IYGB GCE

Core Mathematics C4

Advanced

Practice Paper J

Difficulty Rating: 3.3333/1.500

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A curve is given by

$$2\cos x + \tan y = 2\sqrt{3}.$$

a) Show by implicit differentiation that

$$\frac{dy}{dx} = 2\sin x \cos^2 y. \qquad \textbf{(4)}$$

b) Find an equation of the normal to the curve at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$, giving the answer in the form $ax + by = \pi$, where a and b are integers. **(3)**

Question 2

Use integration by parts to find the value of

$$\int_0^{\frac{\pi}{4}} 4x\cos 4x \; dx. \qquad \textbf{(5)}$$

Question 3

A curve C is given by the parametric equations

$$x = 2t^2 - 1, \quad y = 3(t+1), \quad t \in \mathbb{R}.$$

Determine the coordinates of the points of intersection between C and the straight line with equation

$$3x - 4y = 3. \qquad \textbf{(7)}$$

Question 4

$$\frac{27x+2}{(2-x)(1+3x)} \equiv \frac{P}{2-x} + \frac{Q}{1+3x}.$$

a) Find the value of each of the constants P and Q. (3)

b) Hence show that if x is sufficiently small

$$\frac{27x+2}{(2-x)(1+3x)} \approx 1 + 11x - 26x^2 + \frac{163}{2}x^3. \qquad (7)$$

Question 5

$$\int \frac{\cos x}{1-\cos x}\, dx.$$

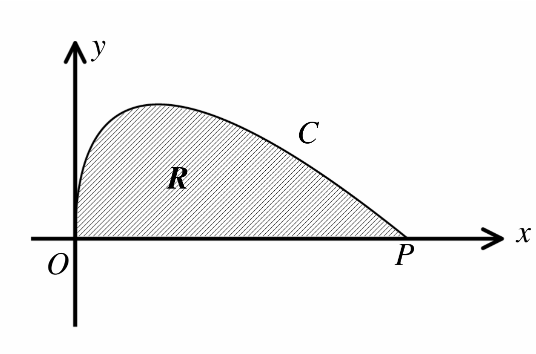
a) Show by multiplying the numerator and denominator of the integrand by $(1+\cos x)$, that the above integral can eventually be written as

$$\int \cot x \operatorname{cosec} x + \cot^2 x \, dx. \qquad (5)$$

b) Hence show further that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{1-\cos x}\, dx = \frac{1}{4}\left(4\sqrt{2} - \pi\right). \qquad (5)$$

Question 6



The figure above shows the curve C, given parametrically by

$$x = 6t^2, \quad y = t - t^2, \quad t \geq 0.$$

The curve meets the x axis at the origin O and at the point P.

a) Show that the x coordinate of P is 6. **(2)**

The finite region R, bounded by C and the x axis, is revolved in the x axis by 2π radians to form a solid of revolution, whose volume is denoted by V.

b) Show clearly that

$$V = \pi \int_0^T 12t\left(t - t^2\right)^2 \, dt,$$ **(3)**

stating the value of T.

c) Hence find the value of V. **(4)**

Question 7

Relative to a fixed origin O the following position vectors are given.

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 8 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 13 \\ 1 \end{pmatrix}.$$

a) Find a vector equation for the line straight l_1 which passes through A and B. **(3)**

The straight line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 do not intersect. **(4)**

c) Find the position vector of C, given it lies on l_2 and $\measuredangle ABC = 90°$. **(7)**

Question 8

Gas is kept in a sealed container whose volume, V cm^3, can be varied as needed.

The pressure of the gas P, in suitable units, is such so that at any given time the product of P and V remains constant.

The container is heated up so that the volume of the gas begins to expand at a rate inversely proportional to the volume of the gas at that instant.

Let t, in seconds, be the time since the volume began to expand.

a) Show that

$$\frac{dP}{dt} = -AP^3 \ ,$$

where A is a positive constant. **(6)**

When $t = 0$, $P = 1$ and when $t = 2$, $P = \frac{1}{3}$.

b) Solve the differential equation to show that

$$P^2 = \frac{1}{4t+1} \ .$$ **(7)**