IYGB GCE

Core Mathematics C3

Advanced

Practice Paper T

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x)=\frac{e^x-1}{e^x+1},\ x\in\mathbb{R}.$$

a) Show clearly that …

i. … $f(-x)=-f(x)$. **(3)**

ii. … $f'(x)=\dfrac{2e^x}{\left(e^x+1\right)^2}$. **(3)**

b) Explain how the results of part **(a)** show that $f^{-1}(x)$ exists. **(1)**

c) Find an expression for $f^{-1}(x)$. **(3)**

The function $g(x)$ is defined in a suitable domain, so that

$$fg(x)=\frac{x^2+6x+8}{x^2+6x+10}.$$

d) Determine the equation of $g(x)$, in its simplest form. **(4)**

Question 2

Prove, by showing full workings that …

i. … $\dfrac{d}{dx}\left(\dfrac{x-4}{\sqrt{x}+2}\right)=\dfrac{1}{2\sqrt{x}}$ **(4)**

ii. … $\dfrac{d}{dx}\left(\dfrac{4x-8\sqrt{x}+3}{\left(\sqrt{x}-1\right)^2}\right)=\dfrac{1}{\sqrt{x}\left(\sqrt{x}-1\right)^3}$. **(6)**

Question 3

Show by a clear method that

$$4\operatorname{arccot}2+\arctan\left(\frac{24}{7}\right)=\pi\,. \qquad (\mathbf{9})$$

Question 4

Solve the following modulus inequality

$$3|x+1|-|x-4|\le 11,\ x\in\mathbb{R}\,. \qquad (\mathbf{8})$$

Question 5 (*****)

A function f is defined as

$$f(x)=\mathrm{e}^{ax}+b,\quad -\ln 2\le x\le \ln 2,$$

where a and b are positive constants.

It is given that $f\left(\ln\left(\frac{3}{2}\right)\right)=\frac{13}{4}$.

a) Show clearly that

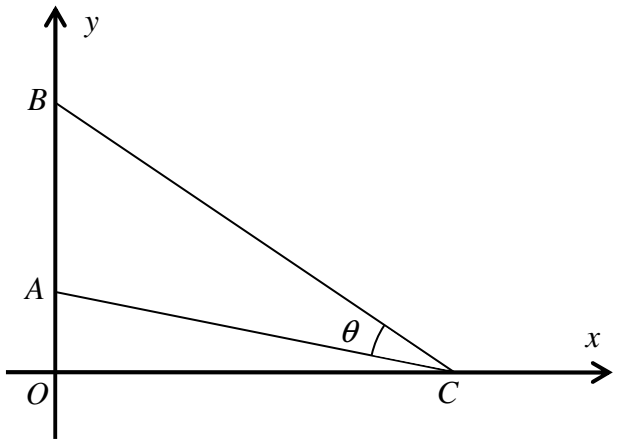
$$b=\frac{13}{4}-\left(\frac{3}{2}\right)^{a}. \qquad (\mathbf{2})$$

It is further given that

$$f\left(\ln\left(\frac{2}{3}\right)\right)=\frac{13}{9}.$$

b) Find the value of a and the value of b. $(\mathbf{10})$

Question 6



The figure above shows the triangle ABC, where $\measuredangle ACB = \theta$.

The points A and B have respective coordinates $(0,1)$ and $(0,3)$, while the **variable** point $C(x,0)$ lies on the positive x axis.

Show that as C varies, the maximum value of θ is $\dfrac{\pi}{6}$. **(11)**

Question 7

It is given that if $x \neq y$, $x \neq 0$, $y \neq 0$,

$$\tan(x+y) = 2\tan(x-y).$$

Show by detailed workings that

$$\frac{\sin 2x}{\sin 2y} = 3. \qquad \textbf{(11)}$$
