IYGB – FP3 PAPER R – QUESTION 1

a) SETTING UP A TABLE OF VALUES WITH GAP OF 0.5

x	0	0.5	1	1.5	2	2.5	3
y	$\ln 4$	$\ln 4.25$	$\ln 5$	$\ln 6.25$	$\ln 8$	$\ln 10.25$	$\ln 13$
	FIRST	ODD	EVEN	ODD	EVEN	ODD	LAST

BY SIMPSON RULE

$$\text{AREA} \approx \frac{\text{THICKNESS}}{3}\Big[\text{FIRST} + \text{LAST} + (2\times\text{EVEN}) + (4\times\text{ODD})\Big]$$

$$\approx \frac{0.5}{3}\Big[\ln 4 + \ln 13 + 4[\ln 4.25 + \ln 6.25 + \ln 10.25] + 2[\ln 5 + \ln 8]\Big]$$

$$\approx \frac{1}{6} \times 33.75611524\ldots$$

$$\approx 5.626$$

b) USING THE ANSWER FROM PART (a)

$$\int_0^3 \ln\left(\tfrac{1}{4}x^2+1\right)\,dx = \int_0^3 \ln\left[\tfrac{1}{4}(x^2+4)\right]\,dx = \ln$$

$$= \int_0^3 \ln\tfrac{1}{4} + \ln(x^2+4)\,dx$$

$$= \int_0^3 \ln(x^2+4)\,dx + \int_0^3 \ln\tfrac{1}{4}\,dx$$

$$\approx 5.626 - \ln 4\int_0^3 1\,dx$$

$$\approx 5.626 - (\ln 4)\Big[x\Big]_0^3$$

$$\approx 5.626 - 3\ln 4$$

$$\approx 1.47$$

IYGB - FP3 PAPER R - QUESTION 2

SINCE x APPEARS IN THE EXPONENT THE USE OF LOGS MIGHT BE NECESSARY, HOWEVER L'HOSPITAL RULE WORKS WELL

$$\lim_{x \to \infty}\left[x\left(2^{\frac{1}{x}}-1\right)\right] = \lim_{x \to \infty}\left[\frac{2^{\frac{1}{x}}-1}{\frac{1}{x}}\right]$$

THIS IS AN INDETERMINATE FORM OF THE TYPE ZERO OVER ZERO

$$\dots = \lim_{x \to \infty}\left[\frac{\frac{d}{dx}\left(2^{\frac{1}{x}}-1\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}\right] = \lim_{x \to \infty}\left[\frac{2^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\times \ln 2}{-\frac{1}{x^2}}\right]$$

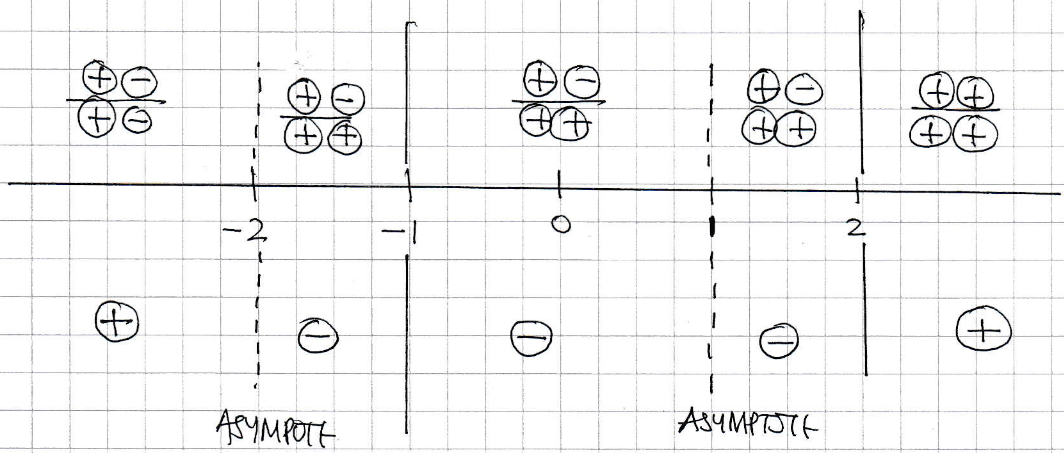
$$= \lim_{x \to \infty}\left[2^{\frac{1}{x}} \times \ln 2\right] = \ln 2$$

IYGB - FP3 PAPER R - QUESTION 3

PICK UP THE CRITICAL VALUES AS L.H.S IS FULLY FACTORIZED

$$\frac{(x+1)^2(x-2)}{(x-1)^2(x+2)} < 0 \implies x = \begin{cases} -1 & \text{REPEATED} \\ 2 & \\ 1 & \text{REPEATED} \\ -2 & \end{cases}$$

USING A NUMBER LINE



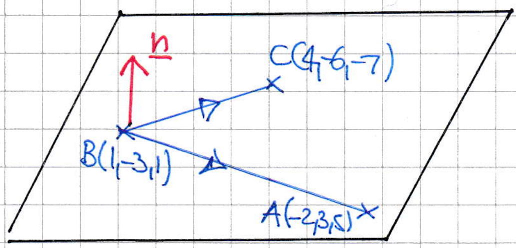
$\therefore \quad -2 < x < 2 \,, \; x \neq 1$

IYGB - FP3 PAPER R - QUESTION 4

a) START BY FINDING A NORMAL TO THE PLANE

$\overrightarrow{BC} = \underline{c} - \underline{b} = (4,-6,-7) - (1,-3,1) = (3,-3,-8)$

$\overrightarrow{BA} = \underline{a} - \underline{b} = (-2,3,5) - (1,-3,1) = (-3,6,4)$



$$\overrightarrow{BC} \wedge \overrightarrow{BA} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -3 & -8 \\ -3 & 6 & 4 \end{vmatrix} = (36,12,9)$$

SCALING THE NORMAL VECTOR

$$\underline{n} = (12,4,3)$$

EQUATION OF PLANE USING $B(1,-3,1)$

$\Rightarrow 12x + 4y + 3z = \text{CONSTANT}$

$\Rightarrow (12\times1) + 4(-3) + (3\times1) = \text{CONSTANT}$

$\Rightarrow \text{CONSTANT} = 3$

$\therefore \ 12x + 4y + 3z = 3$

b) EQUATION OF A LINE PASSING THROUGH $P(26,2,7)$, PERPENDICULAR TO THE PLANE

$\underline{r} = (26,2,7) + \lambda(12,4,3)$

$(x,y,z) = (12\lambda+26,\ 4\lambda+2,\ 3\lambda+7)$

SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE PLANE

$\Rightarrow 12(12\lambda+26) + 4(4\lambda+2) + 3(3\lambda+7) = 3$

$\Rightarrow 144\lambda + 312 + 16\lambda + 8 + 9\lambda + 21 = 3$

$\Rightarrow 169\lambda = -338$

$\Rightarrow \lambda = -2$

$\therefore \ Q(2,-6,1)$

IYGB - FP3 PAPER R - QUESTION 5

WRITE THE O.D.E AS FOLLOWS

$$y = y_0, \quad \frac{dy}{dx} = y_1, \quad \frac{d^2y}{dx^2} = y_2, \quad \frac{d^3y}{dx^3} = y_3 \quad \cdots \quad \frac{d^ny}{dx^n} = y_n$$

$$\Rightarrow y_2(x^2+1) + y_1 x - 4y_0 = 0$$

DIFFERENTIATE n TIMES BY LEIBNIZ RULE

$$\Rightarrow \frac{d^n}{dx^n}\Big[\overset{u}{y_2}\overset{v}{(x^2+1)}\Big] + \frac{d^n}{dx^n}\Big[\overset{u}{y_1}\overset{v}{x}\Big] - 4\frac{d^n}{dx^n}\Big[y_0\Big] = \cancel{\frac{d^n}{dx^n}}\Big[0\Big]$$

$$\Rightarrow y_{n+2}(x^2+1) + ny_{n+1}(2x) + \frac{n(n-1)}{2!}y_n(2) + \ldots \text{ ZERO TERMS}$$

$$+ y_{n+1}x + ny_n(1) + \ldots \text{ ZERO TERMS}$$

$$- 4y_n \qquad = 0$$

$$\Rightarrow y_{n+2}(x^2+1) + (2nx+x)y_{n+1} + (n(n-1)+n-4)y_n = 0$$

$$\Rightarrow y_{n+2}(x^2+1) + (2n+1)xy_{n+1} + (n^2-4)y_n = 0$$

FINALLY SET $x=0$

$$\Rightarrow y_{n+2} + (n^2-4)y_n = 0$$

$$\Rightarrow \frac{d^{n+2}y}{dx^{n+2}} - (4-n^2)\frac{d^ny}{dx^n} = 0$$

$$\Rightarrow \frac{d^{n+2}y}{dx^{n+2}} = (4-n^2)\frac{d^ny}{dx^n}$$

// AS REQUIRED

IYGB – FP3 PAPER R – QUESTION 6

$$\boxed{\frac{dy}{dx} = x + \ln x \qquad x=1 \ , \ y=0}$$

USING THE RESULT $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_0}{h}$ FIRST

$$y_0' = \frac{y_1 - y_0}{h}$$

$$y_1 = hy_0' + y_0$$

HERE WE HAVE $x_0 = 1$, $y_0 = 0$, $h = 0.1$

$$y_1 = h\left[x_0 + \ln x_0\right] + y_0$$

$$y_1 = 0.1\left[1 + \ln 1\right] + 0$$

$$y_1 = 0.1$$

NEXT WE ARE USING $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_{-1}}{2h}$

REWRITE AS

$$y_0' = \frac{y_1 - y_{-1}}{2h}$$

$$y_n' = \frac{y_{n+1} - y_{n-1}}{2h}$$

$$2hy_n' = y_{n+1} - y_{n-1}$$

$$y_{n+1} = 2hy_n' + y_{n-1}$$

$$y_{n+2} = 2hy_{n+1}' + y_n$$

$$\boxed{y_{n+2} = 2h\left[x_{n+1} + \ln x_{n+1}\right] + y_n}$$

WITH $x_0 = 1 \quad y_0 = 0$
$x_1 = 1.1 \quad y_1 = 0.1$
$x_2 = 1.2$
$x_3 = 1.3$

IYGB - FP3 PAPER R - QUESTION 6

APPLYING THE FORMULA IN SUCCESSION

- $y_2 \approx 2h\left[x_1 + \ln x_1\right] + y_0$

  $y_2 \approx 2\times 0.1\left[1.1 + \ln(1.1)\right] + 0$

  $y_2 \approx 0.239062... \approx \underline{0.2391}$

- $y_3 \approx 2h\left[x_2 + \ln x_2\right] + y_1$

  $y_3 \approx 2(0.1)\left[1.2 + \ln(1.2)\right] + 0.1$

  $y_3 \approx 0.376464... \approx \underline{0.3765}$

- $y_4 \approx 2h\left(x_3 + \ln x_3\right) + y_2$

  $y_4 \approx 2(0.1)\left[1.3 + \ln(1.3)\right] + 0.239062...$

  $y_4 \approx 0.551534... \approx \underline{0.5515}$

IYGB – FP3 PAPER R – QUESTION 7

a) DIFFERENTIATE THE EQUATION WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}\left[e^{-x}\frac{d^2y}{dx^2}\right] = \frac{d}{dx}\left[2y\frac{dy}{dx}\right] + \frac{d}{dx}\left[y^2+1\right]$$

$$\Rightarrow -e^{-x}\frac{d^2y}{dx^2} + e^{-x}\frac{d^3y}{dx^3} = 2\frac{dy}{dx}\frac{dy}{dx} + 2y\frac{d^2y}{dx^2} + 2y\frac{dy}{dx}$$

$$\Rightarrow e^{-x}\frac{d^3y}{dx^3} = e^{-x}\frac{d^2y}{dx^2} + 2y\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\frac{dy}{dx} + 2y\frac{dy}{dx}$$

$$\Rightarrow e^{-x}\frac{d^3y}{dx^3} = \left(e^{-x}+2y\right)\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\left[\frac{dy}{dx} + y\right]$$

AS REQUIRED

b) EVALUATE AT $x=0$

$x=0$ $\quad y=1$

$\frac{dy}{dx}=2$

$\frac{d^2y}{dx^2}=6 \longrightarrow e^{-0}\left.\frac{d^2y}{dx^2}\right|_{x=0} = 2\times1\times2+1^2+1$

$\frac{d^3y}{dx^3}=30 \longrightarrow e^{-0}\frac{d^3y}{dx^3} = \left(e^{-0}+2\times1\right)\times6 + 2\times2\left[2+1\right]$

HENCE WE HAVE

$$y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + O(x^4)$$

$$y = 1 + 2x + \frac{x^2}{2}\times6 + \frac{x^3}{6}\times30 + O(x^4)$$

$$y = 1 + 2x + 3x^2 + 5x^3 + O(x^4)$$

IYGB - FP3 PAPER R - QUESTION 8

IF $y = xv(x)$ $\Longrightarrow$ $v(x) = \dfrac{y(x)}{x}$

$$\frac{dy}{dx} = 1 \times v + x\frac{dv}{dx}$$

TRANSFORM THE O.D.E.

$$\Longrightarrow 2\frac{dy}{dx} = 1 + \frac{y^2}{x^2}$$

$$\Longrightarrow 2\left[v + x\frac{dv}{dx}\right] = 1 + v^2$$

$$\Longrightarrow 2v + 2x\frac{dv}{dx} = 1 + v^2$$

$$\Longrightarrow 2x\frac{dv}{dx} = v^2 - 2v + 1$$

$$\Longrightarrow 2x\frac{dv}{dx} = (v-1)^2$$

$$\Longrightarrow \frac{2}{(v-1)^2}\,dv = \frac{1}{x}\,dx$$

INTEGRATING BOTH SIDES SUBJECT

$x = e \quad y = -e \quad v = -1$

$$\Longrightarrow \int_{v=-1}^{v} \frac{2}{(v-1)^2}\,dv = \int_{x=e}^{x} \frac{1}{x}\,dx$$

$$\Longrightarrow \left[-\frac{2}{v-1}\right]_{-1}^{v} = \Big[\ln x\Big]_e^x$$

$$\Longrightarrow \left[\frac{2}{v-1}\right]_{v}^{-1} = \Big[\ln x\Big]_e^x$$

$$\Longrightarrow \frac{2}{-2} - \frac{2}{v-1} = \ln x - \ln e$$

$$\Longrightarrow -1 - \frac{2}{v-1} = \ln x - 1$$

$$\Longrightarrow -\frac{2}{v-1} = \ln x$$

$$\Longrightarrow -\frac{2}{\ln x} = v - 1$$

$$\Longrightarrow v = 1 - \frac{2}{\ln x}$$

$$\Longrightarrow \frac{y}{x} = 1 - \frac{2}{\ln x}$$

$$\Longrightarrow y = x - \frac{2x}{\ln x}$$

IYGB – FP3 PAPER R – QUESTION 9

a) DIFFERENTIATE W.R.T x

$$\frac{d}{dx}\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\frac{\cancel{2}x}{a^2}-\frac{\cancel{2}y}{b^2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{b^2x}{a^2y}$$

EVALUATE AT P

$$\left.\frac{dy}{dx}\right|_P=\frac{b^2(a\sec\theta)}{a^2(b\tan\theta)}=\frac{b}{a}\sec\theta\cot\theta=\frac{b}{a}\frac{1}{\cancel{\cos\theta}}\frac{\cancel{\cos\theta}}{\sin\theta}$$

$$=\frac{b}{a\sin\theta}$$

NORMAL EQUATION IS GIVEN BY

$$y-b\tan\theta=-\frac{a\sin\theta}{b}(x-a\sec\theta)$$

$$by-b^2\tan\theta=-ax\sin\theta+a^2\sin\theta\sec\theta$$

$$by+ax\sin\theta=b^2\tan\theta+a^2\sin\theta\times\frac{1}{\cos\theta}$$

$$by+ax\sin\theta=b^2\tan\theta+a^2\tan\theta$$

$$by+ax\sin\theta=(b^2+a^2)\tan\theta$$

AS REQUIRED

b) FIRSTLY FIND THE CO-ORDINATES OF $X \Rightarrow y=0$

$$ax\sin\theta=(a^2+b^2)\tan\theta$$

$$x=\frac{a^2+b^2}{a}\frac{\tan\theta}{\sin\theta}$$

$$x=\frac{a^2+b^2}{a}\sec\theta$$

$$\therefore X\left(\frac{a^2+b^2}{a}\sec\theta,0\right)$$

IYGB - FP3 PAPER R - QUESTION 9

USING THE ECCENTRICITY RELATION

$$b^2 = a^2(e^2-1)$$

$$b^2 = a^2\left(\frac{9}{4}-1\right)$$

$$b^2 = \frac{5}{4}a^2 \implies X\left(\frac{a^2+\frac{5}{4}a^2}{a}\sec\theta, 0\right)$$

$$X\left(\frac{9}{4}a\sec\theta, 0\right)$$

NEXT THE FOCI S WITH POSITIVE x CO-ORDINATE

$$S(ae, 0) \implies S\left(\frac{3}{2}a, 0\right)$$

FINALLY WE HAVE

$$\implies |OX| = 3|OS|$$

$$\implies \frac{9}{4}\cancel{a}\sec\theta = 3\times\frac{3}{2}\cancel{a}$$

$$\implies \frac{9}{4}\sec\theta = \frac{9}{2}$$

$$\implies \sec\theta = 2$$

$$\implies \cos\theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3} \ \& \ \frac{5\pi}{3}$$