C3, IYGB, PAPER D

1. $$\frac{3x-4}{x^2-5x-6} - \frac{2}{x-6} = \frac{3x-4}{(x-6)(x+1)} - \frac{2}{x-6} = \frac{3x-4-2(x+1)}{(x-6)(x+1)}$$

$$= \frac{3x-4-2x-2}{(x-6)(x+1)} = \frac{\cancel{x-6}}{\cancel{(x-6)}(x+1)} = \frac{1}{x+1}$$

2. a) $y = (1-2x)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = -\frac{1}{2}(1-2x)^{-\frac{3}{2}} \times (-2) = (1-2x)^{-\frac{3}{2}}$$

b) $y = e^{3x}(\sin x + \cos x)$

$$\frac{dy}{dx} = 3e^{3x}(\sin x + \cos x) + e^{3x}(\cos x - \sin x)$$

$$\text{OR } e^{3x}(3\sin x + 3\cos x + \cos x - \sin x)$$

$$= e^{3x}(2\sin x + 4\cos x)$$

$$= 2e^{3x}(\sin x + 2\cos x)$$

c) $y = \dfrac{\ln x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2\left(\frac{1}{x}\right) - \ln x \times 2x}{(x^2)^2} = \frac{x - 2x\ln x}{x^4} = \frac{1-2\ln x}{x^3}$$

3. $$\text{LHS} = \frac{1+\cot^2\theta}{2\cot\theta} = \frac{\operatorname{cosec}^2\theta}{2\cot\theta} = \frac{\frac{1}{\sin^2\theta}}{\frac{2\cos\theta}{\sin\theta}} = \frac{\sin\theta}{\sin^2\theta \times 2\cos\theta} = \frac{1}{2\sin\theta\cos\theta}$$

$$= \frac{1}{\sin 2\theta} = \operatorname{cosec}2\theta = \text{RHS}$$

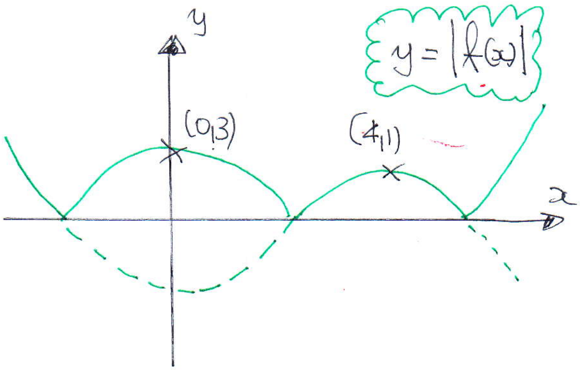
ALTERNATIVE

$$\text{LHS} = \frac{1+\cot^2\theta}{2\cot\theta} = \frac{1+\frac{\cos^2\theta}{\sin^2\theta}}{\frac{2\cos\theta}{\sin\theta}} = \cdots$$ MULTIPLY TOP/BOTTOM BY $\sin^2\theta$ OR ADD $$\cdots = \frac{\sin^2\theta+\cos^2\theta}{2\cos\theta\sin\theta}$$

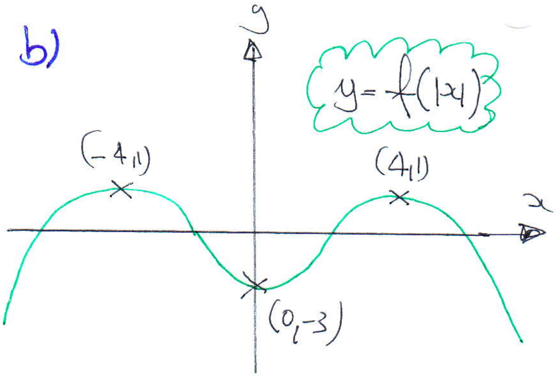
$$= \frac{1}{2\sin\theta\cos\theta} = \frac{1}{\sin 2\theta} = \operatorname{cosec}2\theta = \text{RHS}$$

C3, IYGB, PAPER D

4. a)



b)

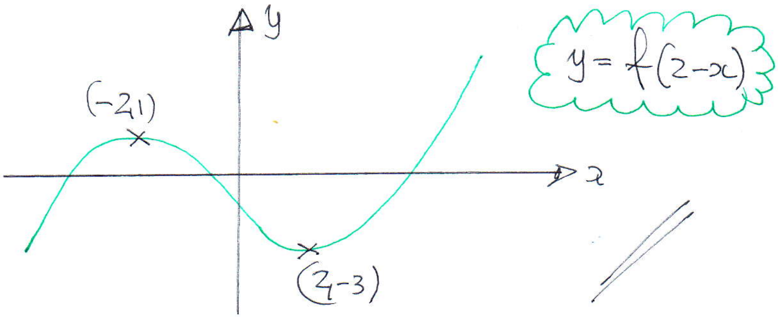


c) $f(x) \longmapsto f(x+2) \longmapsto f((-x)+2)$

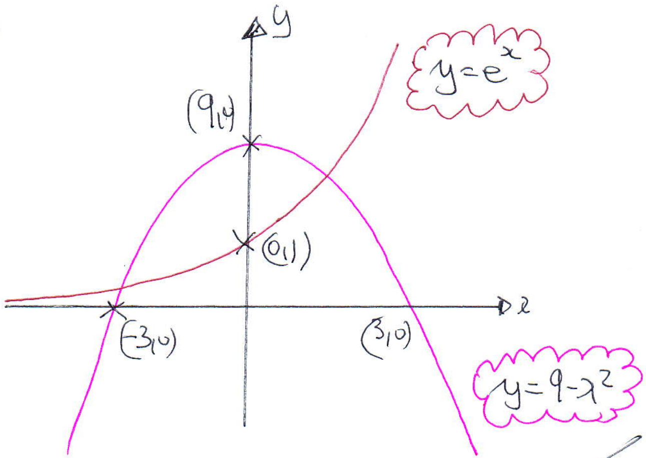
OR $f(x) \longmapsto f(-x) \longmapsto f(-(x-2))$

SO TRANSLATION "LEFT BY 2", THEN REFLECTION IN THE y AXIS

OR REFLECTION IN THE y AXIS, THEN ~~TRAN~~ TRANSLATION "RIGHT BY 2"



5. (a)



b) $(9-x^2)e^{-x} = 1$

$9-x^2 = \dfrac{1}{e^{-x}}$

$9-x^2 = e^x$

TWO INTERSECTIONS BETWEEN THE GRAPHS ONE POSITIVE & ONE NEGATIVE

c) $x_{n+1} = -\sqrt{9 - e^{x_n}}$

$x_1 = -3$

$x_2 = -2.99169$

$x_3 = -2.99162$

$x_4 = -2.99162$ 5 d.p.

d) $x_1 = 2$

$x_2 = 1.26922\ldots$

$x_3 = 2.3327\ldots$

NEXT ITERATION WILL FAIL BECAUSE

$9 - e^{2.3327} < 0$

6. a) $t=0$

$P = 8 + 32e^0$

$P = 8 + 32$

$P = 40$

b) $t=2 \quad P=20$

Thus

$\Rightarrow 20 = 8 + 32e^{-k \times 2}$

$\Rightarrow 12 = 32e^{-2k}$

$\Rightarrow \frac{3}{8} = e^{-2k}$

$\Rightarrow e^{2k} = \frac{8}{3}$

$\Rightarrow 2k = \ln\frac{8}{3}$

$\Rightarrow k = \frac{1}{2}\ln\frac{8}{3} \approx 0.4904$

c) $P=12 \quad P = 8 + 32e^{-0.4904t}$

$\Rightarrow 12 = 8 + 32e^{-0.4904t}$

$\Rightarrow 4 = 32e^{-0.4904t}$

$\Rightarrow \frac{1}{8} = e^{-0.4904t}$

$\Rightarrow 8 = e^{0.4904t}$

$\Rightarrow \ln 8 = 0.4904t$

$\Rightarrow t \simeq 4.24$ min

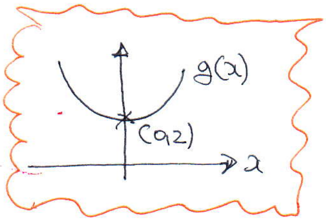
d) $P = 8 + 32e^{-0.4904t}$

$\frac{dP}{dt} = -15.6928e^{-0.4904t}$

$\left.\frac{dP}{dt}\right|_{t=1} = -156928 \times e^{-0.4904 \times 1}$

$= -9.61$

7. (a) $g(x) \geqslant 2$



b) $f(g(x)) = f(x^2+2) = \dfrac{2(x^2+2)+3}{2(x^2+2)-3} = \dfrac{2x^2+7}{2x^2+1}$

c) LET $y = \dfrac{2x+3}{2x-3}$

$\Rightarrow 2xy - 3y = 2x+3$

$\Rightarrow 2xy - 2x = 3y+3$

$\Rightarrow x(2y-2) = 3y+3$

$\Rightarrow x = \dfrac{3y+3}{2y-2}$

$\therefore f^{-1}(x) = \dfrac{3x+3}{2x-2}$ OR $\dfrac{3(x+1)}{2(x-1)}$

d) $\dfrac{2x+3}{2x-3} = \dfrac{3x+3}{2x-2}$

$\Rightarrow (2x+3)(2x-2) = (3x+3)(2x-3)$

$\Rightarrow 4x^2 - 4x + 6x - 6 = 6x^2 - 9x + 6x - 9$

$\Rightarrow 0 = 2x^2 - 5x - 3$

$\Rightarrow (2x+1)(x-3) = 0$

$x = 3$ or $x = -\frac{1}{2}$

Both ok

C3, IYGB, PAPER D

8. $f(x) = \dfrac{\sin x}{2-\cos x}$

$$f'(x) = \frac{(2-\cos x)(\cos x) - \sin x(\sin x)}{(2-\cos x)^2} = \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$$

• SOLVE FOR ZERO

$$\Rightarrow \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2} = 0$$

$$\Rightarrow 2\cos x - \cos^2 x - \sin^2 x = 0$$

$$\Rightarrow 2\cos x = \cos^2 x + \sin^2 x$$

$$\Rightarrow 2\cos x = 1$$

$$\Rightarrow \cos x = \frac{1}{2}$$

$x = \frac{\pi}{3}$ ONLY SOLUTION

$$y = \frac{\sin\frac{\pi}{3}}{2-\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{2-\frac{1}{2}} = \frac{\sqrt{3}}{3} \qquad \therefore P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$$

9. a)

$$2\cos x + 2\sin x \equiv R\cos(x-\alpha)$$
$$\equiv R\cos x\cos\alpha + R\sin x\sin\alpha$$
$$\equiv (R\cos\alpha)\cos x + (R\sin\alpha)\sin x$$

THUS

$\begin{cases} R\cos\alpha = 2 \\ R\sin\alpha = 2 \end{cases}$

• SQUARE & ADD $R = \sqrt{2^2+2^2} = \sqrt{8}$

• DIVIDE EQUATIONS $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{2}{2}$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$$\therefore 2\cos x + 2\sin x \equiv \sqrt{8}\cos\left(x - \frac{\pi}{4}\right)$$

b) $f(x) = \dfrac{6}{\sqrt{8}\cos\left(x - \frac{\pi}{4}\right)}$

- VERTICAL ASYMPTOTE OCCURS WHEN WE DIVIDE BY ZERO

$\therefore \cos\left(x - \frac{\pi}{4}\right) = 0$

$x - \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{3\pi}{4}$ // ONLY SOLUTION

c) $f(3x) - \sqrt{6} = 0$

$\Rightarrow \dfrac{6}{\sqrt{8}\cos\left(3x - \frac{\pi}{4}\right)} - \sqrt{6} = 0$

$\Rightarrow \dfrac{6}{\sqrt{8}\cos\left(3x - \frac{\pi}{4}\right)} = \sqrt{6}$

$\Rightarrow 6 = \sqrt{48}\cos\left(3x - \frac{\pi}{4}\right)$

$\Rightarrow \cos\left(3x - \frac{\pi}{4}\right) = \frac{6}{\sqrt{48}}$

$\Rightarrow \cos\left(3x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

- $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\Rightarrow \begin{cases} 3x - \frac{\pi}{4} = \frac{\pi}{6} \pm 2n\pi \\ 3x - \frac{\pi}{4} = \frac{11\pi}{6} \pm 2n\pi \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} 3x = \frac{5\pi}{12} \pm 2n\pi \\ 3x = \frac{25\pi}{12} \pm 2n\pi \end{cases}$

$\Rightarrow \begin{cases} x = \frac{5\pi}{36} \pm \frac{2}{3}n\pi \\ x = \frac{25\pi}{36} \pm \frac{2}{3}n\pi \end{cases}$

$x_1 = \frac{5\pi}{36}$

$x_2 = \frac{29\pi}{36}$

$x_3 = \frac{25\pi}{36}$

$x_4 = \frac{\pi}{36}$ //