IYGB - FP4 PAPER N - QUESTION 1

a) INTRODUCE SOME SENSIBLE LABELLING

- a = ANTICLOCKWISE ROTATION BY $90°$ (ANTICLOCKWISE BY $270°$)
- b = ROTATION BY $180°$
- c = ANTICLOCKWISE ROTATION BY $270°$ (CLOCKWISE BY $90°$)
- d = ROTATION BY $360°$ (IDENTITY)

HENCE THE TABLE WILL BE

	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

b) CONSTRUCTING THE TABLE

	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

BOTH HAVE 3 ELEMENTS & IDENTITY

$a^4 = d$ $\qquad 3^4 = 81 \mapsto 6$

$b^2 = d$ $\qquad 9^2 = 81 \mapsto 6$

$c^4 = d$ $\qquad 12^4 = \dots \mapsto 6$

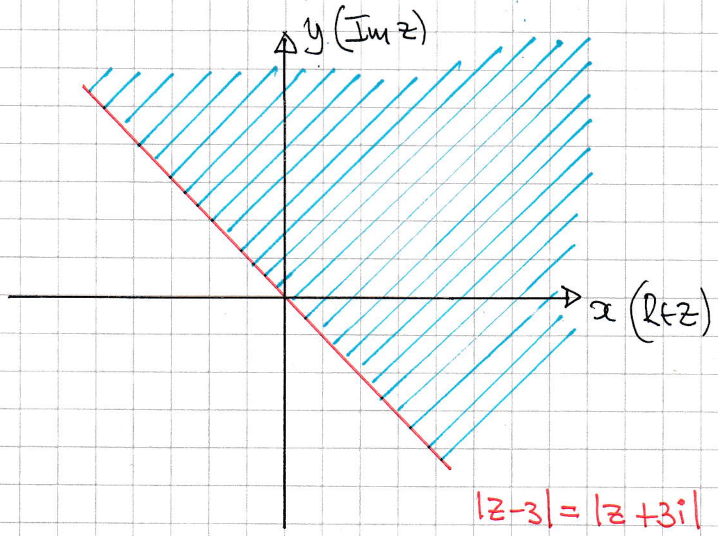
$(12^4 = 12^2\, 12^2 = 9 \times 9 = 81 = 6)$

$\therefore$ ISOMORPHIC WITH $\{a, b, c, d\} \longmapsto \{3, 9, 12, 6\}$

IYGB - FP4 PAPER N - QUESTION 2

LET $z = x + iy$ & "SOLVE" THE CORRESPONDING EQUATION

$$\Rightarrow |z-3| = |z+3i|$$
$$\Rightarrow |x+iy-3| = |x+iy+3i|$$
$$\Rightarrow |(x-3)+iy| = |x+(y+3)i|$$
$$\Rightarrow \sqrt{(x-3)^2+y^2} = \sqrt{x^2+(y+3)^2}$$
$$\Rightarrow (x-3)^2+y^2 = x^2+(y+3)^2$$
$$\Rightarrow x^2-6x+9+y^2 = x^2+y^2+6y+9.$$
$$\Rightarrow -6x = 6y$$
$$\Rightarrow y = -x$$



TO ESTABLISH THE REGION

- EITHER

  TRY A POINT $z = 1$

  $$|1-3| \leqslant |1+3i|$$
  $$|-2| \leqslant |1+3i|$$
  $$2 \leqslant \sqrt{10}$$

  WHICH IS TRUE, SO THE POINT $(1,0)$ LIES IN THE REGION

- OR BE CAREFUL

  $$|z-3| \leqslant |z+3i|$$
  $$\vdots \qquad \vdots$$
  $$-6x \leqslant 6y$$
  $$-x \leqslant y$$
  $$y \geqslant -x$$

IYGB - FP4 PAPER N - QUESTION 3

USING THE STANDARD SURFACE FORMULA FOR $y = x^3$

$$S = \int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = 2\pi \int_0^1 x^3 \sqrt{1 + (3x^2)^2}\, dx$$

$$S = 2\pi \int_0^1 x^3 (1 + 9x^4)^{\frac{1}{2}}\, dx.$$

NOW BY SUBSTITUTION OR RECOGNITION

$$S = 2\pi \left[\frac{1}{54}(1 + 9x^4)^{\frac{3}{2}}\right]_0^1 = \frac{\pi}{27}\left[10^{\frac{3}{2}} - 1\right] = \frac{\pi}{27}\left[10\sqrt{10} - 1\right]$$

AS REQUIRED

IYGB - FP4 PAPER N - QUESTION 4

a) IF THERE IS NO RESTRICTION IN THE GENDER

"TEAMS OF 8 OUT OF 15" $= \binom{15}{8} = \frac{15!}{8!\,7!} = 6435$

b) IF THERE IS GENDER RESTRICTION

"MORE GIRLS THAN BOYS"

GIRLS (7)	BOYS (8)
7	1
6	2
5	3

$$\binom{7}{7} \times \binom{8}{1} = 1 \times 8 = 8$$

$$\binom{7}{6} \times \binom{8}{2} = 7 \times 28 = 196$$

$$\binom{7}{5} \times \binom{8}{3} = 21 \times 56 = 1176$$

$$1380$$

IYGB – FP4 PAPER N – QUESTION 5

PROCEED BY INTEGRATION BY PARTS

x^n	nx^{n-1}
$-\frac{2}{5}(1-x)^{\frac{5}{2}}$	$(1-x)^{\frac{3}{2}}$

$$I_n = \int_0^1 x^n(1-x)^{\frac{3}{2}}\,dx$$

$$I_n = \cancel{\left[-\frac{2}{5}(1-x)^{\frac{5}{2}}x^n\right]_0^1} - \int_0^1 -\frac{2}{5}nx^{n-1}(1-x)^{\frac{5}{2}}\,dx$$

ZERO AT BOTH ENDS

$$I_n = \frac{2}{5}n\int_0^1 x^{n-1}(1-x)^{\frac{5}{2}}\,dx$$

$$I_n = \frac{2}{5}n\int_0^1 x^{n-1}(1-x)^1(1-x)^{\frac{3}{2}}\,dx$$

$$I_n = \frac{2}{5}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}} - x^n(1-x)^{\frac{3}{2}}\,dx$$

$$I_n = \frac{2}{5}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}}\,dx - \frac{2}{5}n\int_0^1 x^n(1-x)^{\frac{3}{2}}\,dx$$

$$I_n = \frac{2}{5}nI_{n-1} - \frac{2}{5}nI_n$$

$$5I_n = 2nI_{n-1} - 2nI_n$$

$$(2n+5)I_n = 2nI_{n-1}$$

$$I_n = \frac{2n}{2n+5}I_{n-1}$$

AS REQUIRED

FINALLY TO EVALUATE I_3

$$I_3 = \frac{2\times3}{2\times3+5}I_2 = \frac{6}{11}I_2$$

$$= \frac{6}{11}\left[\frac{2\times2}{2\times2+5}I_1\right] = \frac{6}{11}\times\frac{4}{9}I_1$$

IYGB - FP4 PAPER N - QUESTION 5

$$I_3 = \frac{6}{11} \times \frac{4}{9} I_1$$

$$I_3 = \frac{24}{99} I_1 = \frac{8}{11} I_1$$

$$I_3 = \frac{8}{33}\left[\frac{2\times1}{2\times1+5} I_0\right] = \frac{8}{33} \times \frac{2}{7} I_0 = \frac{16}{231} I_0$$

THUS WE NOW HAVE

$$I_3 = \frac{16}{231}\int_0^1 (1-x)^{\frac{3}{2}}\,dx$$

$$= \frac{16}{231}\left[-\frac{2}{5}(1-x)^{\frac{5}{2}}\right]_0^1$$

$$= \frac{16}{231}\left[\frac{2}{5}(1-x)^{\frac{5}{2}}\right]_1^0$$

$$= \frac{32}{1155}$$

IYGB - FP4 PAPER N - QUESTION 6

a) • $\frac{a}{b} = 20 + \frac{17}{b}$ OR $a = 20b + 17$, $a, b \in \mathbb{N}$

THEN

• $\frac{a}{5} = \frac{20b}{5} + \frac{17}{5}$ OR $a = 20b + 15 + 2$

$\frac{a}{5} = 4b + 3 + \frac{2}{5}$ $a = 5(4b) + (5\times3) + 2$

$\frac{a}{5} = (4b+3) + \frac{2}{5}$ $a = 5(4b+3) + 2$

∴ THE REMAINDER IS 2

b) SUPPOSE THERE EXIST AN INTEGER a, SO THAT...

• ITS DIVISION BY 8, YIELDS REMAINDER 6

$\Rightarrow a = 8n + 6$, $n \in \mathbb{N}$

$\Rightarrow a = 2[4n+3]$

∴ a IS EVEN

• ITS DIVISION BY 18, YIELDS REMAINDER 3

$\Rightarrow a = 18m + 3$, $m \in \mathbb{N}$

$\Rightarrow a = 18m + 2 + 1$

$\Rightarrow a = 2(9m+1) + 1$

∴ a IS ODD

HENCE THERE IS NO SUCH INTEGER

IYGB - FP4 PAPER N - QUESTION 7

a) $\underline{M} = \underline{P}\,\underline{D}\,\underline{P}^{-1} = \begin{pmatrix} -1 & 4 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 27 \end{pmatrix}\frac{1}{-13}\begin{pmatrix} 1 & -4 \\ -3 & -1 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} -1 & 4 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 27 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 3 & 1 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} -1 & 108 \\ 3 & 27 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 3 & 1 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} 325 & 104 \\ 78 & 13 \end{pmatrix}$

$= \begin{pmatrix} 25 & 8 \\ 6 & 1 \end{pmatrix}$ //

b) EIGENVALUE $\lambda = 1$, WITH EIGENVECTOR $\alpha\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

EIGENVALUE $\lambda = 27$, WITH EIGENVECTOR $b\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ //

c) THIS CORRESPONDS TO EIGENVALUE $\lambda = 1 \Rightarrow \begin{pmatrix} -1 \\ 3 \end{pmatrix} \Rightarrow y = -3x$ //

d) AS M HAS INTEGER ENTRIES M^n MUST ALSO HAVE INTEGER ENTRIES, ALL OF WHICH MUST BE DIVISIBLE BY 13

HENCE: $\underline{M}^n_{11} - \underline{M}^n_{21} = \frac{1}{13}\left[(4\times3^{3n+1}+1) - (3^{3n+1}-3)\right]$

$= \frac{1}{13}\left[4\times3^{3n+1} - 3^{3n+1} + 4\right]$

$= \frac{1}{13}\left[3\times3^{3n+1} + 4\right]$

$= \frac{1}{13}\left[3^{3n+2} + 4\right]$

AS REQUIRED

IYGB - FP4 PAPER N - QUESTION 8

WORK AS FOLLOWS

$$\Rightarrow f(z) = (z+2i)^2$$
$$\Rightarrow w = (z+2i)^2$$
$$\Rightarrow u+iv = (x+iy+2i)^2$$
$$\Rightarrow u+iv = [x+(y+2)i]^2$$
$$\Rightarrow u+iv = x^2+2x(y+2)i-(y+2)^2$$
$$\Rightarrow u+iv = [x^2-(y+2)^2]+[2x(y+2)]i$$

BUT $y = x-1$

$$\Rightarrow u+iv = [x^2-(x-1+2)^2]+[2x(x-1+2)]i$$
$$\Rightarrow u+iv = [x^2-(x+1)^2]+[2x(x+1)]i$$
$$\Rightarrow u+iv = [x^2-x^2-2x-1]+[2x^2+2x]i$$
$$\Rightarrow u+iv = [-2x-1]+[2x^2+2x]i$$

ELIMINATE x AS A PARAMETER

$$u = -2x-1$$
$$2x = -u-1$$
$$x = -\frac{u+1}{2}$$

$$\Rightarrow v = 2x^2+2x$$
$$\Rightarrow v = 2\left(-\frac{u+1}{2}\right)^2+(-u-1)$$
$$\Rightarrow v = 2\frac{(u+1)^2}{4}-u-1$$
$$\Rightarrow v = \frac{1}{2}(u^2+2u+1)-u-1$$
$$\Rightarrow v = \frac{1}{2}u^2+\cancel{u}+\frac{1}{2}-\cancel{u}-1$$
$$\Rightarrow v = \frac{1}{2}u^2-\frac{1}{2}$$
$$\Rightarrow v = \frac{1}{2}(u^2-1)$$

IYGB – FP4 PAPER N – QUESTION 9

REWRITE THE EQUATION

$$u_n = 5u_{n-1} - 4u_{n-2} - 12n + 31$$

$$u_n - 5u_{n-1} + 4u_{n-2} = -12n + 31$$

"AUXILIARY" EQUATION

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda - 1)(\lambda - 4) = 0$$

$$\lambda = \begin{cases} 1 \\ 4 \end{cases}$$

"COMPLIMENTARY" FUNCTION

$$u_n = A \times 1^n + B \times 4^n$$

$$u_n = A + B \times 4^n$$

NEXT LOOK FOR "THE PARTICULAR INTEGRAL" – TRY A POLYNOMIAL

$$u_n = Pn + Qn^2$$ (AS THE CONSTANT "A" IS ALREADY THERE)

$$u_{n-1} = P(n-1) + Q(n-1)^2$$

$$u_{n-2} = P(n-2) + Q(n-2)^2$$

SUBSTITUTE BACK INTO THE LHS OF THE DIFFERENCE EQUATION

$$\Rightarrow Pn + Qn^2 - 5\left[P(n-1) + Q(n-1)^2\right] + 4\left[P(n-2) + Q(n-2)^2\right] \equiv -12n + 31$$

$$\Rightarrow Pn + Qn^2 - 5\left[Pn - P + Qn^2 - 2Qn + Q\right] + 4\left[Pn - 2P + Qn^2 - 4Qn + 4Q\right] \equiv -12n + 31$$

$$\Rightarrow \cancel{Pn} + \cancel{Qn^2} - \cancel{5Pn} + 5P - \cancel{5Qn^2} + 10Qn - 5Q + \cancel{4Pn} - 8P + \cancel{4Qn^2} - 16Qn + 16Q \equiv -12n + 31$$

$$\Rightarrow -3P - 6Qn + 11Q \equiv -12n + 31$$

$$\Rightarrow -6Qn + (11Q - 3P) \equiv -12n + 31$$

IYGB - FP4 PAPER N - QUESTION 9

COMPARING COEFFICIENTS

- $-6Q = -12$
  $Q = 2$

- $11Q - 3P = 31$
  $22 - 3P = 31$
  $-3P = 9$
  $P = -3$

$$\therefore \quad u_n = A + B \times 4^n - 3n + 2n^2$$

FINALLY APPLY CONDITIONS

- $u_0 = 7$
  $A + B = 7$

- $u_1 = 9$
  $A + 4B - 3 + 2 = 9$
  $A + 4B = 10$

$$A + B = 7$$
$$A + 4B = 10$$

$$3B = 3$$
$$B = 1 \quad \& \quad A = 6$$

$$\therefore \quad u_n = 6 + 4^n - 3n + 2n^2$$