1. BY LONG DIVISION

$$\begin{array}{r|l} & 4x^2+1 \\ x^2+x-6 & 4x^4+4x^3-23x^2+0x-4 \\ & -4x^4-4x^3+24x^2 \\ \hline & x^2+0x-4 \\ & -x^2-x+6 \\ \hline & -x+2 \end{array}$$

$$\therefore \frac{4x^4+4x^3-23x^2-4}{x^2+x-6} = 4x^2+1+\frac{-x+2}{x^2+x-6} = 4x^2+1+\frac{-x+2}{(x-2)(x+3)}$$

$$= 4x^2+1-\frac{\cancel{x-2}}{\cancel{(x-2)}(x+3)} = 4x^2+1-\frac{1}{x+3}$$

2. a)

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

b)

$$2\tan^2 x + \sec^2 x = 5\sec x$$

$$2(\sec^2 x - 1) + \sec^2 x = 5\sec x$$

$$2\sec^2 x - 2 + \sec^2 x - 5\sec x = 0$$

$$3\sec^2 x - 5\sec x - 2 = 0$$

$$(3\sec x + 1)(\sec x - 2)$$

$$\sec x = \begin{cases} -1/3 \\ 2 \end{cases}$$

$$\cos x = \begin{cases} \cancel{-3} \\ \frac{1}{2} \end{cases}$$

$$\arccos\left(\tfrac{1}{2}\right) = 60°.$$

$$\left(\begin{array}{l} x = 60 \pm 360n \\ x = 300 \pm 360n \end{array} \right. \quad n = 0,1,2,3\ldots$$

$$x_1 = 60°$$

$$x_2 = 300°$$

3. $$y = \frac{8x^2+8x+3}{(2x+1)^2}$$

BY THE QUOTIENT RULE

$$\frac{dy}{dx} = \frac{(2x+1)^2(16x+8)-(8x^2+8x+3)\times 2(2x+1)\times 2}{(2x+1)^4}$$

$$\frac{dy}{dx} = \frac{(2x+1)^{\cancel{2}\,1}(16x+8)-4\cancel{(2x+1)}(8x^2+8x+3)}{(2x+1)^{\cancel{4}\,3}}$$

$$\frac{dx}{dx} = \frac{(2x+1)(16x+8)-4(8x^2+8x+3)}{(2x+1)^3}$$

$$\frac{dy}{dx} = \frac{\cancel{32x^2}+\cancel{16x}+\cancel{16x}+8-\cancel{32x^2}-\cancel{32x}-12}{(2x+1)^3} = \frac{-4}{(2x+1)^3}$$

AS REQUIRED

ALTERNATIVE

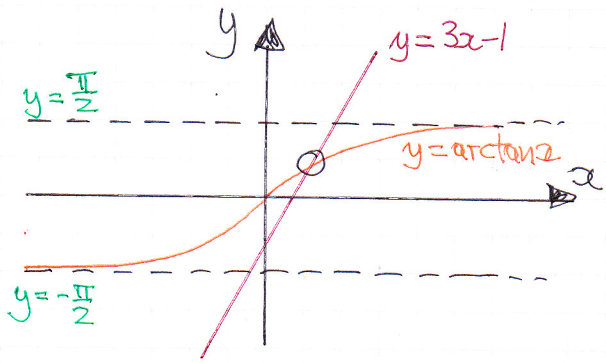
$$y = \frac{8x^2+8x+3}{(2x+1)^2} = \frac{2(4x^2+4x+1)+1}{(2x+1)^2} = \frac{2(2x+1)^2+1}{(2x+1)^2}$$

$$\therefore y = 2 + \frac{1}{(2x+1)^2} = 2 + (2x+1)^{-2}$$

$$\frac{dy}{dy} = 0 - 2(2x+1)^{-3}\times 2 = -4(2x+1)^{-3} = -\frac{4}{(2x+1)^3}$$

AS BEFORE

4. a) b)



- $3x - \arctan x = 1$

$3x - 1 = \arctan x$

DRAWN IN

ONE INTERSECTION FOR $x>0$, SO ONE POSITIVE REAL ROOT

C3, IYGB, PAPER P

c) $3x - \arctan x = 1$

$3x - 1 - \arctan x = 0$

$f(x) = 3x - 1 - \arctan x$

$f(0.45) = -0.073 < 0$

$f(0.5) = 0.036 > 0$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN BETWEEN 0.45 & 0.5, THERE MUST BE A ROOT BETWEEN 0.45 & 0.5

d) $x_{n+1} = \frac{1}{3}(1 + \arctan x_n)$

$x_0 = 0.475$

$x_1 = 0.481$

$x_2 = 0.483$

$x_3 = 0.483$

5. a) $y = e^{2x} - 4e^{x} - 16x$

$\frac{dy}{dx} = 2e^{2x} - 4e^{x} - 16$

SOLVE FOR ZERO

$0 = 2e^{2x} - 4e^{x} - 16$

$0 = e^{2x} - 2e^{x} - 8$

$e^{2x} - 2e^{x} - 8 = 0$

AS REQUIRED

b) $e^{2x} - 2e^{x} - 8 = 0$

$(e^{x} - 4)(e^{x} + 2) = 0$

$e^{x} = 4, \cancel{-2}$

$x = \ln 4 = 2\ln 2$

$y = e^{2\ln 4} - 4e^{\ln 4} - 16\ln 4$

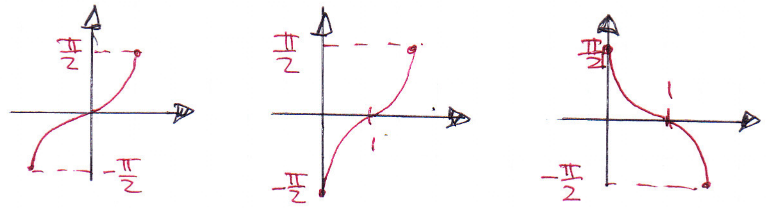
$y = e^{\ln 16} - 4\times 4 - 16(2\ln 2)$

$y = 16 - 16 - 32\ln 2$

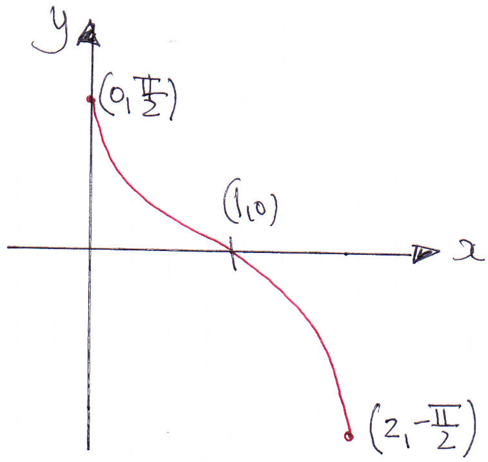
$y = -32\ln 2$

$\therefore (2\ln 2, -32\ln 2)$

6. a) • TRANSLATION, TO THE "RIGHT" BY 1 UNIT
• REFLECTION IN THE x AXIS $(y=0)$) EITHER ORDER

b) 

$\arcsin x$ $\qquad$ $\arcsin(x-1)$ $\qquad$ $-\arcsin(x-1)$



7. $\boxed{x = y^2 \ln y}$

$$\Rightarrow \frac{dx}{dy} = 2y\ln y + y^2\left(\frac{1}{y}\right)$$

$$\Rightarrow \frac{dx}{dy} = 2y\ln y + y$$

$$\Rightarrow \frac{dx}{dy} = y(2\ln y + 1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{y(2\ln y+1)}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{y=e} = \frac{1}{e(2+1)} = \frac{1}{3e}$$

SO "NORMAL" GRADIENT IS $-3e$

WHEN $y=e$ $\quad x = e^2\ln e = e^2$

$\therefore$ (e^2, e) GRADIENT $-3e$

$$y - y_0 = m(x - x_0)$$

$$y - e = -3e(x - e^2)$$

$$y - e = -3ex + 3e^3$$

$$y + 3ex = 3e^3 + e$$

$$y + 3ex = e(3e^2+1)$$

AS REQUIRED

8. a) $f(y) = 6 + 3\cos y + 4\sin y$

$$3\cos y + 4\sin y \equiv a\cos(y-b)$$
$$3\cos y + 4\sin y \equiv a\cos y\cos b + a\sin y\sin b$$
$$3\cos y + 4\sin y \equiv (a\cos b)\cos y + (a\sin b)\sin y$$

$\left.\begin{array}{l} a\cos b = 3 \\ a\sin b = 4 \end{array}\right\} \Longrightarrow$ SQUARE & ADD $a = \sqrt{3^2+4^2} = 5$

DIVIDING $\tan b = \frac{4}{3}$

$b \simeq 0.9273^c$

$\therefore\ 3\cos y + 4\sin y \approx 5\cos(y - 0.9273^c)$

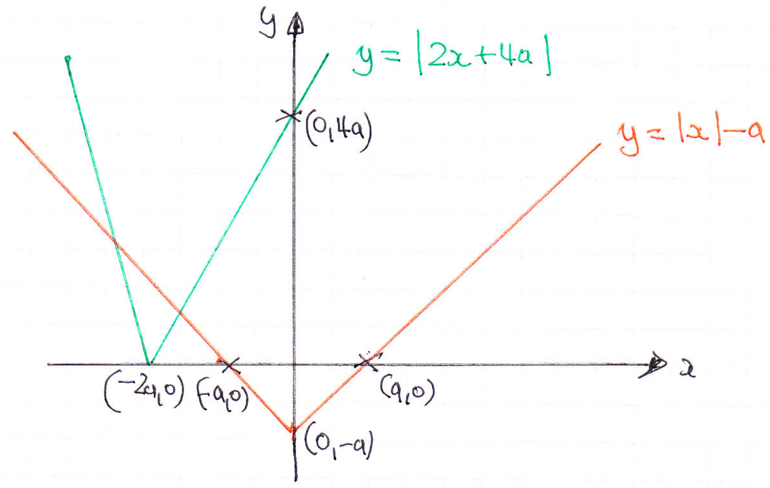
b)

$$-5 \leq 5\cos(y-0.9273^c) \leq 5$$
$$1 \leq 6 + 5\cos(y-0.9273^c) \leq 11$$
$$1 \leq f(y) \leq 11$$
$1 \leq f(2y) \leq 11$ (DOESN'T AFFECT THE "HEIGHT")
$$2 \leq 2f(2y) \leq 22$$

$\therefore\ A = 2$
$B = 22$

(P.T.O)

9. a)



b) LOOKING AT THE ABOVE GRAPHS WITH $a=3$

$|x|-3 = |2x+12|$

- $-x-3 = 2x+12$
  $-15 = 3x$
  $x = -5$

- $-x-3 = -2x-12$
  $x = -9$

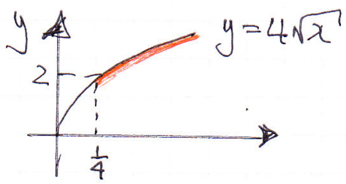
$\therefore\ x = \begin{cases} -9 \\ -5 \end{cases}$

10. a) $f(g(x)) = f(\ln 4x) = 2e^{\frac{1}{2}\ln 4x} = 2e^{\ln\sqrt{4x}} = 2\times\sqrt{4x}$

$= 2\times 2\sqrt{x} = 4\sqrt{x}$

b) $x>\frac{1}{4} \longrightarrow \boxed{g(x)} \longrightarrow \cdots\cdots \ x\in\mathbb{R} \longrightarrow \boxed{f(x)} \longrightarrow$ $\quad\therefore$ DOMAIN $x>\frac{1}{4}$

RANGE OF $f(g(x)) = 4\sqrt{x} \quad x>\frac{1}{4}$



$\therefore$ RANGE $f(g(x)) > 2$

c) $4\sqrt{x} = 3x+1$

$0 = 3x - 4\sqrt{x} + 1$

$0 = 3(\sqrt{x})^2 - 4(\sqrt{x}) + 1$ $\quad$ i.e. $3a^2 - 4a + 1 = 0$

$0 = (3\sqrt{x}-1)(\sqrt{x}-1) = 0$

$\sqrt{x} = \begin{cases} \frac{1}{3} \\ 1 \end{cases}$

$x = \begin{cases} \cancel{\frac{1}{9}} \\ 1 \end{cases}$ $\quad$ DOMAIN OF $f(g(x))$, IS $x > \frac{1}{4}$