C4, IYGB, PAPER R

1.

$$\int \frac{12x}{(1-x^2)^{\frac{3}{2}}}\,dx = \dots \text{ BY SUBSTITUTION } \dots$$

$$u = 1-x^2$$
$$\frac{du}{dx} = -2x$$
$$dx = \frac{du}{-2x}$$

$$= \int \frac{12x}{u^{\frac{3}{2}}}\left(\frac{du}{-2x}\right) = \int -\frac{6}{u^{\frac{3}{2}}}\,du$$

$$= \int -6u^{-\frac{3}{2}}\,du = 12u^{-\frac{1}{2}} + C$$

$$= \frac{12}{u^{\frac{1}{2}}} + C = \frac{12}{\sqrt{1-x^2}} + C$$

2.

$$\frac{dr}{dt} = 3 \text{ (GIVEN)}$$

CIRCLE
$$A = \pi r^2$$
$$\frac{dA}{dr} = 2\pi r$$

$$\Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$
$$\Rightarrow \frac{dA}{dt} = 2\pi r \times 3$$
$$\Rightarrow \frac{dA}{dt} = 6\pi r$$
$$\Rightarrow \left.\frac{dA}{dt}\right|_{r=13.5} = 6\times\pi\times 13.5 = 81\pi \simeq 254 \text{ cm}^2\text{s}^{-1}$$

3.

$$\boxed{\begin{aligned} &\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \\ &\frac{\pi}{3} \div 4 = \frac{\pi}{12} \end{aligned}} \Rightarrow$$

x	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
y	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	$\sqrt{6}-\sqrt{2}$	1

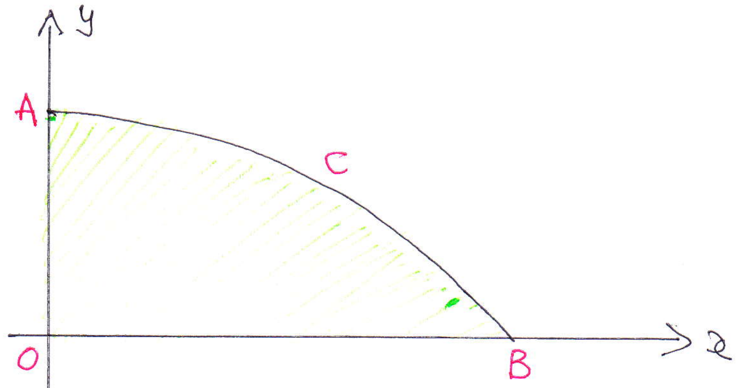
$$I = \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$I = \frac{\pi/12}{2}\left[2 + 1 + 2\left[\cancel{\sqrt{2}} + \frac{2}{3}\sqrt{3} + \sqrt{6} - \cancel{\sqrt{2}}\right]\right]$$

$$I = \frac{\pi}{24} \times 10.208\ldots \simeq 1.34$$

C4, IYGB, PAPER R

4.



$$x = 36t^2 - \pi^2$$
$$y = \frac{\sin 3t}{8}$$
$$\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$$

$t_1 = \frac{\pi}{6}$

$x = 0$
$36t^2 - \pi^2 = 0$
$36t^2 = \pi^2$
$t = \frac{\pi}{6}$

$t_2 = \frac{\pi}{3}$

$y = 0$
$\frac{\sin 3t}{8} = 0$
$\sin 3t = 0$
$3t < \begin{matrix} 0 \pm 2n\pi \\ \pi \pm 2n\pi \end{matrix}$
$t = < \begin{matrix} 0 \pm \frac{2}{3}n\pi \\ \frac{\pi}{3} \pm \frac{2}{3}n\pi \end{matrix}$

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \underbrace{\left(\frac{\sin 3t}{8}\right)}_{y(t)} \underbrace{72t}_{\frac{dx}{dt}}\,dt$$

$$\text{AREA} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9t\sin 3t$$

BY PARTS AND IGNORING LIMITS

$\dots -3t\cos 3t - \int -3\cos 3t\,dt$

$\dots -3t\cos 3t + \int 3\cos 3t\,dt$

$\dots -3t\cos 3t + \sin 3t + C$

$9t$	9
$-\frac{1}{3}\cos 3t$	$\sin 3t$

$$\text{AREA} = \left[-3t\cos 3t + \sin 3t\right]_{\pi/6}^{\pi/3} = \left[\left(-3\left(\frac{\pi}{3}\right)(-1) + 0\right) - (0 + 1)\right]$$

$= \pi - 1$

AS REQUIRED

5. a) $f(x) = \dfrac{8x^2+17}{(1-x)(3+2x)^2} \equiv \dfrac{A}{1-x} + \dfrac{B}{(3+2x)^2} + \dfrac{C}{3+2x}$

$$8x^2+17x \equiv A(3+2x)^2 + B(1-x) + C(3+2x)(1-x)$$

If $x=1$, $25 = 25A \implies \boxed{A=1}$

If $x=-\frac{3}{2}$, $8\times\frac{9}{4}+17\left(-\frac{3}{2}\right) = \left(\frac{5}{2}\right)B \implies \boxed{B=-3}$

If $x=0$, $0 = 9A+B+3C$

$0 = 9-3+3C$

$-6 = 3C$

$\boxed{C=-2}$

$\therefore f(x) = \dfrac{1}{1-x} - \dfrac{3}{(3+2x)^2} - \dfrac{2}{3+2x}$ //

b) • $\dfrac{1}{1-x} = (1-x)^{-1} = 1 + \frac{-1}{1}(-x)^1 + \frac{-1(-2)}{1\times2}(-x)^2 + O(x^3)$

$= \boxed{\cancel{1} + x + x^2 + O(x^3)}$

• $-\dfrac{3}{(3+2x)^2} = -3(3+2x)^{-2} = -3\times3^{-2}\left(1+\frac{2}{3}x\right)^{-2} = -\frac{1}{3}\left(1+\frac{2}{3}x\right)^{-2}$

$= -\frac{1}{3}\left[1 + \frac{-2}{1}\left(\frac{2}{3}x\right)^1 + \frac{-2(-3)}{1\times2}\left(\frac{2}{3}x\right)^2 + O(x^3)\right]$

$= -\frac{1}{3}\left[1 - \frac{4}{3}x + \frac{4}{9}x^2 + O(x^3)\right.$

$= \boxed{-\cancel{\tfrac{1}{3}} + \frac{4}{9}x - \frac{4}{9}x^2 + O(x^3)}$

• $-\dfrac{2}{3+2x} = -2(3+2x)^{-1} = -2\times3^{-1}\left(1+\frac{2}{3}x\right)^{-1} = -\frac{2}{3}\left(1+\frac{2}{3}x\right)^{-1}$

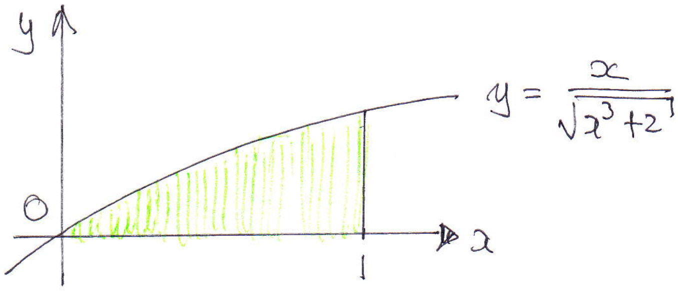
$= -\frac{2}{3}\left[1 + \frac{-1}{1}\left(\frac{2}{3}x\right)^1 + \frac{-1(-2)}{1\times2}\left(\frac{2}{3}x\right)^2 + O(x^3)\right]$

$= -\frac{2}{3}\left[1 - \frac{2}{3}x + \frac{4}{9}x^2 + O(x^3)\right]$

$= \boxed{-\cancel{\tfrac{2}{3}} + \frac{4}{9}x - \frac{8}{27}x^3 + O(x^3)}$

ADDING GIVES $f(x) = \frac{17}{9}x + \frac{7}{27}x^2 + O(x^3) = \frac{1}{27}x(7x+51)$ //

6.



$$V = \pi \int_0^1 [y(x)]^2 \, dx = \pi \int_0^1 \left(\frac{x}{\sqrt{x^3+2}} \right)^2 dx = \pi \int_0^1 \frac{x^2}{x^3+2} \, dx$$

$$V = \frac{1}{3}\pi \int_0^1 \frac{3x^2}{x^3+2} \, dx$$

$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)|$

"BOTTOM" DIFFERENTIATES TO "TOP" OR USE $u = x^3+2$

$$V = \frac{1}{3}\pi \left[\ln|x^3+2| \right]_0^1$$

$$V = \frac{1}{3}\pi \left[\ln 3 - \ln 2 \right]$$

$$V = \frac{\pi}{3} \ln \frac{3}{2}$$

AS REQUIRED

7.

$$ax(2x-y) = b - 3y^2$$

$$\Rightarrow 2ax^2 - axy = b - 3y^2$$

Diff w.r.t x

$$\Rightarrow 4ax - ay - ax\frac{dy}{dx} = 0 - 6y\frac{dy}{dx}$$

$$\Rightarrow (6y - ax)\frac{dy}{dx} = ay - 4ax$$

$$\Rightarrow \frac{dy}{dx} = \frac{ay - 4ax}{6y - ax}$$

Now $\left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{3}{2} \implies \frac{2a-8a}{12-2a} = -\frac{3}{2}$

$\implies \frac{-6a}{12-2a} = -\frac{3}{2}$

$\implies \frac{\cancel{-}3a}{6-a} = \cancel{-}\frac{3}{2}$

$\implies \frac{a}{6-a} = \frac{1}{2}$

$\implies 2a = 6-a$

$\implies 3a = 6$

$\implies a = 2$

FINALLY $2a(2x-y) = b - 3y^2$

$2\times2(2\times2-2) = b - 3\times2^2$

$8 = b - 12$

$b = 20$

8. a) $\underline{a} = (6,2,0)$
$\underline{b} = (5,0,5)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (5,0,5) - (6,2,0) = (-1,-2,5)$

$\underline{r}_1 = (6,2,0) + \lambda(-1,-2,5)$

$(x,y,z) = (6-\lambda,\ 2-2\lambda,\ 5\lambda)$

b) $\underline{r}_2 = (-7,6,-4) + \mu(-5,0,2) = (-5\mu-7,\ 6,\ 2\mu-4)$

- FROM $\underline{j}$: $2-2\lambda = 6$
  $-4 = 2\lambda$
  $\lambda = -2$

- FROM $\underline{i}$: $6-\lambda = -5\mu-7$
  $8 = -5\mu-7$
  $5\mu = -15$
  $\mu = -3$

- CHECK $\underline{k}$
  $5\lambda = 5(-2) = -10$
  $2\mu - 4 = 2(-3) - 4 = -10$

AS ALL 3 COMPONENTS AGREE IF $\lambda=-2$, $\mu=-3$, THE LINES INTERSECT

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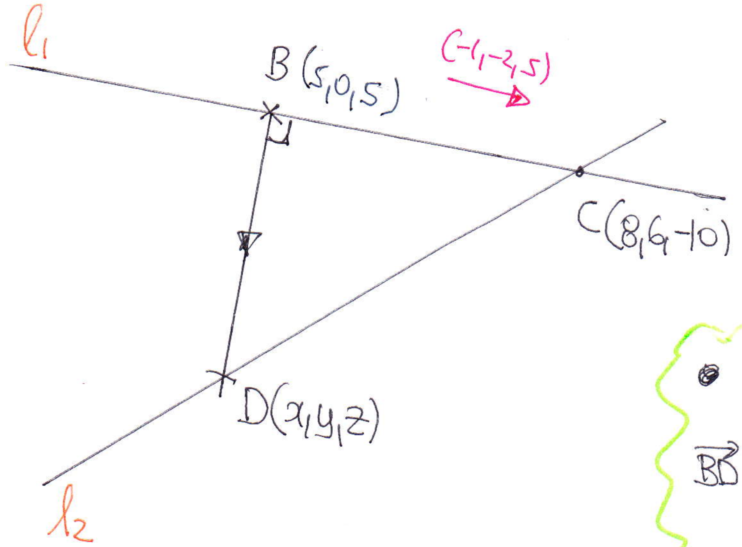
USING $\lambda = -2$ (OR $\mu = -3$) INTO THE EQUATION OF THE LINE WE OBTAIN

$C(8,6,-10)$

d)



- LET $\underline{d} = (x,y,z)$

$\overrightarrow{BD} = \underline{d} - \underline{b} = (x,y,z) - (5,0,5)$

$= (x-5, y, z-5)$

$\hat{DBC} = 90°$

$\Rightarrow (x-5, y, z-5) \cdot (-1,-2,5) = 0$

$\Rightarrow -x+5-2y+5z-25=0$

$\Rightarrow -x-2y+5z = 20$

$\Rightarrow \boxed{x+2y-5z=-20}$

D LIES ON l_2

$\Rightarrow \boxed{\begin{array}{l} x = -5\mu-7 \\ y = 6 \\ z = 2\mu-4 \end{array}}$

$(-5\mu-7)+2\times6-5(2\mu-4) = -20$

$-5\mu-7+12-10\mu+20 = -20$

$-15\mu = -45$

$\boxed{\mu = 3}$

$\therefore D(-22,6,2)$

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9. a) $\frac{dy}{dt} = -6(y-7)^{\frac{2}{3}}$

$\Rightarrow \int \frac{1}{(y-7)^{\frac{2}{3}}}\,dy = \int -6\,dt$

$\Rightarrow \int (y-7)^{-\frac{2}{3}}\,dt = \int -6\,dt$

$\Rightarrow 3(y-7)^{\frac{1}{3}} = -6t + C$

$(y-7)^{\frac{1}{3}} = A - 2t$

$t=0,\ y=132$

$125^{\frac{1}{3}} = A - 0$

$A = 5$

$\Rightarrow (y-7)^{\frac{1}{3}} = 5 - 2t$

Now with $y = 34$

$27^{\frac{1}{3}} = 5 - 2t$

$2t = 2$

$t = 1$

ie 1 min

b) $\frac{dy}{dt} = 0 \Rightarrow y = 7$

$\Rightarrow 5 - 2t = 0$

$\Rightarrow t = \frac{5}{2}$

ie $2\frac{1}{2}$ minutes