IYGB GCE

Core Mathematics C4

Advanced

Practice Paper V

Difficulty Rating: 3.9467/1.9481

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Use integration by parts to find the value of

$$\int_1^{e} \ln x \ dx .$$ **(4)**

Question 2

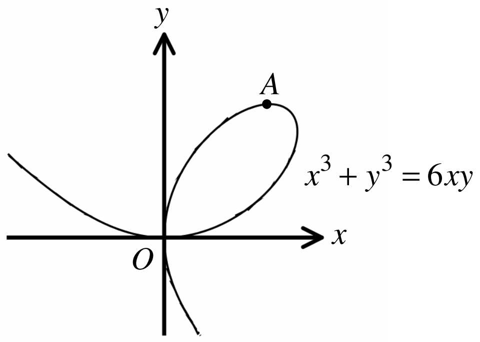
$$(125 - 27x)^{\frac{1}{3}}, \ |x| < \frac{125}{27}$$

a) Find the first three terms in the series expansion of $f(x)$. **(4)**

b) Use first three terms in the series expansion of $f(x)$ to show that **(3)**

$$\sqrt[3]{120} \approx \frac{5549}{1125}$$

Question 3



The diagram above shows a curve known as "the folium of Descartes", with equation

$$x^3 + y^3 = 6xy .$$

The curve is stationary at the origin O and at the point A.

Find the exact coordinates of A in the form $\left(2^n, 2^m\right)$, where n and m are fractions to be found. **(8)**

Question 4

The straight lines L_1 and L_2 have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 2 \\ 10 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} a \\ 8 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ b \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters, and a and b are scalar constants.

L_1 and L_2 intersect at the point P whose z coordinate is 6, and the acute angle between L_1 and L_2, is θ.

a) Determine the coordinates of P. **(2)**

b) Find the value of a and the value of b. **(2)**

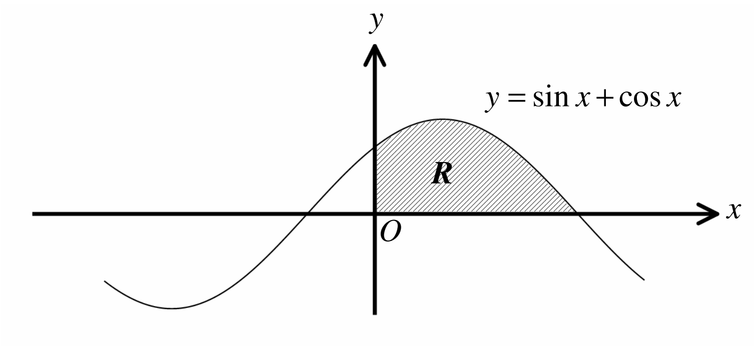
c) Show that $\cos\theta = \frac{5}{18}\sqrt{3}$. **(3)**

The point Q lies on L_1 where $\lambda = 1$.

The point T lies on L_2 so that $\overrightarrow{QT}$ is perpendicular to L_2.

d) Determine the exact distance PT. **(5)**

Question 5



The figure above shows the graph of the curve with equation

$$y = \sin x + \cos x \,, \quad -\pi \le x \le \pi \,.$$

The finite region R, shown shaded in the figure, is bounded by the curve and the coordinate axes.

When R is revolved by a full turn in the x axis it traces a solid of volume V.

Show clearly that

$$V = \frac{1}{4}\pi\left(3\pi + 2\right). \qquad \mathbf{(8)}$$

Question 6

A population P, in millions, at a given time t years, satisfies the differential equation

$$\frac{dP}{dt} = P(1-P).$$

Initially the population is one quarter of a million.

a) Solve the differential equation to show that

$$\frac{3P}{1-P} = e^{t}. \qquad \textbf{(8)}$$

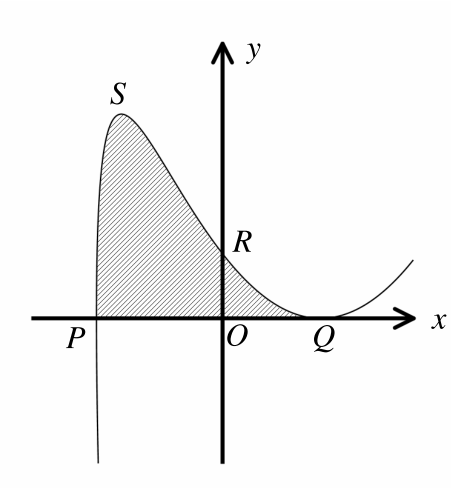
b) Show further that

$$P = \frac{1}{1+3e^{-t}}. \qquad \textbf{(3)}$$

c) Show mathematically that the limiting value for this population is one million. **(1)**

d) Find, to two decimal places, the time it takes for the population to reach three quarters of its limiting value. **(3)**

Question 7



The figure above shows part of the curve with parametric equations

$$x = t^2 - 9, \ \ y = t(4-t)^2, \ t \in \mathbb{R}.$$

The curve meets the x axis at the points P and Q, and the y axis at the points R and T. The point T is not shown in the figure.

a) Find the coordinates of each of the points P, Q, R and T. **(4)**

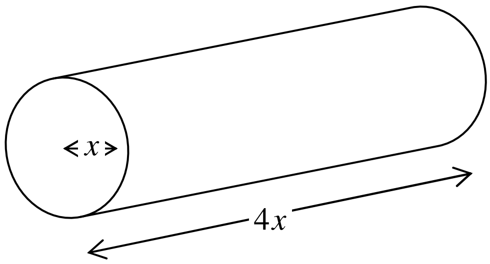
The point S is a stationary point of the curve.

b) Show that the coordinates of S are $\left(-\frac{65}{9}, \frac{256}{27}\right)$. **(5)**

The region bounded by the curve and the x axis is shown shaded in the figure above.

c) Determine an exact value for the area of the shaded region. **(5)**

Question 8



A metal bolt is in the shape of a right circular cylinder, with radius x cm and length $4x$ cm.

The bolt is heated so that the area of its circular cross section is expanding at the constant rate of $0.036 \text{ cm}^2 \text{s}^{-1}$.

Find the rate at which the volume of the bolt is increasing, when the radius of the bolt has reached 1.25 cm. **(7)**

(*You may assume that the bolt is expanding uniformly when heated.*)