IYGB GCE

Core Mathematics C4

Advanced

Practice Paper T

Time: 3 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

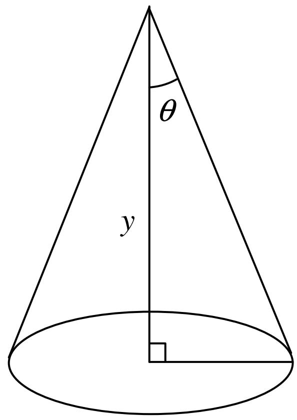
Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



Fine sand starts falling onto a horizontal floor at the constant rate of $3.2 \text{ cm}^3\text{s}^{-1}$.

A heap is formed in the shape of a right circular cone of height y cm, where t is the time in seconds since the sand started falling. The angle θ between the vertical height and the slant height of the cone is such so that $\tan\theta = \frac{1}{\sqrt{3}}$, as shown in the figure.

a) Show with detailed workings that

$$y^3 = \frac{144t}{5\pi}. \qquad (3)$$

The curved surface area of the heap is $A \text{ cm}^2$.

b) Show further that when $t = 60$, …

i. … $\frac{dy}{dt} = \frac{1}{15}\pi^{-\frac{1}{3}}$. **(7)**

ii. … $\frac{dA}{dt} = \frac{16}{15}\pi^{\frac{1}{3}}$. **(7)**

You may assume that the volume V and curved surface area A of a cone of radius r and height h are given by $V = \frac{1}{3}\pi r^2 h$ and $A = \pi r\sqrt{r^2 + h^2}$.

Question 2

Use the substitution $\tan x = \frac{1}{2}\left(-1+\sqrt{3}\tan\theta\right)$ to find the exact value of

$$\int_{0}^{\frac{\pi}{4}} \frac{\sqrt{3}}{2+\sin 2x}\ dx.$$

(10)

Question 3

Relative to a fixed origin O, the straight line l passes through the points $A(a,-3,6)$, $B(2,b,2)$ and $C(3,3,0)$, where a and b are constants.

a) Find the value of a and the value of b, and hence find a vector equation of l. **(4)**

The points P and Q lie on the l so that $|OP| = |OQ|$ and $\measuredangle POQ = 90°$.

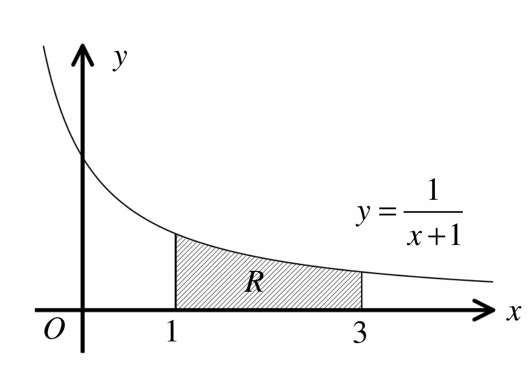
b) Find the coordinates of P and the coordinates of Q. **(9)**

Question 4

$$f(x) \equiv \frac{3x^4 + x^3 + 2x^2 + x - 2}{(x+1)x^5},\ x \in \mathbb{R},\ x \neq 0, x \neq -1$$

Express $f(x)$ into partial fractions in their simplest form. **(6)**

Question 5



The figure above shows the graph of the curve with equation

$$y=\frac{1}{x+1},\ x\in\mathbb{R},\ x=-1.$$

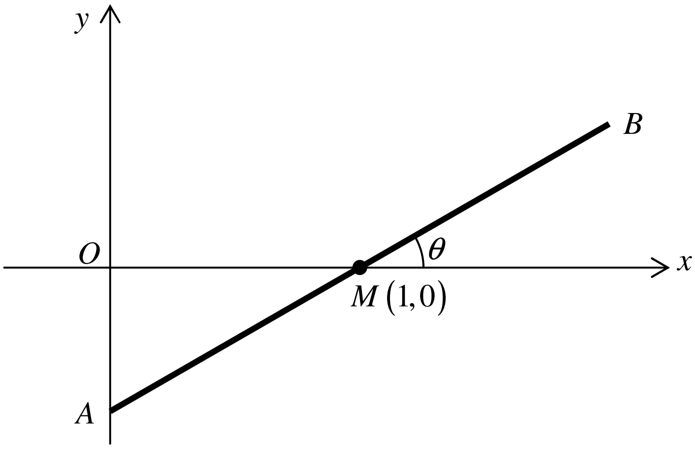
The finite region R is bounded by the curve, the x axis and the lines with equations $x=1$ and $x=3$.

Determine the exact volume of the solid formed when the region R is revolved by 2π radians about …

a) … the y axis. **(8)**

b) … the straight line with equation $x=3$. **(8)**

Question 6



The figure above shows a rigid rod AB of a length 4 units which can slide through a hinge located at the point $M(1,0)$.

The hinge allows the rod to turn in any direction in the x-y plane. The end of the rod marked as A can slide on the y axis so that $|OA| \le 4$.

Let θ be the angle of inclination of the rod to the positive x axis.

a) Show that as A slides on the on the y axis, the locus of B satisfies the parametric equations

$$x = 4\cos\theta, \quad y = 4\sin\theta - \tan\theta, \quad -\theta_0 \le \theta \le \theta_0,$$

stating the exact value of θ_0. **(7)**

b) Show further that a Cartesian equation of this locus is given by

$$y^2 = \frac{\left(16 - x^2\right)(x-1)^2}{x^2}.$$ **(7)**

[*You may **not** use verification in this part*]

Question 7

Initially a tank contains 25 litres of fresh water.

At time $t=0$ salt water of concentration 0.2 kg of salt per litre begins to pour into the tank, at the rate of 1 litre per minute, and at the same time the salt water mix begins to leave the tank at the rate of 1.5 litre per minute.

The concentration of the salt water mix in the tank is thereafter maintained uniform, by constant stirring.

Let x kg be the mass of salt dissolved in the water in the tank, at time t minutes.

a) Show by detailed workings that

$$\frac{dx}{dt}=\frac{1}{5}-\frac{3x}{50-t}.$$ **(7)**

b) Verify by differentiation that the general solution of the differential equation of part **(a)**, is

$$x=\frac{1}{10}(50-t)+A(50-t)^3,$$

where A is an arbitrary constant. **(4)**

c) Determine as an exact simplified surd the maximum value of x. **(6)**

Question 8

Show by considering a suitable binomial expansion that

$$1+\frac{1}{24}+\frac{1\cdot 4}{24\cdot 48}+\frac{1\cdot 4\cdot 7}{24\cdot 48\cdot 72}+\frac{1\cdot 4\cdot 7\cdot 10}{24\cdot 48\cdot 72\cdot 96}-\ldots=\frac{2}{\sqrt[3]{7}}.$$ **(7)**
