IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper P

Difficulty Rating: 3.6867/1.7291

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

$$f(x) = \ln(1+\sin x), \ \sin x \neq -1.$$

a) Find the Maclaurin expansion of $f(x)$ up and including the term in x^3. **(5)**

b) Hence show that

$$\int_0^{\frac{1}{4}} \ln(1+\sin x) \ dx \approx 0.028809.$$ **(3)**

Question 2

Use the method of differences to show that

$$\frac{1}{1\times2\times3}+\frac{4}{2\times3\times4}+\frac{7}{3\times4\times5}+\ldots+\frac{3n-2}{n(n+1)(n+2)} = \frac{n^2}{(n+1)(n+2)}$$ **(7)**

Question 3

The points A and B have respective coordinates $(-1,0)$ and $(1,0)$.

The locus of the point $P(x,y)$ traces a curve in such a way so that $|AP||BP| = 1$.

a) By forming a Cartesian equation of the locus of P, show that the polar equation of the curve is

$$r^2 = 2\cos 2\theta, \ 0 \leq \theta < 2\pi.$$ **(6)**

b) Sketch the curve. **(3)**

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Question 4

The function f is defined by.

$$f(x) \equiv \frac{ax+b}{e^x}, \ x \in \mathbb{R},$$

where a and b are non zero constants.

The mean value of f in the interval $(\ln 2, \ln 4)$ is $\frac{1}{4\ln 2}$.

Given further that

$$\int_1^{\infty} f(x) \ dx = \frac{3}{e},$$

determine the value of a and the value of b. **(10)**

Question 5

The curve with the following equation is defined in the largest real domain.

$$y = (4x-3)\sqrt{-8\left(2x^2-3x+1\right)} + \arcsin(4x-3).$$

a) Show that

$$\frac{dy}{dx} = k\sqrt{-2x^2+3x-1},$$

where k is an exact constant to be found. **(7)**

b) Hence find the exact value of the following integral.

$$\int_{\frac{1}{2}}^{1} \sqrt{-2x^2+3x-1} \ dx. \quad \textbf{(4)}$$

Question 6

$$f(x) \equiv \operatorname{artanh} x \;,\; x \in \mathbb{R} \,,\, |x| < 1$$

a) Use the definition of the hyperbolic tangent to prove that

$$f(x) \equiv \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]. \qquad (5)$$

b) Use a method involving complex numbers and the trigonometric identity

$$1+\tan^2 x \equiv \sec^2 x \,,$$

to obtain the hyperbolic equivalent

$$1-\tanh^2 x \equiv \operatorname{sech}^2 x \,. \qquad (3)$$

c) Hence solve the equation

$$6\operatorname{sech}^2 x - \tanh x = 4 \,,$$

giving the two solutions in the form $\pm\frac{1}{2}\ln k$, where k are two distinct integers.

(6)

Question 7

The complex number is defined as

$$z = (1 + \mathrm{i}\tan\theta)^3, \; -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

By considering the real part of z, or otherwise, prove the validity of the following trigonometric identity

$$1 - 3\tan^2\theta \equiv \frac{\cos 3\theta}{\cos^3\theta}. \qquad (6)$$

Question 8

$$\frac{dx}{dt} + y = e^{-t} \qquad \text{and} \qquad \frac{dy}{dt} - x = e^{t} .$$

Given that $x = 0$, $y = 0$ at $t = 0$, solve the differential equations to obtain simplified expressions for $x = f(t)$ and $y = g(t)$.

(10)

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