IYGB GCE

Core Mathematics C4

Advanced

Practice Paper S

Time: 3 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 90.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve C has implicit equation

$$xy + x^3y + ay = 1,$$

where a is a positive constant.

Use implicit differentiation to show that the gradient at every point on C is negative. **(6)**

Question 2

In the convergent expansion of

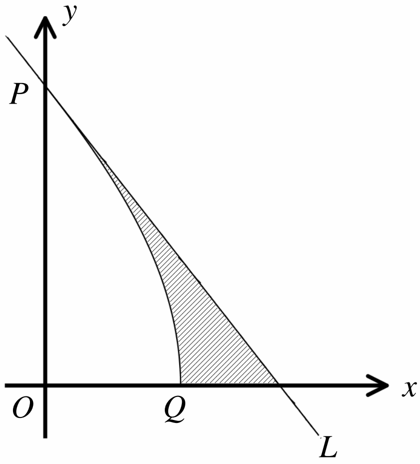
$$\left(1+\tfrac{4}{7}nx\right)^n, \; n \in \mathbb{R}, \; n \notin \mathbb{N}, \; n \neq 0,$$

the coefficients of x^2 and x^3 are non zero and equal.

a) Determine the possible values of n. **(6)**

b) State with justification which value, values or indeed if any of the values of n produces a valid expansion for $x = 1$. **(1)**

Question 3



The figure above shows the curve C with parametric equations

$$x = 4\cos 3\theta, \;\; y = 4\sin\theta, \;\; 0 \le \theta \le \frac{\pi}{6}.$$

The curve meets the coordinate axes at $P(0,2)$ and at $Q(4,0)$.

The straight line L is the tangent to C at the point P.

a) Show that an equation of L is

$$6y + x\sqrt{3} = 12. \qquad \textbf{(4)}$$

The finite region bounded by the curve C the tangent L and the x axis is shown shaded in the above figure.

b) Show further that the area of this region is exactly $\sqrt{3}$ square units. **(8)**

Question 4

It is given that

$$x^2 + x + 2 = (u - x)^2 .$$

a) Show by a clear method that …

i. … $x = \dfrac{u^2 - 2}{2u + 1}$. **(1)**

ii. … $\dfrac{dx}{du} = \dfrac{2\left(u^2 + u + 2\right)}{(2u+1)^2}$. **(2)**

b) Find a simplified expression for

$$\int \frac{1}{x\sqrt{x^2 + x + 2}}\, dx .$$ **(9)**

Question 5

Relative to a fixed origin O, the straight lines l_1 and l_2 have vector equations

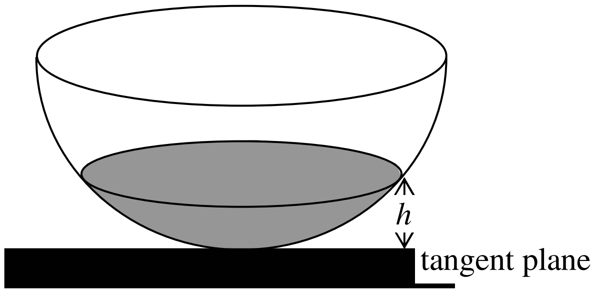
$$\mathbf{r}_1 = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 14 \\ 19 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix},$$

where λ and μ are scalar parameters.

The point A lies on l_1 and the point B lies on l_2, so that the distance AB is least.

Find the coordinates of A and the coordinates of B. **(10)**

Question 6



The figure above shows a hemispherical bowl of radius r cm containing water up to a certain level h cm. The water in the bowl is in the shape of a spherical segment.

It is required to find a formula for the volume of a spherical segment as a function of the radius r cm and the distance of its plane face from the tangent plane, h cm.

The circle with equation

$$x^2 + y^2 = r^2, \; x \geq 0$$

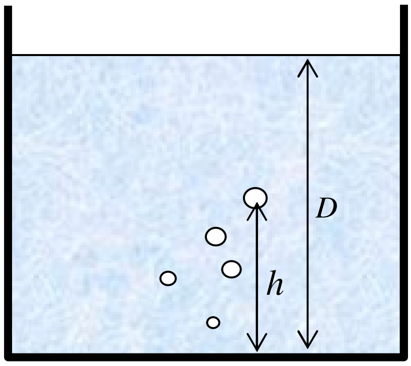
is to be used to find a formula for the volume of a spherical segment.

Show by integration that the volume of the spherical segment V is given by

$$V = \frac{1}{3}\pi h^2 (3r - h),$$

where r is the radius of the sphere or hemisphere and h is the distance of its plane face from the tangent plane. **(10)**

Question 7



An air bubble, rising in a water tank, increases in volume as the pressure of the fluid around it decreases. It is assumed that the shape of the bubble remains spherical at all times.

It is further assumed the volume V cm^3 of an air bubble satisfies the equation

$$V = \frac{k}{D-h},$$

where h cm is the height of the bubble from the bottom of the tank, D cm is the depth of the water in the tank, and k is a positive constant.

The tank is filled up with water to a depth of 800 cm.

A bubble with a volume of 8 cm^3 is created in the water tank at a height of 350 cm from the bottom of the tank.

Show that by the time the bubble has risen by 50 cm, …

a) … the volume of the bubble increases to 9 cm^3. **(2)**

b) … the volume of the bubble increases at the rate of $\frac{9}{400}$ cm^3 per cm risen. **(4)**

c) … the rate at which the radius of the bubble is increasing is

$$\frac{1}{400\sqrt[3]{4\pi}} \text{ cm per cm risen.}$$ **(6)**

Question 8

A shop stays open for 8 hours every Sunday and its sales, £ x, t hours after the shop opens are modelled as follows.

The rate at which the sales are made, is **directly proportional** to the time left until the shop closes and **inversely proportional** to the sales already made until that time.

Two hours after the shop opens it has made sales worth £336 and sales are made at the rate of £72 per hour.

a) Show clearly that

$$x\frac{dx}{dt} = 4032(8-t). \qquad \mathbf{(2)}$$

b) Solve the differential equation to show

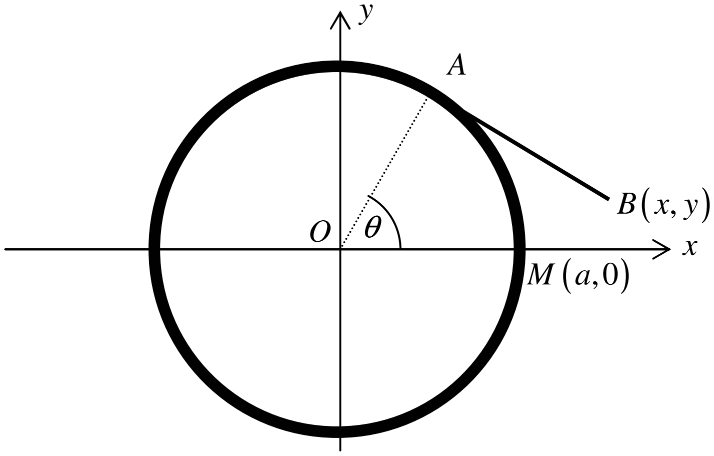
$$x^2 = 4032t(16-t). \qquad \mathbf{(6)}$$

c) Find, to the nearest £, the Sunday sales of the shop according to this model. **(1)**

The shop opens on Sundays at 09.00. The owner knows that the shop is not profitable once the rate at which it makes sales drops under £24 per hour.

d) By squaring the differential equation of part **(a)**, find to the nearest minute, the time the shop should close on Sundays. **(6)**

Question 9



The figure above shows a set of coordinate axes superimposed with a cotton reel.

Cotton thread is being unwound from around the circumference of the fixed circular reel of radius a and centre at O.

The free end of the cotton thread is marked as the point $B(x, y)$ which was originally at $P(a, 0)$.

The unwound part of the cotton thread AB is kept straight and θ is the angle OA subtends at the positive x axis, as shown in the figure above.

Find the parametric equations that satisfy the locus of $B(x, y)$, as the cotton thread is unwound in the fashion described.

(6)