IYGB GCE

Core Mathematics C4

Advanced

Practice Paper I

Difficulty Rating: 3.2200/1.4388

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

By using the substitution $u = 4 + 3x^2$, or otherwise, find

$$\int \frac{2x}{\left(4+3x^2\right)^2}\, dx . \qquad (5)$$

Question 2

$$f(x) = \frac{8x}{\sqrt{4-x}} .$$

Show that if x is small, then

$$f(x) \approx 4x + \frac{1}{2}x^2 + \frac{3}{32}x^3 . \qquad (7)$$

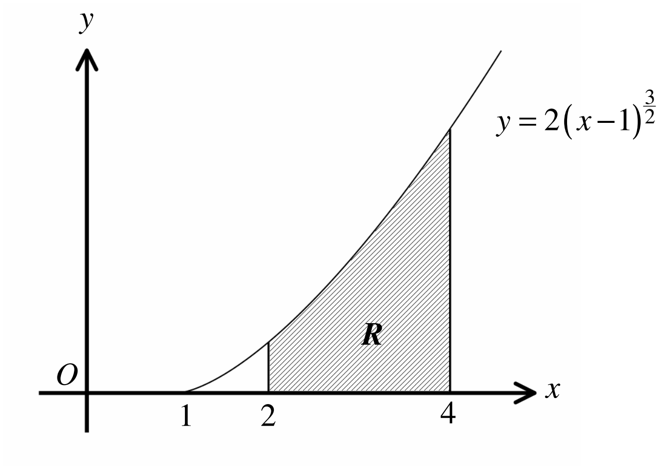
Question 3

$$x = 4\sin\theta + 7\cos\theta .$$

The value of θ is increasing at the constant rate of 0.5, in suitable units.

Find the rate at which x is changing, when $\theta = \frac{\pi}{2}$. **(5)**

Question 4



The figure above shows part of the curve with equation

$$y = 2(x-1)^{\frac{3}{2}}.$$

The shaded region, labelled as R, bounded by the curve, the x axis and the straight lines with equations $x = 2$ and $x = 4$.

This region is rotated by 2π radians in the x axis, to form a solid of revolution S.

Show that the volume of S is 80π. **(5)**

Question 5

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i} + 2\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_2 = 2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 **do not** intersect. **(4)**

The point P lies on l_1 where $\lambda = 4$ and the point Q lies on l_2 where $\mu = -1$.

b) Find the acute angle between PQ and l_1. **(5)**

Question 6

A curve C is defined implicitly by

$$6^x + 6xy + y^2 = 9 .$$

a) Show clearly that **(5)**

$$\frac{dy}{dx} = -\frac{6y + 6^x \ln 6}{6x + 2y} .$$

b) Find the gradient at each of the two points on C, where $x = 2$.

Give the answers in the form $a + b\ln 6$, where a and b are integers. **(4)**

Question 7

Solve the differential equation

$$(2x-3)(x-1)\frac{dy}{dx} = y(2x-1), \quad y \neq 0 ,$$

subject to the condition $y = 1$ at $x = 2$, giving the answer in the form $y = f(x)$. **(10)**

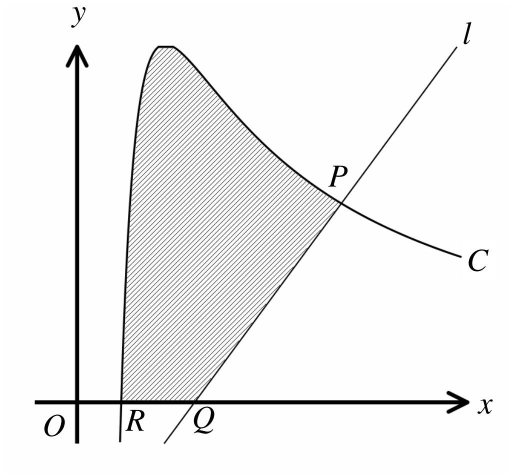
Question 8

Use appropriate integration techniques to show that

$$\int_0^{\frac{1}{4}\pi^2} \sin\sqrt{x} \;\; dx = N ,$$

where N is a positive integer . **(9)**

Question 9



The figure above shows part of the curve C with parametric equations

$$x = \frac{6}{t}, \quad y = 6t - t^2 \ , \quad t \neq 0 .$$

The curve crosses the x axis at the point R.

The point $P(6,5)$ lies on C and the straight line l is the normal to C at P.

This normal crosses the x axis at the point Q.

a) Determine …

i. … the value of t at the points R and P. **(3)**

ii. … an equation for l. **(5)**

iii. … the coordinates of R and Q. **(3)**

The finite region bounded by C, l and the x axis is shown shaded in the figure above.

b) Use parametric integration to find, correct to two decimal places, the area of this region. **(5)**