IYGB GCE

Core Mathematics C4

Advanced

Practice Paper P

Difficulty Rating: 3.7733/1.7964

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x) = \left(\frac{6-x}{1+2x}\right)^2, \ |x| < \frac{1}{2}.$$

Determine the value of the coefficient of x^2 in the binomial expansion of $f(x)$. **(6)**

Question 2

With respect to a fixed origin O, the following points are given

$$A(2,2,5), \ B(12,7,0), \ C(0,0,1) \text{ and } D(9,k,4),$$

where k is a scalar constant.

a) Find a vector equation of the straight line l_1 that passes through A and B. **(3)**

The straight line l_2 passes through C and D, and intersects l_1 at the point P.

b) Determine in any order …

i. … the coordinates of P.

ii. … the value of k.

iii. … the acute angle between l_1 and l_2. **(9)**

Question 3

A curve C has implicit equation

$$(xy-2)(y+5)=10.$$

The curve crosses the y axis at the point A.

The straight line L is the tangent to C at A.

a) State the coordinates of A. **(1)**

b) Find an equation for L. **(5)**

c) Determine the coordinates of the point where L meets C again. **(6)**

Question 4

Water is pouring into a container at a constant rate of $600 \text{ cm}^3\text{s}^{-1}$ and is leaking from a hole at the base of the container at the rate of $\frac{3V}{4} \text{ cm}^3\text{s}^{-1}$, where $V \text{ cm}^3$ is the volume of the water in the container.

a) Show clearly that

$$-4\frac{dV}{dt}=3V-2400,$$

where t is the time measured in seconds. **(2)**

Initially there were 200 cm^3 of water in the container.

b) Show further that

$$V=800-600\mathrm{e}^{-\frac{3}{4}t}.$$ **(7)**

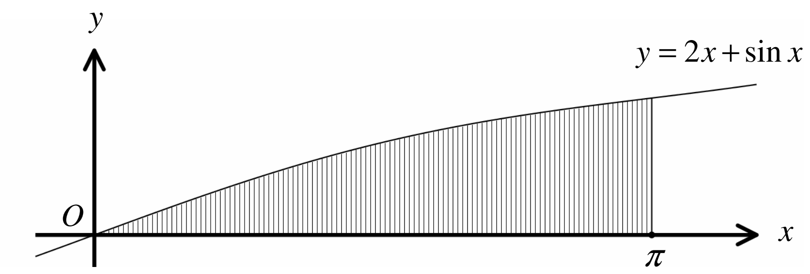
c) State the maximum volume that the water in the container will ever attain. **(1)**

Question 5

Show that

a) $\int_0^{\pi} 4x\sin x \ dx = 4\pi$. **(4)**

b) $\int_0^{\pi} \sin^2 x \ dx = \frac{\pi}{2}$. **(4)**



The figure above shows part of the curve with equation

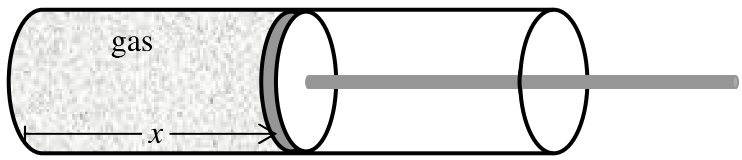
$$y = 2x + \sin x.$$

The shaded region bounded by the curve, the x axis and the line $x = \pi$ is rotated by 2π radians about the x axis to form a solid of revolution.

c) Show that the volume of the solid is

$$\frac{1}{6}\pi^2\left(8\pi^2 + 27\right).$$ **(5)**

Question 6



A piston can slide inside a combustion cylinder which is closed at one end.

The cylinder is filled with gas whose pressure P, in suitable units, is given by

$$P = \frac{60}{x}, \quad x \neq 0$$

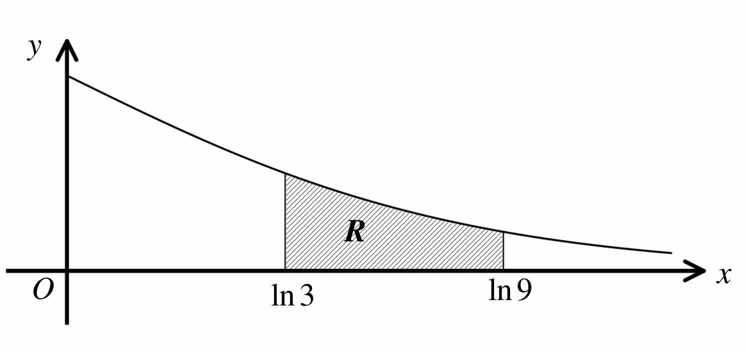
where x is the distance, in cm, of the piston from the closed end.

At a given instant

- the distance of the piston from the closed end is 5 cm.
- its speed is 15 cm s^{-1}, moving away from the closed end.

Determine the rate at which the pressure of the gas is changing at that given instant. **(5)**

Question 7



The figure above shows the curve C with parametric equations

$$x = \ln(t+1), \quad y = \frac{2}{t+2}, \quad t \in \mathbb{R},\ t \geq 0 .$$

The finite region R, shown shaded in the figure above, is bounded by C, the straight lines with equations $x = \ln 3$ and $x = \ln 9$ and the x axis.

a) Show that the area R is given by the integral

$$I = \int_{2}^{8} \frac{2}{(t+1)(t+2)}\, dt . \qquad (3)$$

b) Find an exact value for the above integral. **(7)**

c) Show that a Cartesian equation of C is

$$y = \frac{2}{e^{x}+1}. \qquad (2)$$

d) Use the Cartesian equation of C and the substitution $u = e^{x}+1$ to show that the area of R can also be found by the integral

$$J = \int_{4}^{10} \frac{2}{u(u-1)}\, du . \qquad (3)$$

e) Without evaluating J, show that $J = I$. **(2)**