1. $$1-\frac{1}{x-2}+\frac{3}{x^2-x-2} = 1-\frac{1}{x-2}+\frac{3}{(x-2)(x+1)}$$

$$= \frac{1(x-2)(x+1)-(x+1)+3}{(x-2)(x+1)} = \frac{x^2-x-\cancel{2}-x-\cancel{1}+\cancel{3}}{(x-2)(x+1)}$$

$$= \frac{x^2-2x}{(x-2)(x+1)} = \frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \frac{x}{x+1}$$

ie $a=0$, $b=1$

2. a) $\boxed{f(x)=x^3-6x^2+12x-11}$

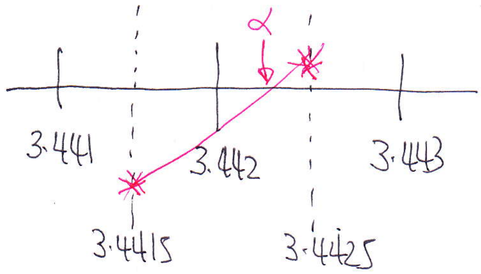
$f(3)=-2<0$
$f(4)=5>0$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN IN THE INTERVAL $[3,4]$, THERE MUST BE A ROOT IN THE INTERVAL

b) $\boxed{x_{n+1}=\sqrt[3]{6x_n^2-12x_n+11}}$

$x_1=3$
$x_2=3.072$
$x_3=3.133$
$x_4=3.185$

c)



$f(3.4415)=-0.0047<0$
$f(3.4425)=0.0016>0$

CHANGE OF SIGN & CONTINUITY $\Rightarrow$ $3.4415<\alpha<3.4425$

$\therefore \alpha = 3.442$

3 d.p

3. a) $f(x) = \dfrac{x}{x^2+4}$

$$f'(x) = \frac{(x^2+4)\times 1 - x(2x^2)}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

b) "DECREASING" $\Rightarrow f'(x) < 0$

$$\frac{4-x^2}{(x^2+4)^2} < 0$$

$$4 - x^2 < 0$$

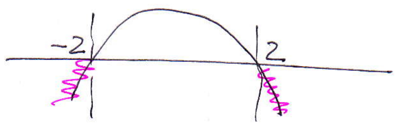
$$-x^2 < -4$$

$$x^2 > 4$$

$x < -2$ OR $x > 2$

OR

$(2-x)(2+x) < 0$



$x < -2$ OR $x > 2$

4. a) $LHS = \dfrac{1+\cos 2\theta}{\sin 2\theta} = \dfrac{1+(2\cos^2\theta - 1)}{2\sin\theta\cos\theta} = \dfrac{2\cos^{\cancel{2}}\theta}{\cancel{2\sin\theta\cos\theta}}$

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta = RHS$

b) $\operatorname{cosec} 4x + \cot 4x = 1$

$\Rightarrow \dfrac{1}{\sin 4x} + \dfrac{\cos 4x}{\sin 4x} = 1$

$\Rightarrow \dfrac{1+\cos 4x}{\sin 4x} = 1$

$\Rightarrow \cot 2x = 1$

$\Rightarrow \tan 2x = 1$

$\arctan 1 = \frac{\pi}{4}$

$\Rightarrow 2x = \frac{\pi}{4} \pm n\pi \qquad n = 0, 1, 2, 3, \ldots$

$\Rightarrow x = \frac{\pi}{8} \pm \frac{n\pi}{2}$

$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

ALTERNATIVE

$\dfrac{1}{\sin 4x} + \dfrac{\cos 4x}{\sin 4x} = 1$

$\dfrac{1+\cos 4x}{\sin 4x} = 1$

$1 + \cos 4x = \sin 4x$

$1 + \cos(2\times 2x) = \sin(2\times 2x)$

$\cancel{1} + (2\cos^2 2x \cancel{-1}) = 2\sin 2x\cos 2x$

$2\cos^2 2x - 2\sin 2x\cos 2x = 0$

$2\cos 2x\,[\cos 2x - \sin 2x]$

EITHER $\cos 2x = 0$

OR $\cos 2x - \sin 2x = 0$, ie $\tan 2x = 0$ ETC

5. a) $f(x) = \sqrt{x} \quad x \geq 0$

$g(x) = x - 2 \quad x \in \mathbb{R}$

$fg(x) = f(g(x)) = f(x-2)$

$= \sqrt{x-2}$

b) $h(3) = 1$

$h(11) = 3$

$\therefore$ RANGE $1 \leq f(g(x)) = h(x) \leq 3$

c) LET $y = \sqrt{x-2}$

$y^2 = x - 2$

$x = y^2 + 2$

$\therefore h^{-1}(x) = x^2 + 2$

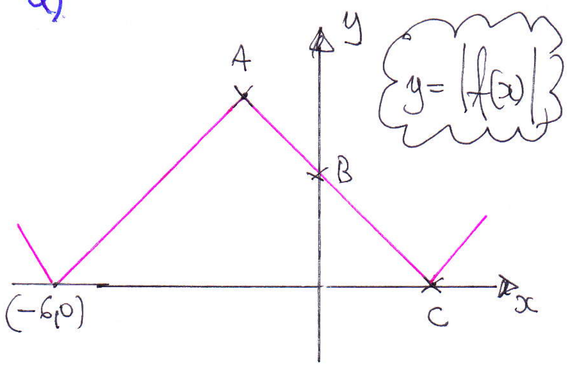
d)

	h	h^{-1}
D	$3 \leq x \leq 11$	$1 \leq x \leq 3$
R	$1 \leq h(x) \leq 3$	$3 \leq h^{-1}(x) \leq 11$

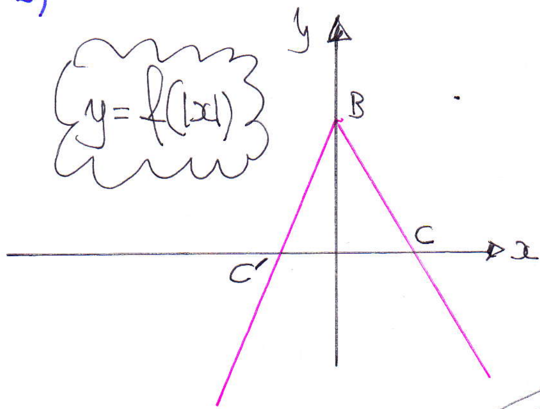
DOMAIN : $1 \leq x \leq 3$

RANGE : $3 \leq h^{-1}(x) \leq 11$

6. a)



b)



c) $f(x) = 4 - |x+2|$

$A(-2, 4)$

$B(0, 2)$

$C(2, 0)$

d) $f(x) = -\frac{1}{2}x$

$4 - |x+2| = -\frac{1}{2}x$

$4 + \frac{1}{2}x = |x+2|$

$$\begin{cases} 4+\frac{1}{2}x = x+2 \\ 4+\frac{1}{2}x = -x-2 \end{cases}$$

$$\begin{cases} 8+x = 2x+4 \\ 8+x = -2x-4 \end{cases}$$

$4 = x$

$3x = -12$

$\therefore x = \begin{cases} -4 \\ 4 \end{cases}$

BOTH ARE OK

7. a) $y = \frac{1}{2}\ln\left(\frac{x}{3}\right) = \frac{1}{2}\ln\left(\frac{1}{3}x\right)$

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\cancel{\frac{1}{3}}x} \times \cancel{\frac{1}{3}} = \frac{1}{2x}$

b) $y = \frac{1}{2}\ln\left(\frac{x}{3}\right)$

$2y = \ln\left(\frac{x}{3}\right)$

$e^{2y} = \frac{x}{3}$

$x = 3e^{2y}$

$\frac{dx}{dy} = 6e^{2y}$

c) $\frac{dy}{dx} \times \frac{dx}{dy} = \frac{1}{2x} \times 6e^{2y}$

$= \frac{3e^{2y}}{x}$

$= \frac{3e^{2y}}{3e^{2y}}$

$= 1$

AS REQUIRED

8. a) $f(\theta) \equiv (\sqrt{3}+1)\cos 2\theta + (\sqrt{3}-1)\sin 2\theta$

$f(\theta) \equiv R\sin(2\theta+\alpha)$

$f(\theta) \equiv R\sin 2\theta\cos\alpha + R\cos 2\theta\sin\alpha$

$f(\theta) \equiv (R\cos\alpha)\sin 2\theta + (R\sin\alpha)\cos 2\theta$

$\therefore \left.\begin{array}{l} R\cos\alpha = \sqrt{3}-1 \\ R\sin\alpha = \sqrt{3}+1 \end{array}\right\} \Rightarrow R = \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}$

$R = \sqrt{3 - \cancel{2\sqrt{3}} + 1 + 3 + \cancel{2\sqrt{3}} + 1}$

$R = \sqrt{8}$

C3, IYGB, PAPER O

$$\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\alpha = 75°$$

$$\therefore f(\theta) = \sqrt{8}\sin(2\theta + 75°)$$

b) $f(\theta) = 2$

$$\sqrt{8}\sin(2\theta + 75°) = 2$$

$$\sin(2\theta + 75) = \frac{\sqrt{2}}{2}$$

$$\arcsin\left(\frac{\sqrt{2}}{2}\right) = 45$$

$$\Rightarrow \begin{cases} 2\theta + 75° = 45 \pm 360n \\ 2\theta + 75° = 135 \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \begin{cases} 2\theta = -30 \pm 360n \\ 2\theta = 60° \pm 360n \end{cases}$$

$$\Rightarrow \begin{cases} \theta = -15° \pm 180n \\ \theta = 30° \pm 180n \end{cases}$$

$$\therefore \theta = 165°, 345°, 30°, 210°$$

9. a) $T = 95 - 75e^{-t}$

$$\Rightarrow 85 = 95 - 75e^{-t}$$

$$\Rightarrow 75e^{-t} = 10$$

$$\Rightarrow e^{-t} = \frac{10}{75}$$

$$\Rightarrow e^{-t} = \frac{2}{15}$$

$$\Rightarrow e^{t} = \frac{15}{2}$$

$$\Rightarrow t = \ln(7.5)$$

$$\Rightarrow t \simeq 2.01$$

b) $\frac{dT}{dt} = 75e^{-t}$

$$\left.\frac{dT}{dt}\right|_{t=0} = 75e^{0} = 75$$

c) WHEN $t=0$, $T=85$ (IMPLIED FROM TEXT)

$$85 = 15 + Ae^{-k \times 0}$$
$$85 = 15 + Ae^{0}$$
$$85 = 15 + A$$
$$A = 70$$

d) AS $t \to \infty$ $\quad e^{-kt} \to 0$

$$Ae^{-kt} \to 0 \quad \text{OR} \quad 70e^{-kt} \to 0$$
$$T \to 15$$

$$\therefore T_0 = 15$$