C4, IYGB, PAPER P

1. $f(x) = \left(\frac{6-x}{1+2x}\right)^2 = (6-x)^2(1+2x)^{-2}$

$= (36-12x+x^2)\left[1 + \frac{-2}{1}(2x)^1 + \frac{-2(-3)}{1\times2}(2x)^2 + \frac{(-2)(-3)(-4)}{1\times2\times3}(2x)^3 + O(x^4)\right]$

$= (36-12x+x^2)(1-4x+12x^2-32x^3+O(x^4))$

x^2

$48x^2$

$432x^2$

$\therefore\ 432+48+1 = 481$

2. a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (12,7,0) - (2,2,5) = (10,5,-5)$

$\underline{r} = (2,2,5) + \lambda(2,1,-1)$ ← SCALED

$(x,y,z) = (2\lambda+2,\ \lambda+2,\ 5-\lambda)$

b) $\overrightarrow{CD} = \underline{d} - \underline{c} = (9,k,4) - (0,0,1) = (9,k,3)$

$\underline{r} = (0,0,1) + \mu(9,k,3)$

$(x,y,z) = (9\mu,\ \mu k,\ 3\mu+1)$

$\underline{i})\quad 2\lambda+2 = 9\mu$

$\underline{j})\quad \lambda+2 = \mu k$

$\underline{k})\quad 5-\lambda = 3\mu+1$

FROM $\underline{i}$ & $\underline{k}$

$\left.\begin{array}{l} 2\lambda+2 = 9\mu \\ 4-3\mu = \lambda \end{array}\right\} \Rightarrow 2(4-3\mu)+2 = 9\mu$

$8-6\mu+2 = 9\mu$

$10 = 15\mu$

$\boxed{\mu = \frac{2}{3}}$

$\boxed{\lambda = 2}$

$\therefore P(2\lambda+2,\ \lambda+2,\ 5-\lambda) \Rightarrow P(6,4,3)$

AND FROM $\underline{j}$: $\lambda+2 = \mu k$

$4 = \frac{2}{3}k \qquad \therefore k = 6$

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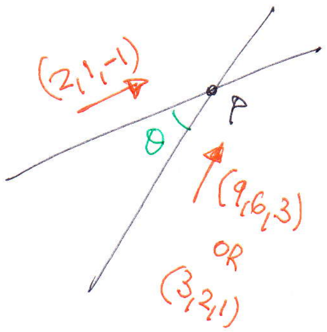
FINALLY



$(2,1,-1)\cdot(3,2,1) = |2,1,-1||3,2,1|\cos\theta$

$6+2-1 = \sqrt{4+1+1}\sqrt{9+4+1}\cos\theta$

$7 = \sqrt{6}\sqrt{14}\cos\theta$

$\cos\theta = \frac{7}{\sqrt{6\times14}}$

$\theta \simeq 40.2°$

3. a) WHEN $x=0$ $\quad (0-2)(y+5) = 10$

$y+5 = -5$

$y = -10$ $\qquad \therefore A(0,-10)$

b) $\boxed{(xy-2)(y+5) = 10 \implies xy^2+5xy-2y-10=10}$

• Differentiate with respect to x

$\implies 1xy^2 + x\left(2y\frac{dy}{dx}\right) + 5y + 5x\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

• AT $(0,-10)$

$\implies 100 + 0 - 50 + 0 - 2\left.\frac{dy}{dx}\right|_{(0,-10)} = 0.$

$\implies 50 = 2\left.\frac{dy}{dx}\right|_{(0,-10)}$

$\implies \left.\frac{dy}{dx}\right|_{(0,-10)} = 25$

$\therefore$ L: $y = 25x - 10$

c) SOLVING SIMULTANEOUSLY WITH THE CURVE

$\implies (x(25x-10)-2)(25x-10+5) = 10$

$\implies (25x^2-10x-2)(25x-5) = 10$

$$\Longrightarrow \quad \begin{array}{r} 625x^3 - 250x^2 - \cancel{50x} \\ -125x^2 + \cancel{50x} + 10 \end{array} = 10$$

$$\Longrightarrow \quad 625x^3 - 375x^2 = 0$$

$$\Longrightarrow \quad 5x^3 - 3x^2 = 0$$

$$\Longrightarrow \quad x^2(5x-3) = 0$$

$$\Longrightarrow \quad x = \begin{cases} 0 & \leftarrow \text{POINT A (POINT OF TANGENCY)} \\ \frac{3}{5} \end{cases}$$

$\therefore \ y = 25 \times \frac{3}{5} - 10 = 5 \qquad \therefore \left(\frac{3}{5}, 5\right)$

4. a)

$$\boxed{\begin{array}{lll} \text{IN :} & \frac{dV}{dt} = 600 \\ \text{OUT :} & \frac{dV}{dt} = -\frac{3}{4}V \\ \text{NET :} & \frac{dV}{dt} = 600 - \frac{3}{4}V \end{array}}$$

THUS

$$\frac{dV}{dt} = 600 - \frac{3}{4}V$$

$$4\frac{dV}{dt} = 2400 - 3V$$

$$-4\frac{dV}{dt} = 3V - 2400$$

b) SOLVING

$$\Longrightarrow \quad -4\frac{dV}{dt} = 3V - 2400$$

$$\Longrightarrow \quad \frac{-4}{3V - 2400}\, dV = 1 \, dt$$

$$\Longrightarrow \quad \int \frac{-4}{3V-2400}\, dV = \int 1 \, dt$$

$$\Longrightarrow \quad -\frac{4}{3}\ln|3V - 2400| = t + C$$

$\Rightarrow \ln|3V-2400| = -\frac{3}{4}t+C$

$\Rightarrow 3V - 2400 = e^{-\frac{3}{4}t+C} = e^{-\frac{3}{4}t} \times e^{C} = Ae^{-\frac{3}{4}t}$

$\Rightarrow 3V = 2400 + Ae^{-\frac{3}{4}t}$

$\Rightarrow V = 800 + Ae^{-\frac{3}{4}t}$

APPLY CONDITION $t=0$ $V=200$ $\Rightarrow 200 = 800 + Ae^{0}$

$-600 = A$

$A = -600$

$\therefore V = 800 - 600e^{-\frac{3}{4}t}$

AS REQUIRED

c) As $t \to \infty$ $e^{-\frac{3}{4}t} \to 0$

$600e^{-\frac{3}{4}t} \to 0$

$\therefore V \to 800$ IE $800\,\text{cm}^3$

5. a) $\int_0^{\pi} 4x\sin x \, dx = \ldots$ BY PARTS ... (IGNORE LIMITS)

$4x$	4
$-\cos x$	$\sin x$

$= -4x\cos x - \int -4\cos x \, dx$

$= -4x\cos x + \int 4\cos x \, dx$

$= -4x\cos x + 4\sin x + C$

$\ldots = \left[4\sin x - 4x\cos x\right]_0^{\pi} = \left[0 - 4\pi(-1)\right] - \left[0 - 0\right] = 4\pi$

AS REQUIRED

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b) $\int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\pi}$

$= \left[\left(\frac{\pi}{2} - 0\right) - (0 - 0)\right] = \frac{\pi}{2}$ AS REQUIRED

c) $V = \pi \int_{x_1}^{x_2} [y(x)]^2 \, dx$

$V = \pi \int_0^{\pi} (2x + \sin x)^2 \, dx = \pi \int_0^{\pi} 4x^2 + 4x\sin x + \sin^2 x \, dx$

$= \pi \int_0^{\pi} 4x^2 \, dx + \pi \int_0^{\pi} 4x\sin x \, dx + \pi \int_0^{\pi} \sin^2 x \, dx$

$= \pi \left[\frac{4}{3}x^3\right]_0^{\pi} + \pi(4\pi) + \pi \times \frac{\pi}{2}$

$= \pi \times \frac{4}{3}\pi^3 + 4\pi^2 + \frac{1}{2}\pi^2$

$= \frac{4}{3}\pi^4 + \frac{9}{2}\pi^2$

$= \frac{8}{6}\pi^4 + \frac{27}{6}\pi^2$

$= \frac{1}{6}\pi^2\left[8\pi^2 + 27\right]$ AS REQUIRED

(P.T.O)

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6.

$$\frac{dP}{dt} = \frac{dP}{dx}\frac{dx}{dt}$$

$$P = \frac{60}{x}$$

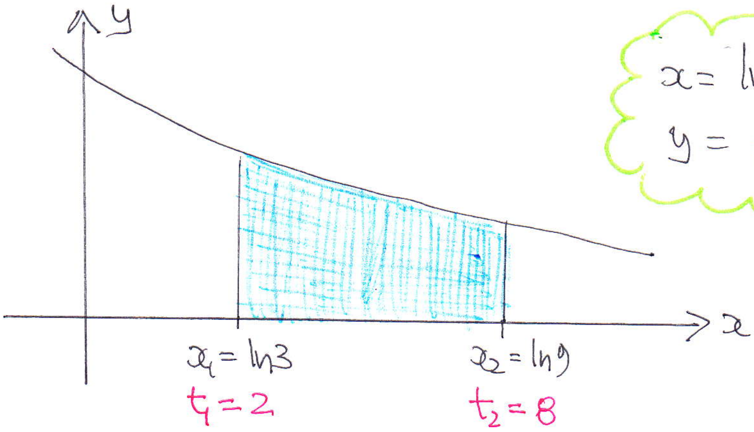
$$\frac{dP}{dx} = -\frac{60}{x^2}$$

$$\frac{dP}{dt} = -\frac{60}{x^2}\frac{dx}{dt} \leftarrow \text{VELOCITY "SPEED"}$$

$$\frac{dP}{dt} = -\frac{60}{5^2}\times(+15)$$

$$\frac{dP}{dt} = -36$$

7. a)



$x = \ln(t+1)$

$y = \frac{2}{t+2}$

$\ln 3 = \ln(t+1)$

$3 = t+1$

$t = 2$

$\ln 9 = \ln(t+1)$

$9 = t+1$

$t = 8$

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt$$

$$\text{AREA} = \int_2^8 \underbrace{\left(\frac{2}{t+2}\right)}_{y(t)}\underbrace{\left(\frac{1}{t+1}\right)}_{\frac{dx}{dt}}\,dt$$

$$\text{AREA} = \int_2^8 \frac{2}{(t+2)(t+1)}\,dt$$

AS REQUIRED

b) BY PARTIAL FRACTIONS

$$\frac{2}{(t+2)(t+1)} \equiv \frac{A}{t+2} + \frac{B}{t+1}$$

$$\boxed{2 \equiv A(t+1) + B(t+2)}$$

If $t=-1$, $\quad 2 = B$

If $t=-2$ $\quad 2 = -A$

$$\therefore I = \int_2^8 \frac{2}{t+1} - \frac{2}{t+2}\,dt = \Big[2\ln|t+1| - 2\ln|t+2|\Big]_2^8$$

$$= (2\ln 9 - 2\ln 10) - (2\ln 3 - 2\ln 4) = 2\big[\ln 9 - \ln 10 - \ln 3 + \ln 4\big]$$

$$= 2\ln\left(\frac{9\times 4}{10\times 3}\right) = 2\ln\left(\frac{36}{30}\right) = 2\ln\frac{6}{5}$$

c) $\left.\begin{aligned} x &= \ln(t+1) \\ y &= \frac{2}{t+2} \end{aligned}\right\} \Longrightarrow \begin{aligned} e^x &= t+1 \\ y &= \frac{2}{(t+1)+1} \end{aligned} \Longrightarrow y = \frac{2}{e^x+1}$

AS REQUIRED

d) $$J = \int_{\ln 3}^{\ln 9} \frac{2}{e^x+1}\,dx = \int_4^{10} \frac{2}{u}\,\frac{du}{e^x}$$

$u = e^x + 1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$x = \ln 3$, $u = e^{\ln 3}+1$

$u = 4$

$x = \ln 9$, $u = e^{\ln 9}+1$

$u = 10$

$e^x = u - 1$

$$J = \int_4^{10} \frac{2}{u} \times \frac{1}{u-1}\,du$$

$$J = \int_4^{10} \frac{2}{u(u-1)}\,du$$

AS REQUIRED

(P.T.O)

e) $$J = \int_4^{10} \frac{2}{u(u-1)} \, du$$

LET $t = v+2$

$$\frac{du}{dt} = 1$$

$u = 4, \ t = 2$

$u = 10, \ t = 8$

$$= \int_2^8 \frac{2}{(t+2)(t+2-1)} \, dt$$

$$= \int_2^8 \frac{2}{(t+2)(t+1)} \, dt$$

$$= I$$