C4, IYGB, PAPER B

1. a) $$\int_0^2 \frac{1}{\sqrt{4x+1}}\,dx = \int_0^2 (4x+1)^{-\frac{1}{2}}\,dx = \left[\frac{1}{2}(4x+1)^{\frac{1}{2}}\right]_0^2$$

$$= \left[\frac{1}{2}\sqrt{4x+1}\right]_0^2 = \frac{3}{2} - \frac{1}{2} = 1$$

b) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos 3x\,dx = \left[\frac{1}{3}\sin 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \cancel{\frac{1}{3}\sin\left(3\times\frac{\pi}{3}\right)} - \frac{1}{3}\sin\left(3\times\frac{\pi}{6}\right)$$

$$= -\frac{1}{3}$$

2. a) $$\frac{5x+3}{(1-x)(1+3x)} \equiv \frac{A}{1-x} + \frac{B}{1+3x}$$

$$5x+3 \equiv A(1+3x) + B(1-x)$$

If $x=1 \Rightarrow 8 = 4A \Rightarrow A = 2$

If $x=0 \Rightarrow 3 = 4 + B \Rightarrow B = 1$

$$\therefore f(x) = \frac{2}{1-x} + \frac{1}{1+3x}$$

b) • $$\frac{2}{1-x} = 2(1-x)^{-1} = 2\left[1 + \frac{-1}{1}(-x)^1 + \frac{-1(-2)}{1\times2}(-x)^2 + \frac{-1(-2)(-3)}{1\times2\times3}(-x)^3 + O(x^4)\right]$$

$$= 2\left[1 + x + x^2 + x^3 + O(x^4)\right]$$

$$= 2 + 2x + 2x^2 + 2x^3 + O(x^4)$$

• $$\frac{1}{1+3x} = (1+3x)^{-1} = 1 + \frac{-1}{1}(3x)^1 + \frac{-1(-2)}{1\times2}(3x)^2 + \frac{-1(-2)(-3)}{1\times2\times3}(3x)^3 + O(x^4)$$

$$= 1 - 3x + 9x^2 - 27x^3 + O(x^4)$$

$$\therefore f(x) = \begin{array}{l} 2 + 2x + 2x^2 + 2x^3 + O(x^4) \\ \underline{1 - 3x + 9x^2 - 27x^3 + O(x^4)} \end{array}$$

$$= 3 - x + 11x^2 - 25x^3 + O(x^4)$$

3. a) $y^2 - 3xy + 4x^2 = 28$

$$\Rightarrow \frac{d}{dx}(y^2) - \frac{d}{dx}(3xy) + \frac{d}{dx}(4x^2) = \frac{d}{dx}(28)$$

$$\Rightarrow 2y\frac{dy}{dx} - \left[3y + 3x\frac{dy}{dx}\right] + 8x = 0$$

$$\Rightarrow 2y\frac{dy}{dx} - 3y - 3x\frac{dy}{dx} + 8x = 0$$

$$\Rightarrow (2y - 3x)\frac{dy}{dx} = 3y - 8x$$

$$\Rightarrow \frac{dy}{dx} = \frac{3y-8x}{2y-3x}$$

b) $\frac{dy}{dx} = 0$

$$\frac{3y-8x}{2y-3x} = 0$$

$$3y - 8x = 0$$

$$3y = 8x$$

$$\boxed{y = \frac{8}{3}x}$$

SUB INTO THE EQUATION OF THE CURVE

$$\Rightarrow \left(\frac{8}{3}x\right)^2 - 3x\left(\frac{8}{3}x\right) + 4x^2 = 28$$

$$\Rightarrow \frac{64}{9}x^2 - 8x^2 + 4x^2 = 28$$

$$\Rightarrow \frac{28}{9}x^2 = 28$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \begin{cases} 3 \\ -3 \end{cases} \qquad y = \begin{cases} \frac{8}{3} \times 3 = 8 \\ \frac{8}{3}(-3) = -8 \end{cases}$$

$\therefore (3, 8) \quad (-3, -8)$

4. $\int_0^{\frac{\pi}{2}} 4x^2 \cos x \, dx$ = IGNORING LIMITS ....

$4x^2$	$8x$
$\sin x$	$\cos x$

... $4x^2 \sin x - \int 8x \sin x \, dx$

BY PARTS AGAIN

$8x$	8
$-\cos x$	$\sin x$

$$= 4x^2 \sin x - \left[-8x\cos x - \int -8\cos x \, dx\right]$$

$$= 4x^2 \sin x + 8x\cos x - \int 8\cos x \, dx$$

$$= 4x^2 \sin x + 8x\cos x - 8\sin x + C$$

C4, IYGB, PAPER B

REINTRODUCE LIMITS $\int_0^{\frac{\pi}{2}} 4x^2\cos x\,dx = \left[4x^2\sin x + 8x\cos x - 8\sin x\right]_0^{\frac{\pi}{2}}$

$= \left[4\left(\frac{\pi}{2}\right)^2\sin\frac{\pi}{2} + \cancel{8\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}} - 8\sin\frac{\pi}{2}\right] - \left[0 + 0 - \cancel{8\sin 0}\right]$

$= 4\times\frac{\pi^2}{4}\times 1 - 8 = \pi^2 - 8$

5. a) $p = (0,-7,4)$
$q = (3,-8,2)$

$\overrightarrow{PQ} = q - p = (3,-8,2) - (0,-7,4)$
$= (3,-1,-2)$

$\therefore\ \underline{r} = (0,-7,4) + \lambda(3,-1,-2)$
$\underline{r} = (3\lambda, -\lambda-7, 4-2\lambda)$

b) $\underline{r}_2 = (7,a,b) + \mu(1,4,-1) = (\mu+7, 4\mu+a, b-\mu)$

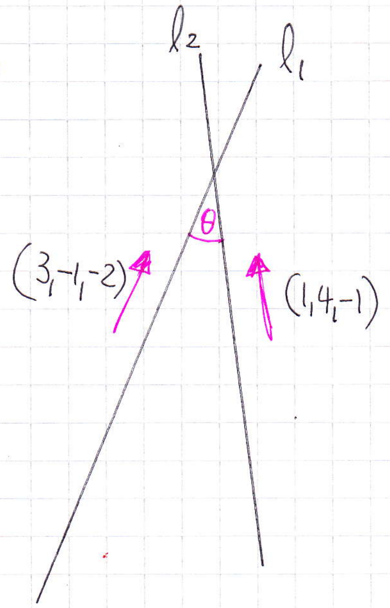
THE POINT Q LIES ON l_2 (THAT'S THE ONLY THING THAT MATTERS!)

$\therefore\ \underline{i}:\ \mu+7=3 \implies \boxed{\mu=-4}$

$\underline{j}:\ 4\mu+a=-8 \implies 4(-4)+a=-8$
$a = 8$

$\underline{k}:\ b-\mu=2 \implies b-(-4)=2$
$b=-2$

c)



DOTTING DIRECTION VECTORS

$(3,-1,-2)\cdot(1,4,-1) = |3,-1,-2||1,4,-1|\cos\theta$

$3-4+2 = \sqrt{9+1+4}\sqrt{1+16+1}\cos\theta$

$1 = \sqrt{14}\sqrt{18}\cos\theta$

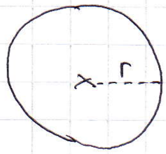
$\cos\theta = \frac{1}{\sqrt{14\times18}}$

$\theta \simeq 86.4^\circ$

$(\simeq 1.51^c)$

6.

$$\boxed{\frac{dA}{dt} = 12}$$



$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dr}{dA} = \frac{1}{2\pi r}$$

$$576\pi = \pi r^2$$

$$576 = r^2$$

$$r = 24$$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$$

$$\frac{dr}{dt} = \frac{1}{2\pi r} \times 12$$

$$\frac{dr}{dt} = \frac{6}{\pi r}$$

$$\left.\frac{dr}{dt}\right|_{A=576\pi} = \left.\frac{dr}{dt}\right|_{r=24} = \frac{6}{\pi \times 24}$$

$$= \frac{1}{4\pi} \approx 0.0796$$

7.

$$-5\frac{dy}{dx} = 2y - 150$$

$$\Rightarrow -5\,dy = (2y-150)\,dx$$

$$\Rightarrow \frac{-5}{2y-150}\,dy = 1\,dx$$

$$\Rightarrow \int \frac{-5}{2y-150}\,dy = \int 1\,dx$$

$$\Rightarrow -\frac{5}{2}\ln|2y-150| = x + C$$

$$\Rightarrow \ln|2y-150| = -\frac{2}{5}x + C$$

$$\Rightarrow 2y - 150 = e^{-\frac{2}{5}x + C}$$

$$\Rightarrow 2y - 150 = e^{-\frac{2}{5}x} \times e^{C}$$

$$\Rightarrow \boxed{2y - 150 = Ae^{-\frac{2}{5}x}}$$

$$(A = e^{C})$$

• APPLY CONDITION

$x=0,\ y=275$

$$2 \times 275 - 150 = Ae^{0}$$

$$\boxed{400 = A}$$

$$2y - 150 = 400e^{-\frac{2}{5}x}$$

$$2y = 150 + 400e^{-\frac{2}{5}x}$$

$$y = 75 + 200e^{-\frac{2}{5}x}$$

8. a)

x	0	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$
y	0	0.1632	0.2620	0.2500

b) $\displaystyle\int_0^{\frac{\pi}{6}} \sin x \cos 2x \, dx \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$

$$\approx \frac{\pi/18}{2}\left[0 + 0.2500 + 2(0.1632 + 0.2620)\right]$$

$$\approx 0.096$$

c) $\displaystyle\int_0^{\frac{\pi}{6}} \sin x \cos 2x \, dx$ ... SUBSTITUTION ...

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$x = 0 \quad u = 1$

$x = \frac{\pi}{6} \quad u = \frac{\sqrt{3}}{2}$

$$= \int_1^{\frac{\sqrt{3}}{2}} \cancel{\sin x} \cos 2x \frac{du}{-\cancel{\sin x}}$$

$$= \int_1^{\frac{\sqrt{3}}{2}} -\cos 2x \, du = \int_{\frac{\sqrt{3}}{2}}^1 \cos 2x \, du$$

$$= \int_{\frac{\sqrt{3}}{2}}^1 2\cos^2 x - 1 \, du = \int_{\frac{\sqrt{3}}{2}}^1 2u^2 - 1 \, du$$

$$= \left[\frac{2}{3}u^3 - u\right]_{\frac{\sqrt{3}}{2}}^1 = \left(\frac{2}{3} - 1\right) - \left[\frac{2}{3}\times\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{\sqrt{3}}{2}\right]$$

$$= \left(\frac{2}{3} - 1\right) - \left[\frac{2}{3}\times\frac{3\sqrt{3}}{8} - \frac{\sqrt{3}}{2}\right]$$

$$= -\frac{1}{3} - \left(\frac{1}{4}\sqrt{3} - \frac{1}{2}\sqrt{3}\right)$$

$$= -\frac{1}{3} - \left(-\frac{1}{4}\sqrt{3}\right)$$

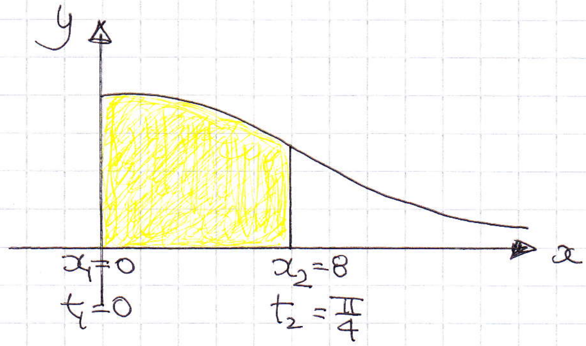
$$= \frac{1}{4}\sqrt{3} - \frac{1}{3}$$

$$\approx 0.099679\ldots$$

NOTE

$\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}\times\sqrt{3}\times\sqrt{3}}{2\times2\times2} = \frac{3\sqrt{3}}{8}$

9. a)



$x = 8\tan t$
$y = \cos^2 t$
$0 \le t < \frac{\pi}{2}$

$$V = \pi \int_{x_1}^{x_2} [y(x)]^2 \, dx = \pi \int_{t_1}^{t_2} [y(t)]^2 \frac{dx}{dt} \, dt$$

$$V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 t)^2 (8\sec^2 t) \, dt$$

$$V = \pi \int_0^{\frac{\pi}{4}} 8\cos^4 t \sec^2 t \, dt$$

$$V = 8\pi \int_0^{\frac{\pi}{4}} \cos^4 t \times \frac{1}{\cos^2 t} \, dt$$

$$V = 8\pi \int_0^{\frac{\pi}{4}} \cos^2 t \, dt$$ // AS REQUIRED

$\boxed{x = 8\tan t}$
- $0 = 8\tan t$
  $\tan t = 0$
  $t = 0$
- $8 = 8\tan t$
  $\tan t = 1$
  $t = \frac{\pi}{4}$

$\therefore t_1 = 0$
$t_2 = \frac{\pi}{4}$

b)

$$V = 8\pi \int_0^{\frac{\pi}{4}} \cos^2 t \, dt = 8\pi \int_0^{\frac{\pi}{4}} \frac{1}{2} + \frac{1}{2}\cos 2t \, dt$$

$$= 8\pi \left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_0^{\frac{\pi}{4}} = 8\pi \left[\left[\frac{1}{2} \times \frac{\pi}{4} + \frac{1}{4}\sin\left(2 \times \frac{\pi}{4}\right)\right] - \left[0 + \cancel{\frac{1}{4}\sin 0}\right]\right]$$

$$= 8\pi \left[\frac{\pi}{8} + \frac{1}{4}\sin\frac{\pi}{2}\right]$$

$$= \pi^2 + 2\pi$$ //