C3, IYGB, PAPER L

1. a)

$x^3 + 3x = 5$

$x^3 + 3x - 5 = 0$

LET $f(x) = x^3 + 3x - 5$

$f(1) = 1 + 3 - 5 = -1 < 0$

$f(2) = 8 + 6 - 5 = 9 > 0$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN IN THE INTERVAL $[1,2]$, THERE MUST BE A SOLUTION IN $[1,2]$

b)

$$\boxed{x_{n+1} = \frac{5 - x_n^3}{3}}$$

$x_1 = 1$

$x_2 \approx 1.33$

$x_3 \approx 0.88$

$x_4 \approx 1.44$

$x_5 \approx 0.67$

$x_6 \approx 1.57$

SEQUENCE OSCILLATES (POSSIBLY DIVERGES POSSIBLY CONVERGES, I.E. WE NEED MORE INVESTIGATION)

c)

$x_{n+1} = \sqrt[3]{5 - 3x_n}$

$x_1 = 1$

$x_2 = 1.26$

$x_3 = 1.07$

$x_4 = 1.22$

$x_5 = 1.11$

$x_6 = 1.19$

2.

$$\boxed{y = (x^2 + 3x + 2)\cos 2x}$$

$\frac{dy}{dx} = (2x+3)\cos 2x + (x^2+3x+2)(-2\sin 2x)$

$\frac{dy}{dx} = (2x+3)\cos 2x - 2(x^2+3x+2)\sin 2x$

$\left.\frac{dy}{dx}\right|_{x=0} = 3\cos 0 - \cancel{2 \times 2 \sin 0}$

$\left.\frac{dy}{dx}\right|_{x=0} = 3 \leftarrow$ TANGENT GRAD

NOW

$x = 0,\ y = 2\cos 0 = 2$

$\therefore (0,2)$

HENCE $y = 3x + 2$

C3, IYGB, PAPER L

3. a) $4 - 3e^{2x} = 3$

$\Rightarrow 1 = 3e^{2x}$

$\Rightarrow \frac{1}{3} = e^{2x}$

$\Rightarrow \ln\frac{1}{3} = 2x$

$\Rightarrow x = \frac{1}{2}\ln\frac{1}{3}$

$\left(x = -\frac{1}{2}\ln 3\right)$

b) $\ln(2w+1) = 1 + \ln(w-1)$

$\Rightarrow \ln(2w+1) - \ln(w-1) = 1$

$\Rightarrow \ln\left(\frac{2w+1}{w-1}\right) = 1$

$\Rightarrow \frac{2w+1}{w-1} = e^1$

$\Rightarrow 2w+1 = ew - e$

$\Rightarrow e+1 = ew - 2w$

$\Rightarrow e+1 \quad w(e-2)$

$\Rightarrow w = \frac{e+1}{e-2}$

4. a) $f(x) = 1 + \frac{4x}{2x-5} - \frac{15}{2x^2-7x+5}$

$= 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)}$

$= \frac{1(2x-5)(x-1) + 4x(x-1) - 15}{(2x-5)(x-1)} = \frac{2x^2-2x-5x+5+4x^2-4x-15}{(2x-5)(x-1)}$

$= \frac{6x^2-11x-10}{(2x-5)(x-1)} = \frac{\cancel{(2x-5)}(3x+2)}{\cancel{(2x-5)}(x-1)} = \frac{3x+2}{x-1}$

ie $k=1$

b) BY LONG DIVISION

$$\begin{array}{r} 3 \\ x-1 \;\overline{)\; 3x+2} \\ -3x+3 \\ \hline 5 \end{array}$$

$\therefore f(x) = \frac{3x+2}{x-1} = 3 + \frac{5}{x-1}$

$A = 3$

$B = 5$

OR BY MANIPULATION

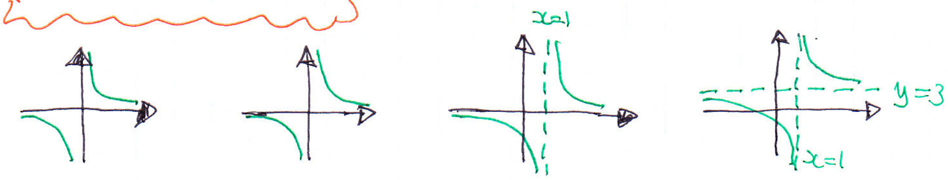
$f(x) = \frac{3x+2}{x-1} = \frac{3(x-1)+5}{(x-1)}$

$= \frac{3(x-1)}{x-1} + \frac{5}{x-1}$

$= 3 + \frac{5}{x-1}$

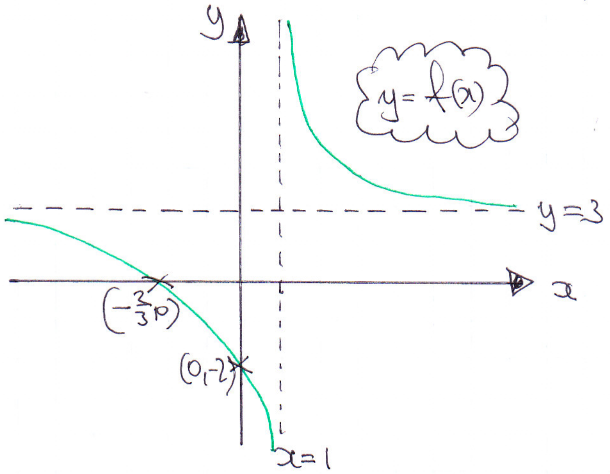
C3, IYGB, PAPER L

c) I) $f(x) = 3 + \dfrac{5}{x-1}$



$\dfrac{1}{x} \longmapsto 5\left(\dfrac{1}{x}\right) \longmapsto \dfrac{5}{(x-1)} \longmapsto \left(\dfrac{5}{x-1}\right)+3$

$\phantom{\dfrac{1}{x} \longmapsto} \dfrac{5}{x} \qquad \dfrac{5}{x-1}$



- $x=0$, $y = 3 + \frac{5}{-1} = -2$

  $\therefore (0,-2)$

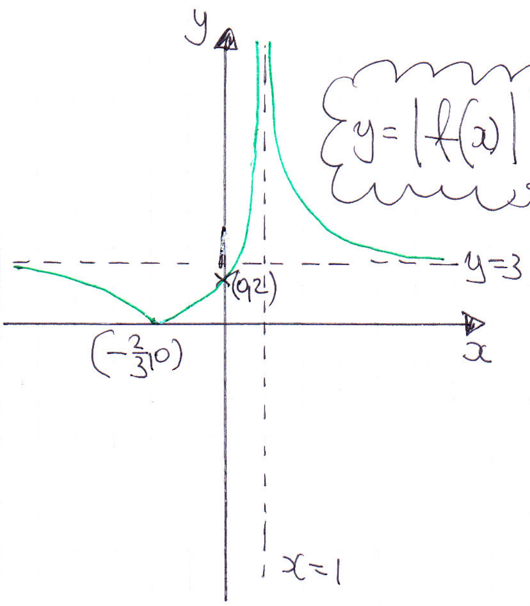
- $y=0$ $\qquad 0 = \dfrac{3x+2}{x-1}$

  $3x+2=0$

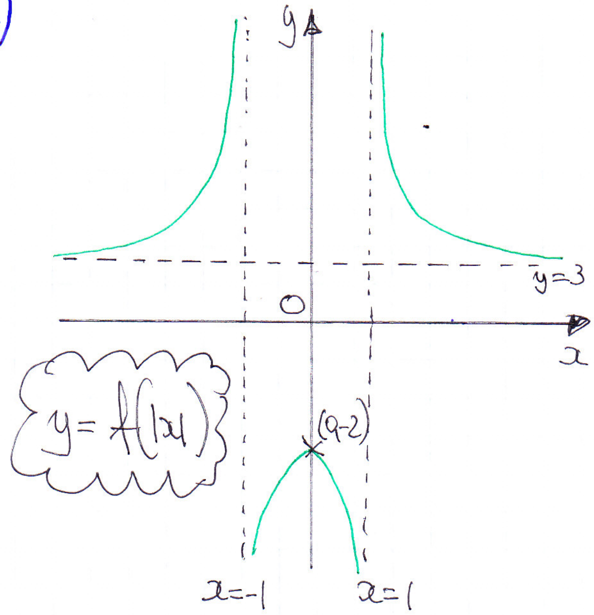
  $x = -\frac{2}{3}$

  $\left(-\frac{2}{3}, 0\right)$

II)



III)



Q5 FOLLOWS AT THE VERY END

6. a) $\boxed{y = \frac{\ln x}{1+\ln x}}$

$$\frac{dy}{dx} = \frac{(1+\ln x)\times\frac{1}{x} - \ln x\left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} + \cancel{\frac{1}{x}\ln x} - \cancel{\frac{1}{x}\ln x}}{(1+\ln x)^2}$$

$$\frac{dy}{dx} = \frac{1}{x(1+\ln x)^2}$$

b) $\boxed{y = \ln\left(\frac{1}{x^2+9}\right)}$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{x^2+9}} \times \frac{\cancel{(x^2+9)\times 0} - 1(2x)}{(x^2+9)^2} = (x^2+9)\times\frac{-2x}{(x^2+9)^2} = -\frac{2x}{x^2+9}$$

ALT $\quad y = \ln\left(\frac{1}{x^2+9}\right) = -\ln(x^2+9)$

$$\frac{dy}{dx} = -\frac{1}{x^2+9}\times 2x = -\frac{2x}{x^2+9}$$

7. $\operatorname{cosec}\theta + 8\cos\theta = 0$

$\Rightarrow \frac{1}{\sin\theta} + 8\cos\theta = 0$

$\Rightarrow 1 + 8\cos\theta\sin\theta = 0$

$\Rightarrow 1 + 4(2\sin\theta\cos\theta) = 0$

$\Rightarrow 1 + 4\sin 2\theta = 0$

$\Rightarrow 4\sin 2\theta = -1$

$\Rightarrow \sin 2\theta = -\frac{1}{4}$

$\arcsin\left(-\frac{1}{4}\right) = -14.48°$.

$2\theta = -14.48 \pm 360n$

$2\theta = 194.48 \pm 360n$ $\quad n = 0, 1, 2, 3, \ldots$

$\theta = -7.24 \pm 180n$

$\theta = 97.24 \pm 180n$

$\theta_1 = 172.8°$

$\theta_2 = 352.8°$

$\theta_3 = 97.2°$

$\theta_4 = 277.2°$

C3, IYGB, PAPER L

8. $\dfrac{2|x|+1}{3} - \dfrac{|x|-1}{2} = 1$

MULTIPLY EQUATION THROUGH BY 6 OR ADD FRACTIONS ON THE LEFT

$\Rightarrow 2\left[2|x|+1\right] - 3\left[|x|-1\right] = 6$

$\Rightarrow 4|x| + 2 - 3|x| + 3 = 6$

$\Rightarrow |x| = 1$

$\Rightarrow x = \begin{cases} 1 \\ -1 \end{cases}$

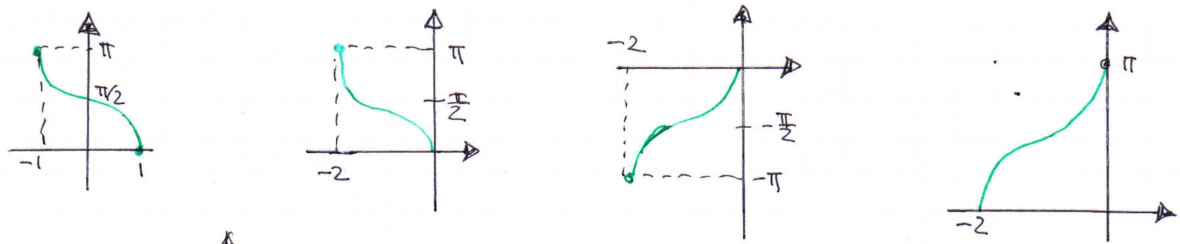
9. a) $\arccos(x) \longmapsto \arccos(x+1) \longmapsto -\arccos(x+1) \longmapsto -\arccos(x+1) + \pi$

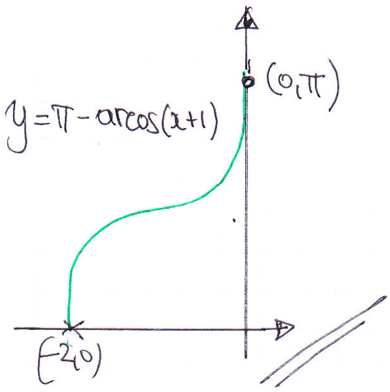
"$f(x+1)$" — TRANSLATION BY 1 UNIT TO THE "LEFT"

"$-f(x)$" — REFLECTION IN THE x AXIS

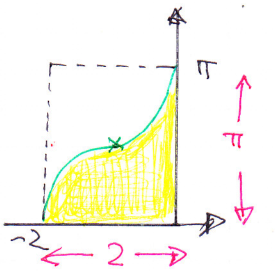
"$f(x) + \pi$" — TRANSLATION, "UPWARDS" BY π

b)





c)



- AREA OF "RECTANGLE" $= 2\pi$
- AS CURVE HAS ROTATIONAL SYMMETRY THE REQUIRED AREA IN YELLOW $= \frac{1}{2} \times 2\pi$

$= \pi$

C3, IYGB, PAPER L

10. a) $\sqrt{3}\cos x - \sin x \equiv R\cos(x+\alpha)$
$$\equiv R\cos x\cos\alpha - R\sin x\sin\alpha$$
$$\equiv (R\cos\alpha)\cos x - (R\sin\alpha)\sin x$$

$\left.\begin{array}{l}\sqrt{3} = R\cos\alpha \\ 1 = R\sin\alpha\end{array}\right\}$ SQUARE & ADD GIVES $R = \sqrt{(\sqrt{3})^2+1^2} = 2$

DIVIDING GIVES $\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x+\frac{\pi}{6}\right)$

b) RANGE: $-2 \le f(x) \le 2$

c) BECAUSE IT IS NOT A ONE TO ONE FUNCTION e.g. $f(x)=0$ HAS TWO DISTINCT VALUES OF x OR IT IS NOT A ONE TO ONE FUNCTION AS IT HAS INCREASING & DECREASING SECTIONS

d) I) MIN VALUE OF $f(x) = -2$, OCCURING WHEN $\cos\left(x+\frac{\pi}{6}\right) = -1$
$$x+\frac{\pi}{6} = \pi$$
$$x = \frac{5\pi}{6}$$

MAX VALUE OF $f(x) = 2$, OCCURING WHEN $\cos\left(x+\frac{\pi}{6}\right) = 1$
$$x+\frac{\pi}{6} = 0$$
$$x = -\frac{\pi}{6} \quad \text{(ADD } 2\pi\text{)}$$
$$x = \frac{11\pi}{6}$$

$\therefore x_1 = \frac{5\pi}{6}$ & $x_2 = \frac{11\pi}{6}$

II)
$$g(x) = 2\cos\left(x+\frac{\pi}{6}\right)$$
$$y = 2\cos\left(x+\frac{\pi}{6}\right)$$
$$\frac{1}{2}y = \cos\left(x+\frac{\pi}{6}\right)$$
$$x+\frac{\pi}{6} = \arccos\left(\frac{1}{2}y\right)$$
$$x = -\frac{\pi}{6} + \arccos\left(\frac{1}{2}y\right)$$
$$\therefore g^{-1}(x) = -\frac{\pi}{6} + \arccos\left(\frac{1}{2}x\right)$$

C3, IYGB, PAPER L

Q5.

$$\cos x = \sin(x-45)$$
$$\cos x = \sin x\cos 45 - \cos x\sin 45$$
$$\cos x = \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x$$
$$2\cos x = \sqrt{2}\sin x - \sqrt{2}\cos x$$
$$\frac{2\cos x}{\cos x} = \frac{\sqrt{2}\sin x}{\cos x} - \frac{\sqrt{2}\cos x}{\cos x}$$
$$2 = \sqrt{2}\tan x - \sqrt{2}$$

$$\sqrt{2}\tan x = 2+\sqrt{2}$$
$$\times\sqrt{2}\ (\quad)\ \times\sqrt{2}$$
$$2\tan x = 2\sqrt{2}+2$$
$$\tan x = \sqrt{2}+1$$

AS REQUIRED

OR

$$\sqrt{2}\tan x = 2+\sqrt{2}$$
$$\tan x = \frac{2+\sqrt{2}}{\sqrt{2}}$$
$$\tan x = \frac{(2+\sqrt{2})\sqrt{2}}{\sqrt{2}\sqrt{2}}$$
$$\tan x = \frac{2\sqrt{2}+2}{2}$$
$$\tan x = \sqrt{2}+1$$