IYGB GCE

Core Mathematics C3

Advanced

Practice Paper O

Difficulty Rating: 3.4200/1.5504

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Show clearly that the expression

$$1-\frac{1}{x-2}+\frac{3}{x^2-x-2},$$

can be written in the form

$$\frac{x+a}{x+b},$$

where a and b are integer constants to be found. **(5)**

Question 2

$$f(x)=x^3-6x^2+12x-11,\ x\in\mathbb{R}.$$

a) Show that the equation $f(x)=0$ has a root α between 3 and 4. **(2)**

An approximation to the value of α is to be found by using the iterative formula

$$x_{n+1}=\sqrt[3]{6{x_n}^2-12x_n+11},\ x_1=3.$$

b) Find, to 3 decimal places, the value of x_2, x_3 and x_4. **(2)**

c) By considering the sign of $f(x)$ in a suitable interval show that $\alpha=3.442$, correct to 3 decimal places. **(2)**

Question 3

$$f(x)=\frac{x}{x^2+4},\ x\in\mathbb{R}.$$

a) Find an expression for $f'(x)$. **(3)**

b) Determine the range of values of x, for which $f(x)$ is decreasing. **(3)**

Question 4

It is given that

$$\frac{1+\cos 2\theta}{\sin 2\theta}\equiv\cot\theta,\ \ \theta\neq\frac{k\pi}{2},\ k\in\mathbb{Z}.$$

a) Prove the validity of the above trigonometric identity. **(3)**

b) Hence solve the equation

$$\operatorname{cosec}4x+\cot 4x=1,\ 0\leq x<2\pi,$$

giving the answers in terms of π. **(7)**

Question 5

The functions f and g are given by

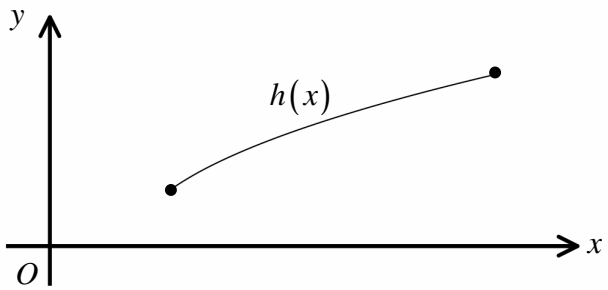
$$f(x)=\sqrt{x},\ x\in\mathbb{R},\ x\geq 0.$$

$$g(x)=x-2,\ x\in\mathbb{R}.$$

a) Find an expression for the function composition $fg(x)$. **(2)**

The function h, whose graph is shown below, is defined by

$$h(x)=fg(x),\ x\in\mathbb{R},\ 3\leq x\leq 11.$$

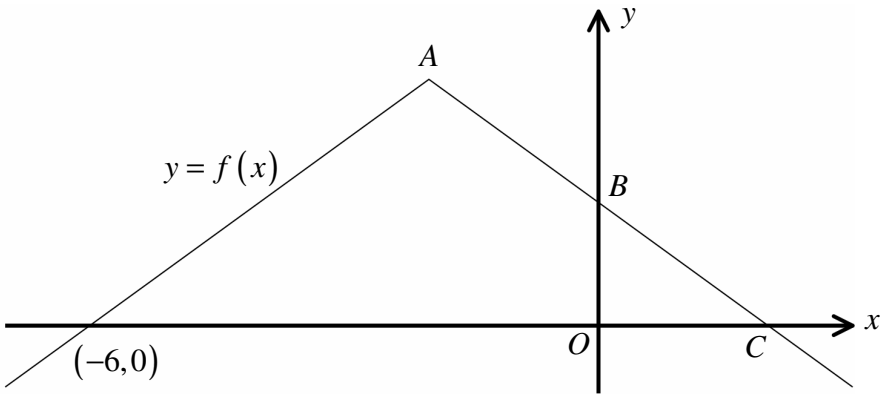


b) State the range of $h(x)$. **(2)**

c) Determine an expression for the inverse function $h^{-1}(x)$. **(2)**

d) State the domain and range of $h^{-1}(x)$. **(2)**

Question 6



The figure below shows the graph of

$$y = f(x),\ x \in \mathbb{R},$$

which consists of two straight line segments which meet at the point A.

The graph of $f(x)$ crosses the coordinate axes at the points $(-6,0)$, B and C.

Sketch, in separate diagrams, the graph of …

a) … $y = |f(x)|$. **(2)**

b) … $y = f(|x|)$. **(2)**

It is now given that

$$f(x) = 4 - |x+2|,\ x \in \mathbb{R}.$$

c) Find the coordinates of the points A, B and C. **(3)**

d) Solve the equation

$$f(x) = -\frac{1}{2}x.$$ **(5)**

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Question 7

A curve has equation

$$y=\frac{1}{2}\ln\left(\frac{x}{3}\right),\ x>0.$$

a) Find an expression for $\frac{dy}{dx}$ in terms of x. **(2)**

b) By making x the subject of the equation and differentiating the resulting equation, find $\frac{dx}{dy}$. **(4)**

c) Use the results of parts **(a)** and **(b)**, to deduce that

$$\frac{dy}{dx}\times\frac{dx}{dy}=1. \quad \textbf{(2)}$$

Question 8

$$f(\theta^\circ)\equiv\left(\sqrt{3}+1\right)\cos 2\theta^\circ+\left(\sqrt{3}-1\right)\sin 2\theta^\circ.$$

a) Express $f(\theta)$ in the form $R\sin(2\theta+\alpha)$, $R>0$, $0\le\theta^\circ<90$. **(4)**

b) Solve the equation

$$f(\theta^\circ)=2,\ 0\le\theta^\circ<360. \quad \textbf{(6)}$$

Question 9

Water is heated in a kettle which is kept in a kitchen. The kitchen is kept at a constant temperature, T_0.

The temperature, T °C, of the water in the kettle satisfies

$$T = 95 - 75\mathrm{e}^{-t}, \ t \geq 0,$$

where t is the time in minutes since the kettle was switched on.

a) Determine the time it takes for the water in the kettle to reach a temperature of 85 °C. **(5)**

b) Find the initial rate of the temperature rise of the water in the kettle. **(2)**

Once the water has reached a temperature of 85 °C the kettle is switched off and is allowed to cool. Its temperature is now given by

$$T = 15 + A\mathrm{e}^{-kt}, \ t \geq 0$$

where A and k are positive constants, and t now represents the time in minutes since the kettle was switched off.

c) Find the value of A. **(2)**

d) State, with a reason, the constant temperature of the kitchen, T_0. **(1)**