IYGB GCE

Core Mathematics C4

Advanced

Practice Paper D

Difficulty Rating: 3.4533 /1.5707

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet “Mathematical Formulae and Statistical Tables” may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^3 \frac{4}{2x+3}\,dx$. **(2)**

b) $\displaystyle\int_0^{\frac{\pi}{6}} \sin\left(4x+\frac{\pi}{6}\right)\,dx$. **(2)**

Question 2

$$\frac{16}{(1-x)(2-x)^2} \equiv \frac{A}{1-x} + \frac{B}{(2-x)^2} + \frac{C}{2-x}.$$

a) Find the value of each of the constants A, B and C. **(4)**

b) Hence show that if x is sufficiently small

$$\frac{16}{(1-x)(2-x)^2} \approx 4+8x+11x^2 .$$ **(5)**

Question 3

Relative to a fixed origin O, the straight lines L and M have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} + s \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix},$$

where t and s are scalar parameters.

a) Show that L and M intersect at some point A and find its coordinates. **(6)**

b) Find the size of the acute angle θ, formed by L and M. **(3)**

The points B and C lie on the L where $t = 3$ and $t = 6$, respectively.

c) State the ratio $AB : BC$. **(2)**

Question 4

$$I = \int (x-1)(4-x)^{\frac{1}{2}} \, dx, \ x \in \mathbb{R}, \ x \leq 4.$$

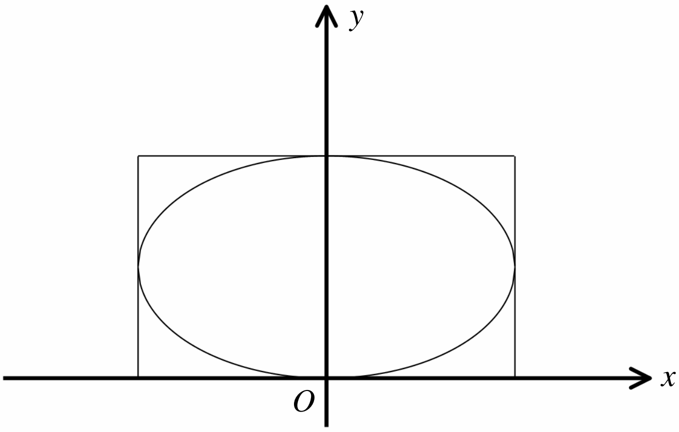
a) Use the substitution $u = (4-x)^{\frac{1}{2}}$ to find an expression for I. **(5)**

b) Show that the answer of part **(a)** can be written as

$$I = -\frac{2}{5}(x+1)(4-x)^{\frac{3}{2}} + C.$$ **(2)**

c) Use integration by parts to verify the answer of part **(b)**. **(4)**

Question 5



The figure above shows the curve with equation

$$x^2 - 8y + 4y^2 = 0.$$

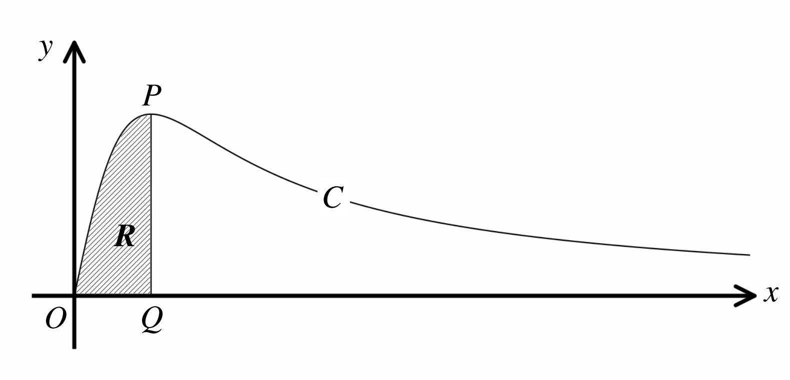
a) Show that

$$\frac{dy}{dx} = \frac{x}{4(1-y)}. \qquad \mathbf{(3)}$$

The curve fits perfectly inside a rectangle whose sides are parallel to the coordinate axes, so they are tangents to the curve.

b) Show further that the area of the rectangle is 8 square units. **(6)**

Question 6



The figure above shows the curve C with parametric equations

$$x = 6\tan\theta, \quad y = \sin 2\theta, \quad 0 \le \theta < \frac{\pi}{2}.$$

The curve has a single stationary point at P.

a) Find the coordinates of P. **(5)**

The point Q lies on the x axis so that PQ is parallel to the y axis. The finite region R is bounded by C, the x axis and the straight line segment PQ. The region R is revolved in the x axis by 2π radians to form a solid of revolution S.

b) Show the volume of S is given by the integral

$$\pi \int_0^{\frac{\pi}{4}} 24\sin^2\theta \ d\theta.$$ **(3)**

c) Hence find an exact value for the volume of S. **(4)**

Question 7

At time t seconds, a spherical balloon has radius r cm and volume V cm^3.

Air is pumped into the balloon so that its volume is increasing at a rate inversely proportional to its volume at that time.

a) Show clearly that

$$\frac{dr}{dt} = \frac{A}{r^5},$$

where A is a positive constant. **(5)**

The initial radius of the balloon is 2 cm and when $t = 1$ it has increased to 3 cm.

b) Show further that

$$r^6 = 665t + 64.$$ **(5)**

c) Find the value of r when $t = 6$. **(1)**

[volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$]

Question 8

Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_0^1 \frac{8}{\left(1+x^2\right)^2}\,dx.$$ **(8)**
