IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 1

WRITE OUT A FEW TERMS & LOOK FOR A PATTERN

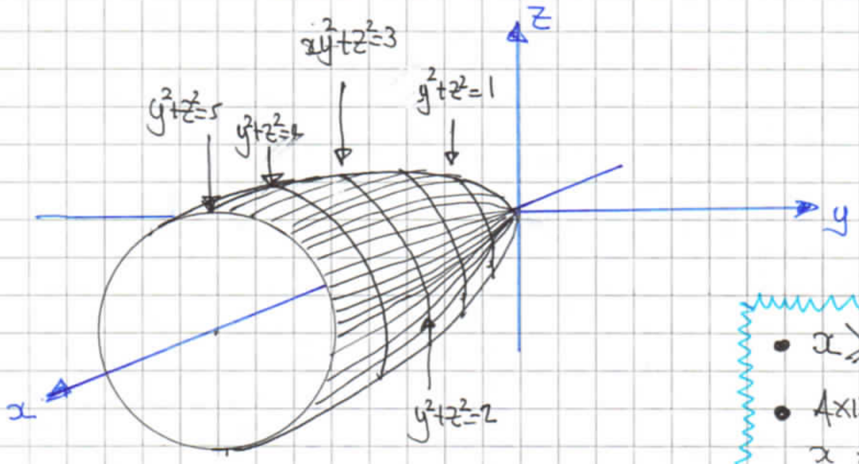
$$\prod_{r=1}^{n}\left(\frac{r+1}{r}\right) = \frac{2}{1}\times\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\dots\times\frac{n+1}{n}$$

$$= \frac{n+1}{1}$$

$$= n+1$$

IYGB - MATHEMATICAL METHODS I - PAPER E - QUESTION 2

THIS EQUATION REPRESENTS A PARABOLOID



- $x \geqslant 0$
- AXIS OF SYMMETRY, THE POSITIVE x AXIS
- VERTEX AT THE ORIGIN

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$$\boxed{T_{n+1} = 2T_n - 5 \quad , \quad T_1 = 6}$$

- "AUXILIARY EQUATION"

$$T_{n+1} - 2T_n = -5$$

$$\Rightarrow \lambda - 2 = 0$$

$$\Rightarrow \lambda = 2$$

- "COMPLEMENTARY FUNCTION"

$$T_n = A \times 2^n$$

- "PARTICULAR INTEGRAL"

$$T_n = P \leftarrow \text{CONSTANT}$$

$$T_{n+1} = P$$

- SUB INTO THE RELATION

$$\Rightarrow P - 2P = -5$$

$$\Rightarrow -P = -5$$

$$\Rightarrow P = 5$$

GENERAL SOLUTION: $T_n = A \times 2^n + 5$

APPLY THE BOUNDARY CONDITION, $T_1 = 6$

$$\Rightarrow 6 = A \times 2^1 + 5$$

$$\Rightarrow 1 = 2A$$

$$\Rightarrow A = \frac{1}{2}$$

HENCE WE FINALLY HAVE

$$\Rightarrow T_n = \frac{1}{2} \times 2^n + 5$$

$$\Rightarrow T_n = 2^{n-1} + 5$$

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- FIND THE SOLUTION BY THE JORDAN-GAUSS ALGORITHM

$$\left.\begin{aligned} x+2y+z&=1\\ x+y+3z&=2\\ 3x+5y+5z&=4 \end{aligned}\right\} \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 1 & 1 & 3 & 2\\ 3 & 5 & 5 & 4 \end{array}\right] \xrightarrow[r_{13}(-3)]{r_{12}(-1)}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & -1 & 2 & 1\\ 0 & -1 & 2 & 1 \end{array}\right] \xrightarrow{r_2(-1)} \left[\begin{array}{cccc} 1 & 2 & 1 & 1\\ 0 & 1 & -2 & -1\\ 0 & -1 & 2 & 1 \end{array}\right] \xrightarrow{r_{23}(1)}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 1 & -2 & -1\\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{r_{21}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 3\\ 0 & 1 & -2 & -1\\ 0 & 0 & 0 & 0 \end{array}\right]$$

- EXTRACT THE SOLUTION

$$\left.\begin{aligned} x+5z&=3\\ y-2z&=-1 \end{aligned}\right\} \Longrightarrow \begin{aligned} x&=3-5z\\ y&=-1+2z \end{aligned}$$

$$\Longrightarrow \text{LET } z=t$$

$$\Longrightarrow \begin{aligned} x&=3-5t\\ y&=2t-1\\ z&=t \end{aligned}$$

AS REQUIRED

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 5

a) $$\frac{1}{D^2-4D+3}\left\{e^{2x}\right\} = \frac{1}{2^2-4\times2+3}\times e^{2x} = -e^{2x}$$

b) $$\frac{1}{D^2-4D+3}\left\{e^{3x}\right\} = \frac{1}{3^2-4\times3+3}\times e^{3x} \ \ldots$$ THERE IS A PROBLEM

INTRODUCE A FUNCTION $V(x) = 1 = e^{0x}$

$$= \frac{1}{D^2-4D+3}\left\{1\times e^{3x}\right\} = \frac{1\times e^{3x}}{(D+3)^2-4(D+3)+3}\left\{1\right\} = \frac{e^{3x}}{D^2+2D}\left\{1\right\}$$

$$= \frac{e^{3x}}{D^2+2D}\left\{e^{0x}\right\} = \frac{e^{3x}}{D(D+2)}\left\{e^{0x}\right\}$$

$$= \frac{e^{3x}}{D}\cdot\frac{1}{D+2}\left\{e^{0x}\right\} = \frac{e^{3x}}{D}\left\{\frac{1}{2}e^{0x}\right\}$$

$$= e^{3x}\frac{1}{D}\left\{\frac{1}{2}\right\} = \frac{1}{2}xe^{3x}$$

c) $$\frac{1}{D^2-4D+3}\left\{\sin 2x\right\} = \frac{1}{-2^2-4D+3}\left\{\sin 2x\right\} = \frac{1}{-1-4D}\left\{\sin 2x\right\}$$

$$= \frac{-1}{4D+1}\left\{\sin 2x\right\} = \frac{-1(4D-1)}{(4D+1)(4D-1)}\left\{\sin 2x\right\}$$

$$= \frac{1-4D}{16D^2-1}\left\{\sin 2x\right\} = \frac{1-4D}{16(-2^2)-1}\left\{\sin 2x\right\}$$

$$= \frac{1-4D}{-65}\left\{\sin 2x\right\} = \frac{4D-1}{65}\left\{\sin 2x\right\}$$

$$= \frac{1}{65}(4D-1)\left\{\sin 2x\right\} = \frac{1}{65}\left[4\times2\cos 2x - \sin 2x\right]$$

$$= \frac{1}{65}\left(8\cos 2x - \sin 2x\right)$$

IYGB – MATHEMATICAL METHODS 1 – PAPER E – QUESTION 5

d) $\frac{1}{D^2+1}\{\cos x\} = \frac{1}{-1^2+1}\{\cos x\} = \ldots$ FAILS

PROCEED BY COMPLEX NUMBERS AS FOLLOWS

$$\frac{1}{D^2+1}\{\cos x\} = \frac{1}{D^2+1}\{\underbrace{1}_{V(x)}\times \mathrm{Re}(e^{ix})\} = \mathrm{Re}\left[\frac{1}{D^2+1}\{\underbrace{1}_{V(x)}\times e^{ix}\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{(D+i)^2+1}\{1\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{D^2+2Di}\{1\}\right] = \mathrm{Re}\left[\frac{e^{ix}}{D(D+2i)}\{1\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{D}\cdot\frac{1}{D+2i}\{1\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{D}\cdot\frac{1}{D+2i}\{e^{0x}\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{D}\left\{\frac{1}{0+2i}e^{0x}\right\}\right]$$

$$= \mathrm{Re}\left[\frac{e^{ix}}{2i}\frac{1}{D}\{1\}\right] = \mathrm{Re}\left[\frac{e^{ix}}{2i}x\right]$$

$$= \frac{1}{2}x\,\mathrm{Re}\left[\frac{e^{ix}}{i}\right] = \frac{1}{2}x\,\mathrm{Re}\left[-ie^{ix}\right]$$

$$= \frac{1}{2}x\,\mathrm{Re}\left[-i\cos x - i(i\sin x)\right]$$

$$= \frac{1}{2}x\sin x$$

//

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 6

$$\boxed{f(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x}$$

- FIND THE FIRST ORDER DERIVATIVES AND SET THEM EQUAL TO ZERO

$$\left.\begin{array}{l}\frac{\partial f}{\partial x} = 6x^2 + 6y^2 - 150 \\ \frac{\partial f}{\partial y} = 12xy - 9y^2\end{array}\right\} \Rightarrow \left.\begin{array}{l}6x^2 + 6y^2 - 150 = 0 \\ 12xy - 9y^2 = 0\end{array}\right\} \Rightarrow$$

$$x^2 + y^2 = 25$$

$$3y(4x - 3y) = 0$$

- FROM THE SECOND EQUATION EITHER $y=0$ OR $y = \frac{4}{3}x$

IF $y = 0$ $\quad x = \left\langle\begin{array}{l}5 \\ -5\end{array}\right.$

IF $y = \frac{4}{3}x$ $\quad x^2 + \frac{16}{9}x^2 = 25$

$$9x^2 + 16x^2 = 225$$

$$25x^2 = 225$$

$$x^2 = 9$$

$$x = \left\langle\begin{array}{l}3 \\ -3\end{array}\right. \qquad y = \left\langle\begin{array}{l}4 \\ -4\end{array}\right.$$

- THUS WE HAVE

x	y	$z = f(x,y)$
5	0	−500
−5	0	500
3	4	−300
−3	−4	300

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 6

- OBTAIN THE SECOND DERIVATIVES

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 12x & 12y \\ 12y & 12x-18y \end{bmatrix}$$

- WHICH CAN BE SCALED TO THE MATRIX $\begin{bmatrix} 2x & 2y \\ 2y & 2x-3y \end{bmatrix}$

- CHECKING EACH POINT

$(5, 0, -500)$

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \Longrightarrow \begin{vmatrix} 10-\lambda & 0 \\ 0 & 10-\lambda \end{vmatrix} = 0$$

$$\Longrightarrow (10-\lambda)^2 = 0$$

$$\Longrightarrow \lambda = 10, 10$$

BOTH EIGENVALUES POSITIVE

$\therefore (5, 0, -500)$ IS A LOCAL MIN

$(-5, 0, 500)$

$$\begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} \Longrightarrow \begin{vmatrix} -10-\lambda & 0 \\ 0 & -10-\lambda \end{vmatrix} = 0$$

$$\Longrightarrow (-10-\lambda)^2 = 0$$

$$\Longrightarrow (\lambda+10)^2 = 0$$

$$\Longrightarrow \lambda = -10, -10$$

BOTH EIGENVALUES NEGATIVE

$\therefore (-5, 0, 500)$ IS A LOCAL MAX

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 6

$(3,4,-300)$

$$\begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix} \Rightarrow \begin{vmatrix} 6-\lambda & 8 \\ 8 & -6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (6-\lambda)(-6-\lambda) - 64 = 0$$

$$\Rightarrow (\lambda-6)(\lambda+6) - 64 = 0$$

$$\Rightarrow \lambda^2 - 36 - 64 = 0$$

$$\Rightarrow \lambda^2 = 100$$

$$\Rightarrow \lambda = \pm 10$$

MIXED SIGN EIGENVALUES

$\therefore$ $(3,4,-300)$ IS A "SADDLE"

$(-3,-4,300)$

$$\begin{bmatrix} -6 & -8 \\ -8 & 6 \end{bmatrix} \Rightarrow \begin{vmatrix} -6-\lambda & -8 \\ -8 & 6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-6-\lambda)(6-\lambda) - 64 = 0$$

$$\Rightarrow (\lambda+6)(\lambda-6) - 64 = 0$$

$$\Rightarrow \lambda^2 - 36 - 64 = 0$$

$$\Rightarrow \lambda^2 = 100$$

$$\Rightarrow \lambda = \pm 10$$

MIXED SIGN EIGENVALUES

$\therefore$ $(-3,-4,300)$ IS A "SADDLE"

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 7

START BY FINDING THE COMPLEMENTARY FUNCTION

$$\Rightarrow \lambda^2 - 4\lambda + 4 = 0$$
$$\Rightarrow (\lambda - 2)^2 = 0$$
$$\Rightarrow \lambda = 2 \quad \text{(REPEATED)}$$

COMPLEMENTARY FUNCTION: $y = Ae^{2x} + Bxe^{2x}$

BY THE METHOD OF VARIATION OF PARAMETERS

$$\underbrace{1}_{a(x)}\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \underbrace{\left\{6xe^{2x}\right\}}_{f(x)}$$

- $a(x) = 1$
- $e_1 = e^{2x}$
- $e_2 = xe^{2x}$
- $f(x) = 6xe^{2x}$

PROCEED TO OBTAIN THE WRONSKIAN

$$W(x) = \begin{vmatrix} e_1 & e_2 \\ e_1' & e_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix} = e^{4x} + \cancel{2xe^{4x}} - \cancel{2xe^{4x}} = e^{4x}$$

FINDING THE PARTICULAR INTEGRAL

$$\Rightarrow y_p = -e_1 \int \frac{e_2 f(x)}{a(x)W(x)}\,dx + e_2 \int \frac{e_1 f(x)}{a(x)W(x)}\,dx$$

$$\Rightarrow y_p = -e^{2x} \int \frac{(xe^{2x})(6xe^{2x})}{1 \times e^{4x}}\,dx + xe^{2x} \int \frac{e^{2x}(6xe^{2x})}{1 \times e^{4x}}\,dx$$

$$\Rightarrow y_p = -e^{2x} \int 6x^2\,dx + xe^{2x} \int 6x\,dx$$

$$\Rightarrow y_p = -2x^3e^{2x} + 3x^3e^{2x}$$

$$\Rightarrow y_p = x^3e^{2x}$$

$$\therefore\ y = e^{2x}\left[x^3 + Bx + A\right]$$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 8

a) $\sum_{n=1}^{\infty} \frac{10^n}{n!}$ = ... BY THE RATIO TEST ...

AS ALL THE TERMS ARE POSITIVE WE MAY IGNORE MODULI IN THE TEST

$$\lim_{n\to\infty}\left[\frac{u_{n+1}}{u_n}\right] = \lim_{n\to\infty}\left[\frac{\frac{10^{n+1}}{(n+1)!}}{\frac{10^n}{n!}}\right] = \lim_{n\to\infty}\left[\frac{10^{n+1}}{(n+1)!}\times\frac{n!}{10^n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{10}{n+1}\right] = 0 < 1$$

SERIES CONVERGES BY THE RATIO TEST

b) $$\sum_{k=1}^{\infty}\frac{k^4}{(k+1)^6} < \sum_{k=1}^{\infty}\frac{k^4}{k^6} = \sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}$$

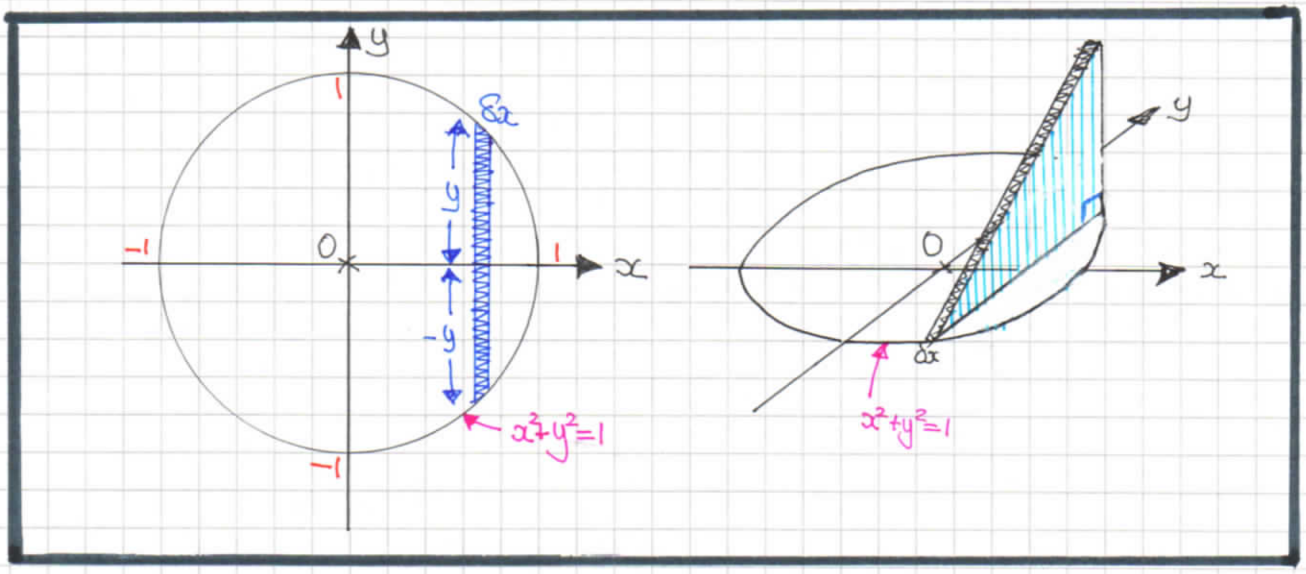
SERIES CONVERGES BY COMPARISON

c) $$\sum_{r=1}^{\infty}\frac{(r+1)(2r+1)(3r+1)}{r^4} > \sum_{r=1}^{\infty}\frac{r\times 2r\times 3r}{r^4} = \sum_{r=1}^{\infty}\frac{6r^3}{r^4}$$

$$= 6\sum_{r=1}^{\infty}\frac{1}{r}$$ WHICH DIVERGES

SERIES DIVERGES BY COMPARISON

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 9



LOOKING AT THE FIGURES ABOVE

- $x^2+y^2=1$
  $y = \pm\sqrt{1-x^2}$
- BOTH THE BASE & HEIGHT OF THE INFINITESIMAL TRIANGLE ARE $2\sqrt{1-x^2}$
- THE VOLUME OF THE INFINITESIMAL TRIANGULAR PRISM IS $\frac{1}{2}\left(2\sqrt{1-x^2}\right)^2 \delta x$

SUMMING ALL THE INFINITESIMAL TRIANGULAR PRISMS FROM $x=-1$ TO $x=1$

$$\Rightarrow V = \int_{-1}^{1} \frac{1}{2}\left(2\sqrt{1-x^2}\right)^2 \, dx = \int_{-1}^{1} 2(1-x^2) \, dx$$

... EVEN INTEGRAND ...

$$= \int_{0}^{1} 4(1-x^2) \, dx = \int_{0}^{1} 4 - 4x^2 \, dx$$

$$= \left[4x - \frac{4}{3}x^3\right]_0^1 = \left(4 - \frac{4}{3}\right) - (0)$$

$$= \frac{8}{3}$$

IYGB, MATHEMATICAL METHODS 1 – PAPER E – QUESTION 10

METHOD A – BY "SURD CONJUGATION"

$$\lim_{x \to 1}\left[\frac{\sqrt{x+3} - 2\sqrt{x}}{\sqrt{x} - 1}\right]$$ ← YIELDS ZERO OVER ZERO

$$= \lim_{x \to 1}\left[\frac{(\sqrt{x+3} - 2\sqrt{x})(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}\right] = \lim_{x \to 1}\underbrace{\left[\frac{(\sqrt{x+3} - 2\sqrt{x})(\sqrt{x}+1)}{x-1}\right]}_{\text{THIS STILL YIELDS ZERO OVER ZERO}}$$

$$= \lim_{x \to 1}\left[\frac{(\sqrt{x+3} - 2\sqrt{x})(\sqrt{x+3}+2\sqrt{x})(\sqrt{x}+1)}{(\sqrt{x+3}+2\sqrt{x})(x-1)}\right]$$

$$= \lim_{x \to 1}\left[\frac{(x+3-4x)(\sqrt{x}+1)}{(\sqrt{x+3}+2\sqrt{x})(x-1)}\right] = \lim_{x \to 1}\left[\frac{(3-3x)(\sqrt{x}+1)}{(\sqrt{x+3}+2\sqrt{x})(x-1)}\right]$$

$$= \lim_{x \to 1}\left[\frac{-3\cancel{(x-1)}(\sqrt{x}+1)}{(\sqrt{x+3}+2\sqrt{x})\cancel{(x-1)}}\right] = \lim_{x \to 1}\left[\frac{-3(\sqrt{x}+1)}{\sqrt{x+3}+2\sqrt{x}}\right]$$

$$= \frac{-3\times 2}{2+2} = -\frac{6}{4} = -\frac{3}{2}$$

METHOD B – BY L'HOSPITAL'S RULE

$$\lim_{x \to 1}\left[\frac{\sqrt{x+3} - 2\sqrt{x}}{\sqrt{x} - 1}\right]$$ ← AS WE HAVE ZERO OVER ZERO, APPLY L'HOSPITAL'S RULE BY SEPARATELY DIFFERENTIATING NUMERATOR AND DENOMINATOR

$$= \lim_{x \to 1}\left[\frac{\frac{1}{2}(x+3)^{-\frac{1}{2}} - x^{-\frac{1}{2}}}{\frac{1}{2}x^{-\frac{1}{2}} - 0}\right]$$ ← SPLIT THE FRACTION

$$= \lim_{x \to 1}\left[\sqrt{\frac{x}{x+3}} - 2\right] = \sqrt{\frac{1}{4}} - 2 = \frac{1}{2} - 2 = -\frac{3}{2}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 11

a) DIRECTLY FROM THE DEFINITIONS

IF $f(x)$ IS PIECEWISE CONTINUOUS IN THE INTERVAL $(-L,L)$, THEN

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]}$$

WHERE $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx \qquad n=0,1,2,3,\ldots$

$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx \qquad n=1,2,3,4,\ldots$

b) LET US START BY NOTING THAT $f(x)=x^2,\ x\in[-1,1]$ IS EVEN

- As $f(x)$ IS EVEN, ALL $b_n=0$, AS THE INTEGRAND OF b_n WILL BE ODD IN A SYMMETRICAL DOMAIN

- $a_0 = \frac{1}{1}\int_{-1}^{1} x^2\,dx = \ldots$ EVEN INTEGRAND $\ldots\ 2\int_0^1 x^2\,dx$

$= \frac{2}{3}\left[x^3\right]_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$

- $a_n = \frac{1}{1}\int_{-1}^{1} x^2\cos\left(\frac{n\pi x}{1}\right)\,dx = \ldots$ EVEN INTEGRAND$\ldots$

$= \int_0^1 2x^2\cos(n\pi x)\,dx$

INTEGRATION BY PARTS

$2x^2$	$4x$
$\frac{1}{n\pi}\sin(n\pi x)$	$\cos(n\pi x)$

$= \cancel{\left[\frac{2}{n\pi}x^2\sin(n\pi x)\right]_0^1} - \frac{4}{n\pi}\int_0^1 x\sin(n\pi x)\,dx$

$= -\frac{4}{n\pi}\int_0^1 x\sin(n\pi x)\,dx$

x	1
$-\frac{1}{n\pi}\cos(n\pi x)$	$\sin(n\pi x)$

INTEGRATION BY PARTS AGAIN

$= -\frac{4}{n\pi}\left\{\left[-\frac{1}{n\pi}x\cos(n\pi x)\right]_0^1 + \frac{1}{n\pi}\int_0^1\cos(n\pi x)\,dx\right\}$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 11

$$= \frac{4}{n^2\pi^2}\Big[x\cos(n\pi x)\Big]_0^1 - \frac{1}{n^2\pi^2}\int_0^1 \cos(n\pi x)\,dx$$

$$= \frac{4}{n^2\pi^2}\Big[x\cos(n\pi x)\Big]_0^1 - \frac{1}{n^2\pi^2}\Big[\cancel{\sin(n\pi x)}\Big]_0^1$$

$$= \frac{4}{n^2\pi^2}\Big[\cos(n\pi) - 0\Big] = \frac{4\cos(n\pi)}{n^2\pi^2} = \frac{4(-1)^n}{n^2\pi^2}$$

$$\therefore \boxed{\begin{aligned} f(x) &= \frac{2/3}{2} + \sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)\right] \\ x^2 &= \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n^2}\cos(n\pi x)\right] \end{aligned}}$$

c) LETTING $x=0$ IN THE ABOVE EXPANSION

$$\Longrightarrow 0^2 = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n^2}\cos 0\right]$$

$$\Longrightarrow 0 = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n^2}\right]$$

$$\Longrightarrow \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2} = -\frac{1}{3}$$

$$\Longrightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}$$

$$\Longrightarrow -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \cdots = -\frac{\pi^2}{12}$$

$$\Longrightarrow 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots = \frac{\pi^2}{12}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 12

LET $I = \int_0^\infty \frac{e^{-kx}\sin x}{x}\,dx$, k A REAL PARAMETER

$$\Rightarrow \frac{\partial I}{\partial k} = \frac{\partial}{\partial k}\left[\int_0^\infty \frac{e^{-kx}\sin x}{x}\,dx\right] = \int_0^\infty \frac{\partial}{\partial k}\left(e^{-kx}\right)\frac{\sin x}{x}\,dx$$

$$\Rightarrow \frac{\partial I}{\partial k} = \int_0^\infty -\cancel{x}e^{-kx}\frac{\sin x}{\cancel{x}}\,dx = \int_0^\infty -e^{-kx}\sin x\,dx$$

PROCEED TO EVALUATE THE INTEGRAL BY COMPLEX NUMBERS (OR LAPLACE TRANSFORMS IF THE TECHNIQUES ARE KNOWN)

$$\frac{\partial I}{\partial k} = -\mathrm{Im}\int_0^\infty e^{-kx}e^{ix}\,dx$$

$$= -\mathrm{Im}\int_0^\infty e^{(-k+i)x}\,dx$$

$$= -\mathrm{Im}\left[\frac{1}{-k+i}e^{(-k+i)x}\right]_0^\infty$$

$$= -\mathrm{Im}\left[\frac{-k-i}{k^2+1}e^{-kx}e^{ix}\right]_0^\infty \qquad \boxed{\left|e^{ix}\right| = 1}$$

$$= -\mathrm{Im}\left[0 - \frac{-k-i}{k^2+1}\right]$$

$$= \mathrm{Im}\left[-\frac{k+i}{k^2+1}\right]$$

$$= -\frac{1}{k^2+1}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 12

FINALLY WE HAVE

$$\Rightarrow \frac{\partial I}{\partial k} = -\frac{1}{k^2+1}$$

$$\Rightarrow I = -\arctan k + C$$

$$\Rightarrow \int_0^\infty \frac{e^{-kx}\sin x}{x}\,dx = C - \arctan k$$

LET $k=0$ IN THE ABOVE EQUATION

$$\int_0^\infty \frac{\sin x}{x}\,dx = C$$

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2} = C$$

OR LET $k \to \infty$

$$0 = C - \arctan(\infty)$$

$$0 = C - \frac{\pi}{2}$$

$$C = \frac{\pi}{2}$$

$$\Rightarrow \int_0^\infty \frac{e^{-kx}\sin x}{x}\,dx = \frac{\pi}{2} - \arctan k$$

$$\Rightarrow \int_0^\infty \frac{e^{-kx}\sin x}{x}\,dx = \operatorname{arccot} k$$

LET $k=2$ IN THE ABOVE EQUATION, YIELDS THE REQUIRED RESULT

$$\Rightarrow \int_0^\infty \frac{e^{-2x}\sin x}{x}\,dx = \operatorname{arccot} 2$$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 13

METHOD A - BY DIRECT EVALUATION

$W = \phi(u,v)$ $\quad x = e^{u}\cos v \ , \ y = e^{-u}\sin v$

$$\left.\begin{aligned} dx &= \frac{\partial x}{\partial u}du + \frac{\partial x}{\partial v}dv \\ dy &= \frac{\partial y}{\partial u}du + \frac{\partial y}{\partial v}dv \end{aligned}\right\} \Rightarrow \begin{aligned} dx &= e^{u}\cos v\, du - e^{u}\sin v\, dv \\ dy &= -e^{-u}\sin v\, du + e^{-u}\cos v\, dv \end{aligned}$$

ELIMINATE du

$$\left.\begin{aligned} e^{-u}\sin v\, dx &= e^{-u}\sin v\, e^{u}\cos v\, du - e^{-u}\sin v\, e^{u}\sin v\, dv \\ e^{u}\cos v\, dy &= -e^{u}\cos v\, e^{-u}\sin v\, du + e^{u}\cos v\, e^{-u}\cos v\, dv \end{aligned}\right\} \Rightarrow \text{TIDY}$$

$$\left.\begin{aligned} e^{-u}\sin v\, dx &= \sin v\cos v\, du - \sin^2 v\, dv \\ e^{u}\cos v\, dy &= -\sin v\cos v\, du + \cos^2 v\, dv \end{aligned}\right\} \Rightarrow \text{ADDING}$$

$\Rightarrow (\cos^2 v - \sin^2 v)\, dv = e^{-u}\sin v\, dx + e^{u}\cos v\, dy$

$\Rightarrow \cos 2v\, dv = e^{-u}\sin v\, dx + e^{u}\cos v\, dy$

$\Rightarrow dv = \dfrac{e^{-u}\sin v}{\cos 2v}dx + \dfrac{e^{u}\cos v}{\cos 2v}dy \leftarrow \boxed{dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy}$

$\therefore \dfrac{\partial v}{\partial x} = \dfrac{e^{-u}\sin v}{\cos 2v}$ & $\dfrac{\partial v}{\partial y} = \dfrac{e^{u}\cos v}{\cos 2v}$ //

IN A SIMILAR FASHION, ELIMINATE dv

$$\left.\begin{aligned} e^{-u}\cos v\, dx &= e^{-u}\cos v\, e^{u}\cos v\, du - e^{-u}\cos v\, e^{u}\sin v\, dv \\ e^{u}\sin v\, dy &= -e^{u}\sin v\, e^{-u}\sin v\, du + e^{u}\sin v\, e^{-u}\cos v\, dv \end{aligned}\right\} \Rightarrow \text{TIDY}$$

$$\left.\begin{aligned} e^{-u}\cos v\, dx &= \cos^2 v\, du - \cos v\sin v\, dv \\ e^{u}\sin v\, dy &= -\sin^2 v\, du + \sin v\cos v\, dv \end{aligned}\right\} \Rightarrow \text{ADDING}$$

IYGB, MATHEMATICAL METHODS 1, PAPER E, QUESTION 13

$$\Rightarrow (\cos^2 v - \sin^2 v)\,du = e^{-u}\cos v\,dx + e^{u}\sin v\,dy$$

$$\Rightarrow \cos 2v\,du = e^{-u}\cos v\,dx + e^{u}\sin v\,dy$$

$$\Rightarrow du = \frac{e^{-u}\cos v}{\cos 2v}dx + \frac{e^{u}\sin v}{\cos 2v}dy \longleftarrow \boxed{du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{e^{-u}\cos v}{\cos 2v} \qquad \frac{\partial u}{\partial y} = \frac{e^{u}\sin v}{\cos 2v}$$

METHOD B - USING JACOBIANS

IF $x = f(u,v)$ & $y = g(u,v)$ THEN.

$$\frac{\partial u}{\partial x} = \frac{\partial y}{\partial v}\Big/J \qquad \frac{\partial u}{\partial y} = -\frac{\partial x}{\partial v}\Big/J$$

$$\frac{\partial v}{\partial x} = -\frac{\partial y}{\partial u}\Big/J \qquad \frac{\partial v}{\partial y} = \frac{\partial x}{\partial u}\Big/J$$

WHERE $J = \frac{\partial(x,y)}{\partial(u,v)}$

- HERE $x = e^{u}\cos v \quad y = e^{-u}\sin v$
- $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} e^{u}\cos v & -e^{u}\sin v \\ -e^{-u}\sin v & e^{-u}\cos v \end{vmatrix}$

$$= \cos^2 v - \sin^2 v = \cos 2v$$

- $\frac{\partial u}{\partial x} = \frac{\partial y}{\partial v}\Big/J = \frac{e^{-u}\cos v}{\cos 2v}$
- $\frac{\partial u}{\partial y} = -\frac{\partial x}{\partial v}\Big/J = \frac{e^{u}\sin v}{\cos 2v}$
- $\frac{\partial v}{\partial x} = -\frac{\partial y}{\partial u}\Big/J = \frac{e^{-u}\sin v}{\cos 2v}$
- $\frac{\partial v}{\partial y} = \frac{\partial x}{\partial u}\Big/J = \frac{e^{u}\cos v}{\cos 2v}$

IYGB - MATHEMATICAL METHODS 1 - PAPER E - QUESTION 14

$$\boxed{\frac{d^2y}{dx^2} - 4y = 24\cos 2x \quad , \quad x \geqslant 0 \quad , \quad x=0, \; y=3, \; \frac{dy}{dx}=4}$$

WRITE THE O.D.E IN COMPACT FORM, & TAKE LAPLACE TRANSFORMS IN x

$$\Longrightarrow y'' - 4y = 24\cos 2x$$

$$\Longrightarrow s^2\bar{y} - sy_0 - y_0' - 4\bar{y} = \mathcal{L}\left[24\cos 2x\right]$$

$$\Longrightarrow s^2\bar{y} - 3s - 4 - 4\bar{y} = 24 \times \frac{s}{s^2+4}$$

$$\Longrightarrow (s^2-4)\bar{y} = 3s + 4 + \frac{24s}{s^2+4}$$

$$\Longrightarrow \bar{y} = \frac{3s+4}{s^2-4} + \frac{24s}{(s^2-4)(s^2+4)}$$

$$\Longrightarrow \bar{y} = \frac{3s+4}{(s-2)(s+2)} + \frac{24s}{(s-2)(s+2)(s^2+4)}$$

PARTIAL FRACTIONS MAINLY BY INSPECTION (COVER UP)

$$\Longrightarrow \bar{y} = \frac{\frac{10}{4}}{s-2} + \frac{\frac{-2}{-4}}{s+2} + \frac{\frac{48}{4\times 8}}{s-2} + \frac{\frac{-48}{-4\times 8}}{s+2} + \frac{As+B}{s^2+4}$$

$$\Longrightarrow \bar{y} = \frac{\frac{5}{2}}{s-2} + \frac{\frac{1}{2}}{s+2} + \underbrace{\frac{\frac{3}{2}}{s-2} + \frac{\frac{3}{2}}{s+2} + \frac{As+B}{s^2+4}}$$

$$24s \equiv \tfrac{3}{2}(s+2)(s^2+4) + \tfrac{3}{2}(s-2)(s^2+4) + (s^2-4)(As+B)$$

- IF $s=0 \Longrightarrow 0 = 12 - 12 - 4B$
  $\Longrightarrow 4B = 0$
  $\Longrightarrow B = 0$

- IF $s=1 \Longrightarrow 24 = \frac{45}{2} - \frac{15}{2} - 3(A)$
  $\Longrightarrow 24 = 15 - 3A$
  $\Longrightarrow 3A = -9$
  $\Longrightarrow A = -3$

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COLLECTING ALL RESULTS

$$\bar{y} = \frac{\frac{5}{2}}{s-2} + \frac{\frac{1}{2}}{s+2} + \frac{\frac{3}{2}}{s-2} + \frac{\frac{3}{2}}{s+2} - \frac{3s}{s^2+4}$$

$$\bar{y} = \frac{4}{s-2} + \frac{2}{s+2} - 3\left(\frac{s}{s^2+4}\right)$$

INVERTING (ALL ONLY SIMPLE STANDARD RESULTS)

$$y = 4e^{2x} + 2e^{-2x} - 3\cos 2x$$

IYGB – MATHEMATICAL METHODS 1 – PAPER E – QUESTION 15

START BY OBTAINING A RELATIONSHIP BETWEEN ψ & EITHER x OR y

$$\Longrightarrow \sin y = e^{x}$$

$$\Longrightarrow y = \arcsin(e^{x})$$

$$\Longrightarrow \frac{dy}{dx} = \frac{e^{x}}{\sqrt{1-e^{2x}}} = \frac{\sin y}{(1-\sin^2 y)^{\frac{1}{2}}} = \frac{\sin y}{\cos y} = \tan y$$

BUT $\frac{dy}{dx} \equiv \tan\psi$ (ALWAYS)

$$\therefore \ y = \psi$$

USING THE ARCLENGTH FORMULA

$$\Longrightarrow s = \int_0^{x} \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = \int_0^{x} (1+\tan^2 y)^{\frac{1}{2}} dx = \int_0^{x} \sec y \, dx$$

FROM $(0, \frac{\pi}{2})$

$$= \int_{y=\frac{\pi}{2}}^{y} \sec y \frac{dx}{dy} \, dy = \int_{\frac{\pi}{2}}^{y} \sec y \left(\frac{1}{\tan y}\right) dy = \int_{\frac{\pi}{2}}^{y} \frac{1}{\cancel{\cos y}} \frac{\cancel{\cos y}}{\sin y} \, dy$$

$$= \int_{\frac{\pi}{2}}^{y} \operatorname{cosec} y \, dy = \Big[\ln\left|\tan \tfrac{y}{2}\right|\Big]_{\frac{\pi}{2}}^{y} = \ln\left|\tan\tfrac{y}{2}\right| - \cancel{\ln\left|\tan\tfrac{\pi}{4}\right|}$$

$$\Longrightarrow s = \ln\left|\tan\frac{y}{2}\right|$$

BUT AS $y = \psi$

$$\Longrightarrow s = \ln\left|\tan\frac{\psi}{2}\right|$$

$$\Longrightarrow e^{s} = \tan\frac{\psi}{2}$$

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VARIATION

$\Rightarrow \sin y = e^{x}$

$\Rightarrow \ln(\sin y) = x$

$\Rightarrow \frac{dx}{dy} = \frac{1}{\sin y} \times \cos y$

$\Rightarrow \frac{dx}{dy} = \cot y$

$\Rightarrow \frac{dy}{dx} = \tan y$ (AS BEFORE) $\Rightarrow$ $y = \psi$

& HENCE USING THE ARCLENGTH FORMULA

$$\Rightarrow s = \int_{y=\frac{\pi}{2}}^{y} \left[1 + \left(\frac{dx}{dy}\right)^2\right]^{\frac{1}{2}} dy$$

$$\Rightarrow s = \int_{\frac{\pi}{2}}^{y} \left(1 + \cot^2 y\right)^{\frac{1}{2}} dy$$

$$\Rightarrow s = \int_{\frac{\pi}{2}}^{y} \operatorname{cosec} y \, dy$$

... WHICH MERGES WITH THE PREVIOUS

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START BY REWRITING THE QUADRIC SURFACE IN MATRIX FORM

$$\Rightarrow x^2 + 2y^2 + z^2 + 2xy + 2yz = 9$$

$$\Rightarrow (x \quad y \quad z) \begin{array}{c|ccc|} & x & y & z \\ \hline x & 1_{x^2} & 1_{xy} & 0_{xz} \\ y & 1_{yx} & 2_{y^2} & 1_{yz} \\ z & 0_{zx} & 1_{zy} & 1_{z^2} \end{array} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 9$$

FINDING THE EIGENVALUES OF THE ABOVE SYMMETRIC MATRIX

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[(\lambda-2)(\lambda-1)-1\right] - (1-\lambda) = 0$$

$$\Rightarrow (1-\lambda)\left[(\lambda-2)(\lambda-1)-1-1\right] = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2-3\lambda+2-2) = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2-3\lambda) = 0$$

$$\Rightarrow \lambda(\lambda-3)(1-\lambda) = 0$$

$$\Rightarrow \lambda = \begin{cases} 0 \\ 1 \\ 3 \end{cases}$$

NEXT FIND EIGENVECTORS FOR EACH EIGENVALUE

- IF $\lambda = 0$

$$\left.\begin{array}{l} 1x + 1y + 0z = 0x \\ 1x + 2y + 1z = 0y \\ 0x + 1y + 1z = 0z \end{array}\right\} \Rightarrow \left.\begin{array}{l} x + y = 0 \\ x + 2y + z = 0 \\ y + z = 0 \end{array}\right\} \Rightarrow \begin{array}{l} x = -y \\ \text{... UNDETERMINED ...} \\ z = -y \end{array}$$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \text{ OR } \alpha\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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- IF $\lambda = 1$

$$\left.\begin{array}{l} 1x + 1y + 0z = 1x \\ 1x + 2y + 1z = 1y \\ 0x + 1y + 1z = 1z \end{array}\right\} \Rightarrow \begin{array}{l} y = 0 \\ x = -z \\ z = z \end{array} \quad \therefore \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ OR } \beta \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

- IF $\lambda = 3$

$$\left.\begin{array}{l} 1x + 1y + 0z = 3x \\ 1x + 2y + 1z = 3y \\ 0x + 1y + 1z = 3z \end{array}\right\} \Rightarrow \begin{array}{l} y = 2x \\ x - y + z = 0 \\ y = 2z \end{array} \quad \therefore \gamma \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

NORMALIZING THE 3 EIGEN-VECTORS

$$\left.\begin{array}{ll} \lambda = 0 & \underline{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \\ \lambda = 1 & \underline{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ \lambda = 3 & \underline{w} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \end{array}\right\} \text{ NORMALIZED TO } \left.\begin{array}{l} \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \\ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \end{array}\right\} \text{ REQUIRED DIRECTION VECTORS OF LINES THROUGH THE ORIGIN}$$

- $\underline{P} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$
- $\underline{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\Rightarrow (X \; Y \; Z) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = 9$$

$$\Rightarrow Y^2 + 3Z^2 = 9$$

I.E AN ELLIPTIC CYLINDER

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AS THE INDEPENDENT VARIABLE (x) IS MISSING, USE THE STANDARD SUBSTITUTION $p = \frac{dy}{dx}$

$$\Rightarrow \frac{dp}{dy} = \frac{d}{dy}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}\frac{dx}{dy} = \frac{d^2y}{dx^2} \times \frac{1}{p}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = p\frac{dp}{dy}}$$

COMPARE WITH THE STANDARD MECHANICS MANIPULATION FOR ACCELERATION

$$\ddot{x} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

TRANSFORMING THE O.D.E

$$\Rightarrow \frac{d^2y}{dx^2} + e^{-y} = 0 \qquad \left[x = \frac{\pi}{2},\ y = 0,\ \frac{dy}{dx} = p = -1\right]$$

$$\Rightarrow p\frac{dp}{dy} = -e^{-y}$$

$$\Rightarrow \int_{p=-1}^{p} p \, dp = \int_{y=0}^{y} -e^{-y} \, dy$$

$$\Rightarrow \left[\frac{1}{2}p^2\right]_{-1}^{p} = \left[e^{-y}\right]_{0}^{y}$$

$$\Rightarrow \frac{1}{2}p^2 - \frac{1}{2} = e^{-y} - 1$$

$$\Rightarrow p^2 - 1 = 2e^{-y} - 2$$

$$\Rightarrow p^2 = \frac{2}{e^y} - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{2 - e^y}{e^y}$$

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$$\Longrightarrow \frac{dy}{dx} = +\frac{\sqrt{2-e^{y}}}{e^{\frac{1}{2}y}}$$

SEPARATING VARIABLES AGAIN

$$\Longrightarrow \frac{e^{\frac{1}{2}y}}{\sqrt{2-e^{y}}}\,dy = 1\,dx$$

$$\Longrightarrow \int_{\frac{\pi}{2}}^{x} 1\,dx = \int_{0}^{y} \frac{e^{\frac{1}{2}y}}{\sqrt{2-e^{y}}}\,dy$$

USING A TRIGONOMETRIC SUBSTITUTION ON THE INTEGRAL IN THE R.H.S

$$e^{y} = 2\sin^2\theta \qquad \left[e^{\frac{1}{2}y} = \sqrt{2}\sin\theta \quad \text{OR} \quad \theta = \arcsin\left(\frac{e^{\frac{1}{2}y}}{\sqrt{2}}\right)\right]$$

$$\Rightarrow e^{y}\,dy = 4\sin\theta\cos\theta\,d\theta$$

$$\Rightarrow dy = \frac{4\sin\theta\cos\theta}{e^{y}}\,d\theta = \frac{4\sin\theta\cos\theta}{2\sin^2\theta}\,d\theta = \frac{2\cos\theta}{\sin\theta}\,d\theta$$

LIMITS TRANSFORM TO

$$y=0 \longmapsto \theta = \frac{\pi}{4}$$

$$y \longmapsto \theta = \arcsin\left(\frac{e^{\frac{1}{2}y}}{\sqrt{2}}\right)$$

RETURNING TO THE O.D.E

$$\Longrightarrow \Big[x\Big]_{\pi/2}^{x} = \int_{\pi/4}^{\arcsin(e^{\frac{1}{2}y}/\sqrt{2})} \frac{\sqrt{2}\sin\theta}{\sqrt{2-2\sin^2\theta}} \times \frac{2\cos\theta}{\sin\theta}\,d\theta$$

$$\Longrightarrow x - \frac{\pi}{2} = \int_{\pi/4}^{\arcsin(e^{\frac{1}{2}y}/\sqrt{2})} \frac{\cancel{\sqrt{2}\sin\theta}}{\cancel{\sqrt{2}\cos\theta}} \times \frac{2\cancel{\cos\theta}}{\cancel{\sin\theta}}\,d\theta$$

$$\Longrightarrow x - \frac{\pi}{2} = \int_{\frac{\pi}{4}}^{\arcsin(e^{\frac{1}{2}y}/\sqrt{2})} 2\,d\theta$$

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$$\Rightarrow x - \frac{\pi}{2} = 2\Big[\theta\Big]_{\pi/4}^{\arcsin\left(e^{\frac{1}{2}y}/\sqrt{2}\right)}$$

$$\Rightarrow x - \frac{\pi}{2} = 2\left[\arcsin\left(e^{\frac{1}{2}y}/\sqrt{2}\right) - \frac{\pi}{4}\right]$$

$$\Rightarrow x = 2\arcsin\left(e^{\frac{1}{2}y}/\sqrt{2}\right)$$

$$\Rightarrow \frac{x}{2} = \arcsin\left(\frac{e^{\frac{1}{2}y}}{\sqrt{2}}\right)$$

$$\Rightarrow \sin\frac{x}{2} = \frac{e^{\frac{1}{2}y}}{\sqrt{2}}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{e^{y}}{2}$$

$$\Rightarrow 2\sin^2\frac{x}{2} = e^{y}$$

$$\Rightarrow 1 - 2\sin^2\frac{x}{2} = 1 - e^{y}$$

$$\Rightarrow \cos x = 1 - e^{y}$$

$$\Rightarrow e^{y} = 1 - \cos x$$

$$\Rightarrow y = \ln(1 - \cos x)$$

AS REQUIRED