IYGB - FS1 PAPER O - QUESTION 1

$$P(X=x) = \begin{cases} kx^2 & x = 3,4,5 \\ 0 & \text{OTHERWISE} \end{cases}$$

a) WRITE THE FORMULA AS A TABLE

x	3	4	5
$P(X=x)$	$9k$	$16k$	$25k$

$$9k + 16k + 25k = 1$$
$$50k = 1$$
$$k = \frac{1}{50}$$

b) I) $E(X) = \sum x P(X=x)$

$$\Rightarrow E(X) = (3 \times 9k) + (4 \times 16k) + (5 \times 25k)$$
$$\Rightarrow E(X) = 27k + 64k + 125k$$
$$\Rightarrow E(X) = 216k$$
$$\Rightarrow E(X) = 4.32$$

II) $E(X^2) = \sum x^2 P(X=x)$

$$\Rightarrow E(X^2) = (3^2 \times 9k) + (4^2 \times 16k) + (5^2 \times 25k)$$
$$\Rightarrow E(X^2) = 81k + 256k + 625k$$
$$\Rightarrow E(X^2) = 962k$$
$$\Rightarrow E(X^2) = 19.24$$

IYGB - FS1 PAPER O - QUESTION 1

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow \text{Var}(X) = 19.24 - 4.32^2$$

$$\Rightarrow \text{Var}(X) = 0.5776$$

c) I) $E(5X-4) = 5E(X) - 4$

$= 5 \times 4.32 - 4$

$= 17.6$

II) $\text{Var}(5X-4) = 5^2 \text{Var}(X)$

$= 25 \times 0.5776$

$= 14.44$

IYGB - FS1 PAPER O - QUESTION 2

a) MODEL AS GEOMETRIC, X = NO OF WINS

$$X \sim \text{Geo}(0.4)$$

I) $P(X=3) = 0.6^2 \times 0.4^1 = 0.144$

II) $P(X>3) = P(X \geqslant 4) = 0.6^3 = 0.216$

"NEEDS" 7 & "HAS" 2 $\Rightarrow$ NEEDS ANOTHER 5 WINS IN 10 BATTLES

MODEL AS NEGATIVE BINOMIAL, $Y \sim \text{Negative } B(5, 0.4)$

$$P(Y=10) = \binom{9}{4}(0.4)^5(0.6)^5 = 0.1003$$

c) WINNING/LOSING A BATTLE IS INDEPENDENT OF ONE ANOTHER

PROBABILITY OF WINNING/LOSING IS CONSTANT

IYGB - FS1 PAPER O - QUESTION 3

SETTING SUITABLE HYPOTHESES

- H_0 : THERE IS NO ASSOCIATION BETWEEN THE EDUCATION LEVEL AND THE EVENTUAL AVERAGE EARNINGS (INDEPENDENCE)
- H_0 : THERE IS ASSOCIATION BETWEEN THE EDUCATION LEVEL AND THE EVENTUAL AVERAGE EARNINGS (NOT INDEPENDENT)

EARNINGS	EDUCATION LEVEL: NON GRADUATES	GRADUATES	POST GRADUATES	TOTAL
UP TO £10000	17 16.56 0.0117	~~3~~ 6 6.44 0.0301	~~3~~	23
£10001 TO £25000	97 83.52 2.1757	~~16~~ 19 32.48 5.5945	~~3~~	116
£25001 TO £40000	42 51.12 1.6270	~~21~~ 29 19.6 4.5081	~~8~~	71
OVER £40000	24 28.8 0.8000	~~10~~ 16 11.2 2.0571	~~6~~	40
TOTAL	180	~~50~~ 70	~~20~~	250

FIRSTLY LOOKING AT SOME OF THE FREQUENCIES, WE SUSPECT THAT THE EXPECTED FREQUENCIES MIGHT FALL BELOW 5

- CHECK "POST GRADS UNDER £10000" : $\frac{23 \times 20}{250} = 1.84 < 5$
- COMBINE THE LAST TWO COLUMNS OF THE TABLE

COMPUTE EXPECTED FREQUENCIES & CONTRIBUTIONS

- ● (black) : OBSERVED FREQUENCIES (ACTUAL DATA), O_i
- ● (red) : EXPECTED FREQUENCIES FOR INDEPENDENCE, E_i
- ● (blue) : CONTRIBUTIONS, $\frac{(O_i - E_i)^2}{E_i}$

IYGB - FS1 PAPER O - QUESTION 3

SUMMARIZING ALL RESULTS

$$\nu = 3$$

$$\chi_3^2(1\%) = 11.345$$

$$\sum_{i=1}^{8} \frac{(O_i - E_i)^2}{E_i} = 18.861$$

AS $18.861 > 11.345$ THERE IS SIGNIFICANT EVIDENCE OF ASSOCIATION (DEPENDENCE) BETWEEN THE LEVEL OF EDUCATION AND THE EXPECTED AVERAGE ANNUAL EARNINGS

THERE IS SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER O - QUESTION 4

a) LOOKING AT THE EXPRESSION

$$(1+t)^3 = 1+3t+3t^2+t^3$$

HENCE WE HAVE

$$G_X(t) = k(1+3t+3t^2+t^3)^3$$
$$G_X(t) = k[(1+t)^3]^3$$
$$G_X(t) = k(1+t)^9$$

COMPARE WITH THE BINOMIAL $(9, \frac{1}{2})$ WITH $G_X(t) = (1-\frac{1}{2}+\frac{1}{2}t)^9$

$$G_X(t) = \left(\frac{1}{2}+\frac{1}{2}t\right)^9 = \frac{1}{2^9}(1+t)^9 = \frac{1}{512}(1+t)^9 \quad \text{IF } k = \frac{1}{512}$$

$$\therefore X \sim B(9, \tfrac{1}{2})$$

b) USING THE RESULTS $E(X) = G_X'(1)$ & $Var(X) = G_X''(1) + G_X'(1) - [G_X'(1)]^2$

USING $G_X(t) = \frac{1}{512}(1+t)^9$

$$G_X'(t) = \frac{9}{512}(1+t)^8 \qquad G_X'(1) = \frac{9}{2}$$
$$G_X''(t) = \frac{9}{64}(1+t)^7 \qquad G_X''(1) = 18$$

$$\therefore E(X) = G_X'(1) = \frac{9}{2}$$

$$Var(X) = G_X''(1) + G_X'(1) - [G_X'(1)]^2$$
$$Var(X) = 18 + \frac{9}{2} - \left(\frac{9}{2}\right)^2$$
$$\therefore Var(X) = 2.25$$

IYGB - FS1 PAPER O - QUESTION 5

- AS THE TEST IS TWO TAILED, THE SIGNIFICANCE MUST BE 2.5% IN EACH TAIL

- LOOKING AT THE POISSON TABLES (ON PAPER OR CALCULATOR)

  - "BOTTOM END"

    $W = 0, 1$ THIS OCCURS FOR INTEGER VALUES OF λ IF $k = 6, 7$

  - "TOP END"

    $W = 12, 13, 14, \ldots$ THIS OCCURS FOR INTEGER VALUES OF λ IF $k = 6$ (ONLY)

- HENCE $k = 6$ AND BY USING TABLES STILL

$$P(W \leq 1) = \ldots \text{tables} = 0.0174 = 1.74\%$$

$$P(W \geq 12) = \ldots \text{tables} = 1 - P(W \leq 11) = 1 - 0.9799$$

$$= 0.0201 = 2.01\%$$

- ACTUAL SIGNIFICANCE IS $1.74\% + 2.01\% = 3.75\%$

IYGB – FS1 PAPER O – QUESTION 6

a) MODEL AS NEGATIVE BINOMIAL

X = NUMBER OF VIDEOS STEVE HAS TO FILM

$X \sim NB(5, 0.25)$

- $P(X=11) = \underbrace{\binom{10}{4}(0.25)^4(0.75)^6}_{\text{4 "GOOD" VIDEOS IN THE FIRST 10}} \times \underbrace{0.25}_{\text{11TH VIDEO IS GOOD}} = 0.0365$

b) THIS IS JUST THE EXPECTATION OF $X \sim NB(r, p)$

- $E(X) = \frac{r}{p} = \frac{5}{0.25} = 20$

c) FIRSTLY CALCULATE THE VARIANCE

$$\text{Var}(X) = \frac{r(1-p)}{p^2} = \frac{5 \times 0.75}{0.25^2} = 60$$

AS THE SAMPLE SIZE IS LARGE ($n > 30$), BY THE C.L.T

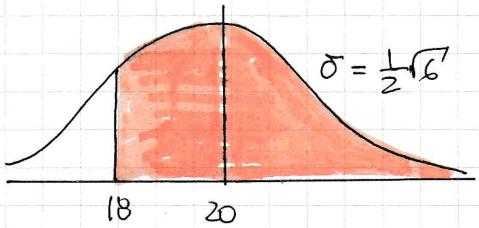
$$\bar{X}_{40} \sim N\left(20, \frac{60}{40}\right) \quad \text{ie} \quad \bar{X}_{40} \sim N\left(20, \left(\tfrac{1}{2}\sqrt{6}\right)^2\right)$$

- $P(\bar{X}_{40} > 18)$

$= P\left(z > \frac{18-20}{\frac{1}{2}\sqrt{6}}\right)$

$= \Phi(-1.6330)$

$= 0.949$

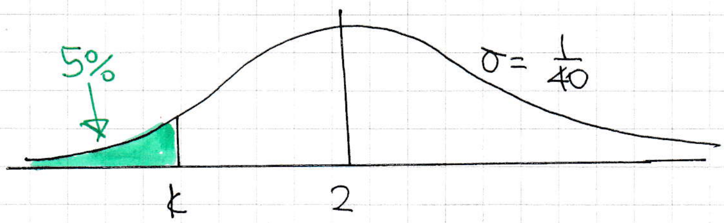


IYGB - FS1 PAPER D - QUESTION 7

START FOR COMPUTING THE CRITICAL REGION

- X = WEIGHT OF BAG OF FLOUR
- $X \sim N(2, 0.05^2)$
- $\bar{X}_4 \sim N\left(2, \frac{0.05^2}{4}\right)$ IE $\bar{X}_4 \sim N\left(2, \left(\frac{1}{40}\right)^2\right)$

LOOKING AT THE DIAGRAM — ONE TAILED TEST AT 5%



$$P(\bar{X}_4 < k) = 0.05$$
$$P(\bar{X}_4 > k) = 0.95$$
$$P\left(Z > \frac{k-2}{1/40}\right) = 0.95$$
$$\frac{k-2}{\frac{1}{40}} = -\Phi^{-1}(0.95)$$
$$40(k-2) = -1.6449$$
$$40k - 80 = -1.6449$$
$$k = 1.95887 5\ldots$$

I.E C.R. $\bar{X}_4 < 1.9589$

PROBABILITY OF A TYPE II ERROR

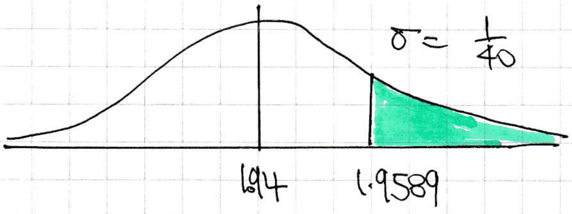
$\Rightarrow$ REJECT H_1 WHEN H_1 IS TRUE

$\Rightarrow$ NOT IN C.R WHEN H_1 IS TRUE

$\Rightarrow$ $\bar{X}_4 > 1.9589$ WHEN $\mu = 1.94$

IYGB - FS1 PAPER O - QUESTION 7

REDRAW THE DIAGRAM



$$P(\bar{X}_4 > 1.9589) = 1 - P(\bar{X}_4 < 1.9589)$$
$$= 1 - P\left(z < \frac{1.9589 - 1.94}{1/40}\right)$$
$$= 1 - \Phi(0.7551)$$
$$= 1 - 0.7749$$
$$= 0.2251 \quad \longleftarrow \quad P(\text{TYPE II ERROR})$$

$\therefore$ POWER OF THE TEST $= 1 - P(\text{TYPE II ERROR})$
$$= 1 - 0.2251$$
$$= 0.7749$$

IYGB - FS1 PAPER O - QUESTION 8

a) START BY DEFINING DISTRIBUTIONS

I) X = no of tube fails per weekday
$X \sim Po(1)$

II) Y = no of tube fails per weekend
$Y \sim Po(0.5)$

$$P(X=4) = \frac{e^{-1} \times 1^4}{4!} = \underline{0.0153}$$

$$P(Y>2) = P(Y \geq 3)$$
$$= 1 - P(Y \leq 2)$$
... tables ...
$$= 1 - 0.9856$$
$$= \underline{0.0144}$$

III)
$X+Y \sim Po(5 \times 1 + 0.5)$
$X+Y \sim Po(5.5)$

$$P(X+Y < 4) = P(X+Y \leq 3) = \text{... tables ...} = \underline{0.2017}$$

b) USING $X+Y \sim Po(5.5)$

$$\Rightarrow P(X+Y > n) < 1\%$$
$$\Rightarrow P(X+Y \geq n+1) < 0.01$$
$$\Rightarrow 1 - P(X+Y) \leq n) < 0.01$$
$$\Rightarrow -P(X+Y \leq n) < -0.99$$
$$\Rightarrow P(X+Y \leq n) > 0.99$$

LOOKING AT THE TABLES OF $Po(5.5)$

$$\Rightarrow \underline{n = 12}$$