IYGB GCE

Core Mathematics C3

Advanced

Practice Paper U

Difficulty Rating: 3.8267/1.8405

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Prove the validity of the following trigonometric identity

$$\cot x - \tan x \equiv 2\cot 2x \,. \qquad (\mathbf{5})$$

Question 2

Given that

$$y = \left(2 + \mathrm{e}^{3x}\right)^{\frac{3}{2}}$$

find the value of $\dfrac{dy}{dx}$ at $x = \dfrac{1}{3}\ln 2$. $(\mathbf{4})$

Question 3

$$f(x) = \mathrm{e}^{-2x} + \frac{\ln 2}{x},\ x \in \mathbb{R},\ x > \ln 4 \,.$$

a) Show that $f(x)$ in a decreasing function. $(\mathbf{4})$

b) Find the range of $f(x)$ in its simplest form. $(\mathbf{4})$

Question 4

$$y = \arcsin x\,,\ -1 \le x \le 1\,.$$

a) By writing $y = \arcsin x$ as $x = \sin y$ show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}\,. \qquad (\mathbf{5})$$

The curve C has equation

$$y = 3\arcsin x - 4x^{\frac{3}{2}} + 5\,,\ 0 \le x \le 1\,.$$

b) Show that the x coordinates of the stationary points of C are the solutions of the equation

$$4x^3 - 4x + 1 = 0\,. \qquad (\mathbf{4})$$

c) Show further that the above equation has a root α in the interval $(0, 0.5)$. $(\mathbf{2})$

The root α can be found by using the iterative formula

$$x_{n+1} = x_n^3 + \frac{1}{4}\,,\ \text{with}\ x_0 = 0.5\,.$$

d) Find, correct to 3 decimal places, the value of x_1, x_2, x_3 and x_4. $(\mathbf{2})$

e) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show that $\alpha = 0.2696$, correct to 4 decimal places. $(\mathbf{3})$

Question 5

The function f is defined by

$$f(x)=\frac{x^2-4}{|x|+2},\quad x\in\mathbb{R}.$$

a) Show that $f(x)$ is even. **(2)**

b) Solve the equation

$$f(x)=-\frac{1}{2}.$$ **(6)**

Question 6

It is given that

$$4\text{cosec}^2 2\theta-\sec^2\theta\equiv\text{cosec}^2\theta,\quad \theta\neq\frac{n\pi}{2},\ n\in\mathbb{Z}.$$

a) Prove the validity of the above trigonometric identity. **(5)**

b) Hence show that if

$$4\left(\text{cosec}^2 2\theta-2\right)=\sec^2\theta-2\,\text{cosec}\,\theta,$$

then either $\sin\theta=\frac{1}{2}$ or $\sin\theta=-\frac{1}{4}$. **(5)**

Question 7

It is given that $\arcsin x=\arccos y$.

Show, by a clear method, that

$$x^2+y^2=1.$$ **(4)**

Question 8

A curve has equation

$$y = \mathrm{e}^{2x}\left(2\cos 3x - \sin 3x\right).$$

Show that

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0. \qquad \textbf{(8)}$$

Question 9

The amount X milligrams, of an anaesthetic drug in the bloodstream of a patient, is given by

$$X = D\mathrm{e}^{-0.2t}, \ t \geq 0,$$

where D is the dose, in milligrams, of the anaesthetic administered and t is the time in hours since the dose was administered.

A patient undergoing an operation is given an initial dose of 20 milligrams.

This patient will remain asleep if there are more than 12 milligrams of anaesthetic in his bloodstream.

a) Show that one hour later $X = 16.37$, correct to two decimal places. **(2)**

b) Show, by calculation, that two hours after the initial dose was administered, the patient should still be asleep. **(1)**

Two hours after the initial dose was administered a further dose of **10** milligrams is given to the patient.

c) Find the amount of the anaesthetic in the patient's bloodstream one hour after the second dose is given. **(3)**

No more anaesthetic is given and the operation lasts for 4 hours.

d) Show by solving a relevant equation that the patient should "wake up" approximately 80 minutes after the end of his operation. **(6)**
