dx} = 12x^2 - 12x + 6$

$$y = \int 12x^2 - 12x + 6 \, dx$$

$$\boxed{y = 4x^3 - 6x^2 + 6x + C}$$

$$\int_0^2 4x^3 - 6x^2 + 6x + C \, dx = 22$$

$$\left[x^4 - 2x^3 + 3x^2 + Cx\right]_0^2 = 22$$

$$(16 - 16 + 12 + 2C) - (0) = 22$$

$$2C + 12 = 22$$

$$2C = 10$$

$$C = 5$$

$$\therefore \; y = 4x^3 - 6x^2 + 6x + 5$$

9. a)

CONSTRAINT

$$3x + y = 54$$

$$y = 54 - 3x$$

$$P = \frac{(54x + 6y - xy - 324)^2}{3x}$$

$$\Rightarrow P = \frac{[54x + 6(54 - 3x) - x(54 - 3x) - 324]^2}{3x}$$

$$\Rightarrow P = \frac{(\cancel{54x} + \cancel{324} - 18x - \cancel{54x} + 3x^2 - \cancel{324})^2}{3x}$$

$$\Rightarrow P = \frac{(3x^2 - 18x)^2}{3x} = \frac{9x^4 - 108x^3 + 324x^2}{3x}$$

$$\Rightarrow P = 3x^3 - 36x + 108x$$

AS REQUIRED

b)

$$P = 108x - 36x^2 + 3x^3$$

$$\frac{dP}{dx} = 108 - 72x + 9x^2 \Longrightarrow \text{SOLVE FOR ZERO}$$

$$\frac{d^2P}{dx^2} = 18x - 72$$

$$9x^2 - 72x + 108 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6)$$

$$x = \begin{cases} 2 \\ 6 \end{cases}$$

- WHEN $x = 2$

$$\left.\frac{d^2P}{dx^2}\right|_{x=2} = 18 \times 2 - 72 = -36 < 0$$

IE MAX

- WHEN $x = 6$

$$\left.\frac{d^2P}{dx^2}\right|_{x=6} = 36 < 0 \quad \text{IE MIN}$$

WHEN $x = 2$

$$P = 108 \times 2 - 36 \times 2^2 + 3 \times 2^3$$

$$P = 216 - 144 + 24$$

$$P = 96$$

c)

$$3x + y = 54$$

$$x, y > 0$$

THUS

$$y > 0$$

$$54 - 3x > 0$$

$$-3x > -54$$

$$x < 18$$

$$\therefore\ 0 < x < 18$$

WHEN $x = 18$, $P = 3 \times 18^3 - 36 \times 18^2 + 108 \times 18$

$P = 7776$ ← ACTUAL PHYSICAL MAX

$P = 96$ ← LOCAL MAXIMUM

NOTE $P = 3x^3 - 36x^2 + 108x$

$P = 3x(x^2 - 12x + 36)$

$P = 3x(x-6)^2$