1. EITHER $\int \frac{2x}{(4+3x^2)^2}\,dx = \int 2x(4+3x^2)^{-2}\,dx = \ldots$ BY REVERSE CHAIN RULE

$$= -\frac{2}{6}(4+3x^2)^{-1} + C = -\frac{1}{3(4+3x^2)} + C$$

OR BY SUBSTITUTION

$$u = 4+3x^2$$
$$\frac{du}{dx} = 6x$$
$$dx = \frac{du}{6x}$$

$$\int \frac{2x}{(4+3x^2)^2}\,dx = \int \frac{\cancel{2x}}{u^2}\cdot\frac{du}{\cancel{6x}} = \int \frac{1}{3u^2}\,du$$

$$= \int \frac{1}{3}u^{-2}\,du = -\frac{1}{3}u^{-1} + C = -\frac{1}{3}(4+3x^2)^{-1} + C$$

$$= -\frac{1}{3(4+3x^2)} + C$$

2. $f(x) = \frac{8x}{\sqrt{4-x}} = 8x(4-x)^{-\frac{1}{2}} = 8x \times 4^{-\frac{1}{2}}(1-\frac{1}{4}x)^{-\frac{1}{2}}$

$$= 8x \times \frac{1}{2}(1-\tfrac{1}{4}x)^{-\frac{1}{2}} = 4x(1-\tfrac{1}{4}x)^{-\frac{1}{2}}$$

$$= 4x\left[1 + \frac{-\frac{1}{2}}{1}(-\tfrac{1}{4}x)^1 + \frac{-\frac{1}{2}(-\frac{3}{2})}{1\times 2}(-\tfrac{1}{4}x)^2 + O(x^3)\right]$$

$$= 4x\left[1 + \frac{1}{8}x + \frac{3}{128}x^2 + O(x^3)\right]$$

$$= 4x + \frac{1}{2}x^2 + \frac{3}{32}x^3 + O(x^3)$$

AS REQUIRED

3. $\frac{d\theta}{dt} = 0.5$ GIVEN

- $x = 4\sin\theta + 7\cos\theta$

$$\frac{dx}{d\theta} = 4\cos\theta - 7\sin\theta$$

$$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = [4\cos\theta - 7\sin\theta] \times 0.5$$

$$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{2}} = \left[\cancel{4\cos\tfrac{\pi}{2}} - 7\sin\tfrac{\pi}{2}\right] \times 0.5 = -\frac{7}{2}$$

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4. $V = \pi\int_{x_1}^{x_2}[y(x)]^2\,dx = \pi\int_2^4\left[2(x-1)^{\frac{3}{2}}\right]^2 dx = \pi\int_2^4 4(x-1)^3\,dx$

$=$ BY RECOGNITION (LINEAR ADJUSTMENT) $= \dots \pi\left[(x-1)^4\right]_2^4$

$= \pi\left[3^4 - 1^4\right] = \pi\left[81-1\right] = 80\pi$

AS REQUIRED

5. a) $r_1 = (2,2,0) + \lambda(1,1,0) = (\lambda+2, \lambda+2, 0)$

$r_2 = (2,5,7) + \mu(2,1,-1) = (2\mu+2, \mu+5, 7-\mu)$

- EQUATE $\underline{k}$

$7-\mu = 0$

$\boxed{\mu = 7}$

- EQUATE $\underline{j}$

$\lambda+2 = \mu+5$

$\lambda+2 = 7+5$

$\boxed{\lambda = 10}$

- CHECK $\underline{i}$

$\lambda+2 = 10+2 = 12$

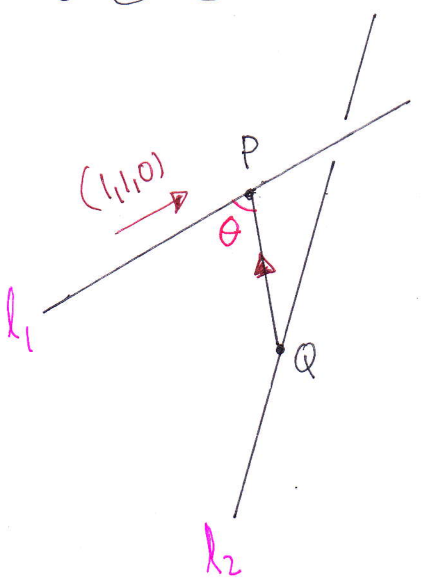
$2\mu+2 = 2\times7+2 = 16$

$12 \neq 16$

LINES DO NOT INTERSECT

b) $\lambda = 4 \Rightarrow P(6,6,0)$

$\mu = -1 \Rightarrow Q(0,4,8)$



- $\overrightarrow{QP} = p - q = (6,6,0)-(0,4,8)$

$= (6,2,-8)$

- BY THE DOT PRODUCT

$(1,1,0)\cdot(6,2,-8) = |1,1,0|\,|6,2,-8|\cos\theta$

$6+2+0 = \sqrt{1+1+0}\sqrt{36+4+64}\cos\theta$

$8 = \sqrt{2}\sqrt{104}\cos\theta$

$\cos\theta = \frac{2}{13}\sqrt{13}$

$\theta \simeq 56.3°$

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6. a) $6^x + 6xy + y^2 = 9$

$$\Rightarrow \frac{d}{dx}(6^x) + \frac{d}{dx}(6xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(9)$$

$$\Rightarrow 6^x \ln 6 + (6 \times y) + \left(6x \times \frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow 6^x \ln 6 + 6y + 6x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow (6x+2y)\frac{dy}{dx} = -6y - 6^x \ln 6$$

$$\Rightarrow \frac{dy}{dx} = \frac{-6y - 6^x \ln 6}{6x + 2y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{6y + 6^x \ln 6}{6x + 2y}$$

AS REQUIRED

b) WHEN $x=2 \Rightarrow 36 + 12y + y^2 = 9$

$$\Rightarrow y^2 + 12y + 27 = 0$$

$$\Rightarrow (y+3)(y+9) = 0$$

$$\Rightarrow y = \begin{cases} -3 \\ -9 \end{cases}$$

$$\left.\frac{dy}{dx}\right|_{(2,-3)} = -\frac{-18 + 36\ln 6}{12 - 6} = \frac{18 - 36\ln 6}{6} = 3 - 6\ln 6$$

$$\left.\frac{dy}{dx}\right|_{(2,-9)} = -\frac{-54 + 36\ln 6}{12 - 18} = \frac{54 - 36\ln 6}{-6} = -9 + 6\ln 6$$

P.T.O

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7.
$$\frac{dy}{dx}(2x-3)(x-1) = y(2x-1)$$
$$\Rightarrow dy\,(2x-3)(x-1) = y(2x-1)\,dx$$
$$\Rightarrow \frac{1}{y}\,dy = \frac{2x-1}{(2x-3)(x-1)}\,dx$$
$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{2x-1}{(2x-3)(x-1)}\,dx$$

PARTIAL FRACTIONS
$$\frac{2x-1}{(2x-3)(x-1)} \equiv \frac{A}{2x-3} + \frac{B}{x-1}$$
$$2x-1 \equiv A(x-1) + B(2x-3)$$
If $x=1$, $1=-B \Rightarrow B=-1$

If $x=\frac{3}{2}$, $2=\frac{1}{2}A \Rightarrow A=4$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{4}{2x-3} - \frac{1}{x-1}\,dx$$
$$\Rightarrow \ln|y| = 2\ln|2x-3| - \ln|x-1| + \ln A$$
$$\Rightarrow \ln|y| = \ln|2x-3|^2 - \ln|x-1| + \ln A$$
$$\Rightarrow \ln|y| = \ln\left|\frac{A(2x-3)^2}{x-1}\right|$$
$$\Rightarrow \boxed{y = \frac{A(2x-3)^2}{x-1}}$$

APPLY CONDITION $x=2$, $y=1$ $\Rightarrow 1 = \frac{A\times 1}{1}$
$$\Rightarrow A=1$$

$$\therefore\ y = \frac{(2x-3)^2}{x-1}$$

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8. $\displaystyle\int_0^{\frac{1}{4}\pi^2} \sin\sqrt{x}\,dx = \dots$ SUBSTITUTION FIRST →

$u = \sqrt{x}$

$u^2 = x$

$x = u^2$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

$x=0,\quad u=0$

$x=\frac{1}{4}\pi^2,\quad u=\frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} \sin u\,(2u\,du) = \int_0^{\frac{\pi}{2}} 2u\sin u\,du$$

... BY PARTS & IGNORING LIMITS ...

$\int 2u\sin u\,du$

$2u$	2
$-\cos u$	$\sin u$

$$= -2u\cos u - \int -2\cos u\,du$$

$$= -2u\cos u + \int 2\cos u\,du$$

$$= -2u\cos u + 2\sin u + C$$

$$\dots \left[-2u\cos u + 2\sin u\right]_0^{\frac{\pi}{2}} = \left[(0+2)-(0+0)\right] = 2$$

IE $N=2$

9. a) $x = \frac{6}{t} \qquad y = 6t - t^2 \qquad t \neq 0$

I ● R is x intercept

$y=0$

$0 = 6t - t^2$

$0 = t(6-t)$

$t = \cancel{0}$ or 6

$\therefore t_R = 6$

● $P(6,5)$

$x = \frac{6}{t}$

$6 = \frac{6}{t}$

$t = 1$

$\therefore t_P = 1$

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II) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6-2t}{-\frac{6}{t^2}} = \frac{\frac{6-2t}{1}}{\frac{-6}{t^2}} = \frac{t^2(6-2t)}{-6} = \frac{-2t^2(t-3)}{-6}$

$\frac{dy}{dx} = \frac{1}{3}t^2(t-3)$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{1}{3}\times 1^2 \times (-2) = -\frac{2}{3}$

∴ NORMAL GRADIENT IS $\frac{3}{2}$

∴ EQUATION OF NORMAL l : $y-5 = \frac{3}{2}(x-6)$

$2y-10 = 3x-18$

$2y+8 = 3x$

III) • AT R, $t=6$

$x = \frac{6}{6} = 1$

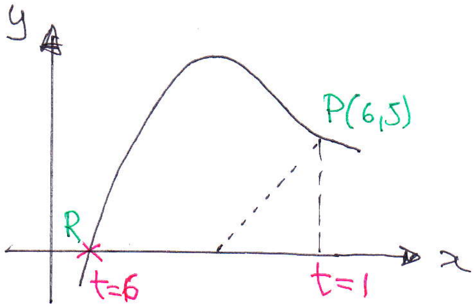
∴ $R(1,0)$

• WHEN $y=0$ (IN NORMAL EQUATION)

$3x = 8$

$x = 8/3$

∴ $Q\left(\frac{8}{3}, 0\right)$

b) 

AREA UNDER PARAMETRIC CURVE

$A = \int_{x_1}^{x_2} y(x)\,dx$

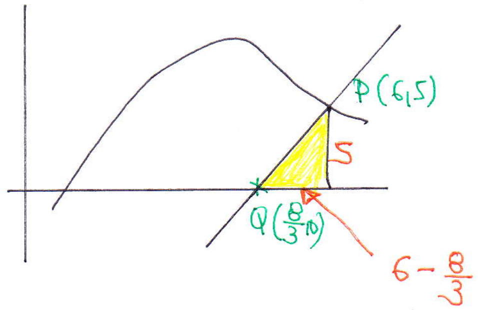
$A = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt$

HERE $\int_6^1 (6t-t^2)\left(-\frac{6}{t^2}\right)dt = \int_6^1 -\frac{36}{t}+6\,dt$

$= \left[6t - 36\ln t\right]_6^1 = (6 - \cancel{36\ln 1}) - (36 - 36\ln 6)$

$= -30 + 36\ln 6$

AREA OF RIGHT ANGLED TRIANGLE $= \frac{1}{2} \times 5 \times \left(6 - \frac{8}{3}\right) = \frac{25}{3}$



$\therefore$ REQUIRED AREA $= \left(-30 + 36\ln 6\right) - \frac{25}{3}$

$= -\frac{115}{3} + 36\ln 6$

≈ 26.17