C3, IYGB, PAPER H

1. a) $x^3 - x^2 = 6x + 6$

$x^3 - x^2 - 6x - 6 = 0$

$f(x) = x^3 - x^2 - 6x - 6$

$f(3) = -6 < 0$

$f(4) = 18 > 0$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN BETWEEN 3 & 4, THERE MUST BE AT LEAST ONE ROOT IN THIS INTERVAL

b) $x^3 - x^2 = 6x + 6$

$x^2(x-1) = 6x + 6$

$x^2 = \dfrac{6x+6}{x-1}$

$x = \pm\sqrt{\dfrac{6x+6}{x-1}}$

$x = \sqrt{\dfrac{6x+6}{x-1}}$

c) $x_{n+1} = \sqrt{\dfrac{6x_n+6}{x_n-1}}$

- $x_0 = 1$ DOES NOT PRODUCE x_1 (ZERO DENOMINATOR)
- $x_0 = 0.5$ DOES NOT PRODUCE x_1 (NEGATIVE IN THE ROOT)

d) $x_0 = 3.3$

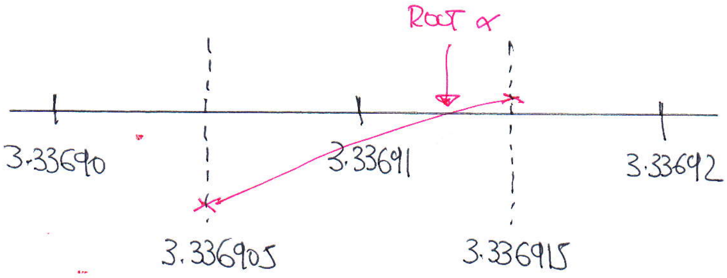
$x_1 \approx 3.349$

$x_2 \approx 3.333$

$x_3 \simeq 3.338$

$x_4 \simeq 3.336$

e)



$f(x) = x^3 - x^2 - 6x - 6$

$f(3.336905) = -0.00014 < 0$

$f(3.336915) = 0.00006 > 0$

CHANGE OF SIGN $\Longrightarrow$ $3.336905 < \alpha < 3.336915$

$\Longrightarrow \alpha \simeq 3.33691$

5 d.p.

2. a) WHEN $t=0$ $\quad T = 20 + 50e^{0} = 20 + 50$

$\therefore T = 70$

b) WHEN $t=30$ $\quad T = 20 + 50e^{-\frac{30}{15}} = 20 + 50e^{-2}$

$\therefore T \approx 27$ $\quad (\simeq 26.76676\ldots)$

c) $T = 35$

$$35 = 20 + 50e^{-\frac{t}{15}}$$
$$15 = 50e^{-\frac{t}{15}}$$
$$\frac{3}{10} = e^{-\frac{t}{15}}$$
$$\frac{10}{3} = e^{\frac{t}{15}}$$
$$\ln\frac{10}{3} = \frac{t}{15}$$
$$t = 15\ln\frac{10}{3} \approx 18$$

3. $y = \frac{1}{6}(x^2+5)^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3}{2} \times \frac{1}{6}(x^2+5)^{\frac{1}{2}} \times 2x$$
$$\frac{dy}{dx} = \frac{1}{2}x(x^2+5)^{\frac{1}{2}}$$
$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{2} \times 2 \times 9^{\frac{1}{2}} = 3$$

WHEN $x=2$

$$y = \frac{1}{6}(2^2+5)^{\frac{3}{2}}$$
$$y = \frac{1}{6} \times 9^{\frac{3}{2}}$$
$$y = \frac{9}{2}$$

$\therefore P\left(2, \frac{9}{2}\right)$

THUS $y - y_0 = m(x - x_0)$

$$y - \frac{9}{2} = 3(x-2)$$
$$y - \frac{9}{2} = 3x - 6$$
$$y = 3x - \frac{3}{2}$$

C3, IYGB, PAPER H

4.

$$5\sin 3x\cos x + 5\cos 3x\sin x = 4$$
$$\Rightarrow 5\left[\sin 3x\cos x + \cos 3x\sin x\right] = 4$$
$$\Rightarrow 5\sin(3x+x) = 4$$
$$\Rightarrow 5\sin 4x = 4$$
$$\Rightarrow \sin 4x = \frac{4}{5}$$

$$\arcsin\left(\tfrac{4}{5}\right) = 0.9273^c$$

$$\left(\begin{array}{l} 4x = 0.9273^c \pm 2n\pi \\ 4x = 2.2143^c \pm 2n\pi \end{array}\right. \quad n = 0,1,2,3,\ldots$$

$$\left(\begin{array}{l} x = 0.2318 \pm \tfrac{1}{2}n\pi \\ x = 0.5536 \pm \tfrac{1}{2}n\pi \end{array}\right.$$

$\therefore$ $x_1 = 0.23^c$

$x_2 = 0.55^c$

$x_3 = 1.80^c$

$x_4 = 2.12^c$

5.

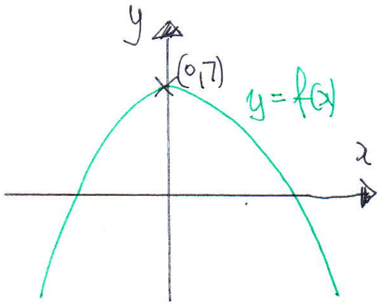
$$f(x) = x - \frac{12}{x^2+2x-3} + \frac{3}{x-1} = x - \frac{12}{(x+3)(x-1)} + \frac{3}{x-1}$$

$$= \frac{x(x+3)(x-1) - 12 + 3(x+3)}{(x+3)(x-1)} = \frac{x^3+2x^2-\cancel{3x}-12+\cancel{3x}+9}{(x+3)(x-1)}$$

$$= \frac{x^3+2x^2-3}{(x+3)(x-1)} = \frac{\cancel{(x-1)}(x^2+3x+3)}{(x+3)\cancel{(x-1)}} = \frac{x^2+3x+3}{x+3}$$

$$(x-1)(x^2+3x+3) = \begin{array}{r} x^3 + 3x^2 + \cancel{3x} \qquad \\ -x^2 - \cancel{3x} - 3 \\ \hline x^3 + 2x^2 \qquad\quad -3 \end{array}$$

6. a)



$\therefore$ RANGE $f(x) \leq 7$

b) $f(g(x)) = f(7-2x^2) = \dfrac{(7-2x^2)+6}{(7-2x^2)+2} = \dfrac{13-2x^2}{9-2x^2}$

c) LET $y = \dfrac{x+6}{x+2}$

$yx + 2y = x+6$

$yx - x = 6-2y$

$x(y-1) = 6-2y$

$x = \dfrac{6-2y}{y-1}$

$\therefore f^{-1}(x) = \dfrac{6-2x}{x-1}$

d) $\dfrac{6-2x}{x-1} = \dfrac{x+6}{x+2}$

$\Rightarrow (6-2x)(x+2) = (x-1)(x+6)$

$\Rightarrow 6x+12-2x^2-4x = x^2+5x-6$

$\Rightarrow 0 = 3x^2+3x-18$

$\Rightarrow 0 = x^2+x-6$

$\Rightarrow 0 = (x-2)(x+3)$

$\Rightarrow x = \begin{cases} 2 \\ -3 \end{cases}$

7. $y = e^{-x}\sin(\sqrt{3}x)$

$\dfrac{dy}{dx} = -e^{-x}\sin(\sqrt{3}x) + e^{-x} \times \sqrt{3}\cos(\sqrt{3}x)$

$= e^{-x}\left[\sqrt{3}\cos\sqrt{3}x - \sin\sqrt{3}x\right]$

NOW $\sqrt{3}\cos\sqrt{3}x - 1\sin\sqrt{3}x \equiv R\cos(\sqrt{3}x+\alpha)$

$\equiv R\cos\sqrt{3}x\cos\alpha - R\sin\sqrt{3}x\sin\alpha$

$\equiv (R\cos\alpha)\cos\sqrt{3}x - (R\sin\alpha)\sin\sqrt{3}x$

$R\sin\alpha = 1$
$R\cos\alpha = \sqrt{3}$

- $R = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{3+1} = 2$
- $\tan\alpha = \dfrac{1}{\sqrt{3}} \qquad \alpha = \dfrac{\pi}{6}$

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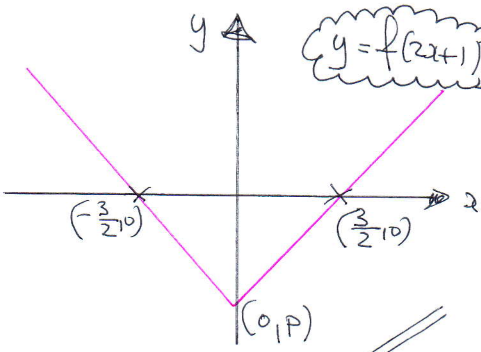
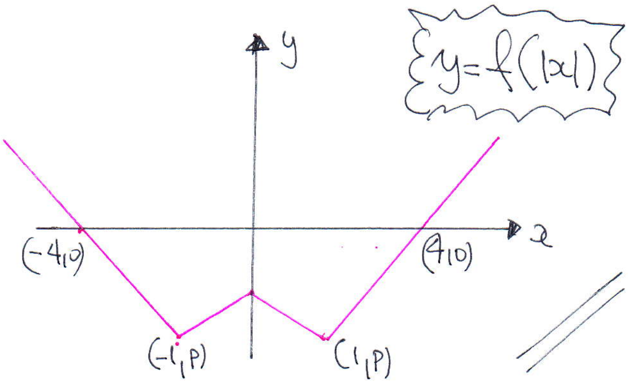
8. (a) TRANSLATION LEFT BY 1 UNIT FOLLOWED BY HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$



(b)



(c) WHEN $x=0$ $\quad |0-1|-3 = 1-3 = -2$ $\quad \therefore C(0,-2)$

WHEN $x=1$ $\quad |1-1|-3 = -3$ $\quad \therefore P(1,-3)$

(d) $f(x) = 4x$

$\Rightarrow |x-1|-3 = 4x$

$\Rightarrow |x-1| = 4x+3$

$$\begin{cases} x-1 = 4x+3 \\ x-1 = -4x-3 \end{cases}$$

$$\begin{cases} -4 = 3x \\ 5x = -2 \end{cases}$$

$$x = \begin{cases} -\frac{4}{3} \\ -\frac{2}{5} \end{cases}$$

CHECK SOLUTIONS

- $\left|-\frac{2}{5}-1\right|-3 = \frac{7}{5}-3 = -\frac{8}{5}$

  $4\left(-\frac{2}{5}\right) = -\frac{8}{5}$

- $\left|-\frac{4}{3}-1\right|-3 = -\frac{2}{3}$

  $4\left(-\frac{4}{3}\right) = -\frac{16}{3}$

$\therefore$ ONLY SOLUTION $x = -\frac{2}{5}$

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9. $\boxed{\tan 2A = \frac{2\tan A}{1-\tan^2 A}}$

$\boxed{\begin{array}{l}\cot\theta = 2\\ \tan\theta = \frac{1}{2}\end{array}}$

- $\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{1-\tan^2\theta}{2\tan\theta} = \frac{1-\left(\frac{1}{2}\right)^2}{2\times\frac{1}{2}} = \frac{3}{4}$

- $\tan 4\theta = \tan[2\times 2\theta] = \frac{2\tan 2\theta}{1-\tan^2 2\theta}$ BUT $\boxed{\begin{array}{l}\cot 2\theta = \frac{3}{4}\\ \tan 2\theta = \frac{4}{3}\end{array}}$

$= \frac{2\times\frac{4}{3}}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = -\frac{24}{7}$

$\therefore \tan\theta\cot 2\theta\tan 4\theta = \frac{1}{2}\times\frac{3}{4}\times\frac{-24}{7} = -\frac{9}{7}$

AS REQUIRED