IYGB

Mathematical Methods 2

Practice Paper A

Time: 3 hours

Candidates may use any non programmable, non graphical calculator which does not have the capability of storing data or manipulating algebraic expressions.

Information for Candidates

This paper follows the most common syllabi of Mathematical Methods used in the United Kingdom Universities for Mathematics, Physics and Engineering degrees.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.

Full marks may be obtained for answers to ALL questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 10 questions in this question paper.

The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

Non exact answers should be given to an appropriate degree of accuracy.

The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

The maximum number of marks on this paper is 100, which can be obtained by attempting all the questions on the paper.

Topics Examined Under Mathematical Methods 2

- Vector Operators: Gradient, Divergence and Curl
- Index Summation Notation
- Double Integrals in Cartesian and Polar Coordinates
- Triple Integrals in Cartesian, Cylindrical and Spherical Coordinates
- Jacobians
- Volume Integrals
- Surface Integrals
- Line Integrals
- Multiple and Vector Integration in Parametric Form
- Applications of Multiple Integration and Vector Integration
  [Mass, Work, Flux, Pressure, Centre of Mass, Moment of Inertia etc.]
- Green's Theorem and Applications
- Divergence Theorem and Applications
- Stokes' Theorem and Applications

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Question 1

Use index summation notation to prove the validity of the following vector identity

$$\nabla \cdot [\nabla \wedge \mathbf{F}] \equiv 0,$$

where $\mathbf{F} = \mathbf{F}(x, y, z)$ is a smooth vector function. **(6)**

Question 2

Evaluate the integral

$$\int_{(1,1,0)}^{(5,3,4)} (3x - 2y)\, dx + (y + z)\, dy + \left(1 - z^2\right) dz,$$

along the straight line segment joining the points with Cartesian coordinates $(1,1,0)$ and $(5,3,4)$. **(7)**

Question 3

A solid sphere has equation

$$x^2 + y^2 + z^2 = a^2.$$

The density, ρ, at the point of the sphere with coordinates (x_1, y_1, z_1) is given by

$$\rho = \sqrt{x_1^2 + y_1^2}.$$

Determine the **average** density of the sphere. **(10)**

Question 4

The finite region R in the x-y plane, is defined as the interior of a parallelogram with vertices at $(4,0)$, $(0,1)$, $(-2,7)$ and $(2,6)$.

Evaluate the integral

$$\int_R x^2 \ dxdy \ . \qquad \mathbf{(12)}$$

Question 5

A Cartesian position vector is denoted by $\mathbf{r}$ and $r = |\mathbf{r}|$.

Show that

$$\nabla \cdot \left(r^n \mathbf{r} \right) = (n+3) r^n \ . \qquad \mathbf{(10)}$$

Question 6

Find the value of I in exact simplified form.

$$I = \int_1^2 \int_2^{2y} \frac{16y}{\left(16 - x^2\right)^{\frac{3}{2}}} \ dx\,dy \ . \qquad \mathbf{(8)}$$

Question 7

It is given that the vector function $\mathbf{F}$ satisfies

$$\mathbf{F} = [x\cos x]\mathbf{i} + \left[15xy + \ln\left(1+y^3\right)\right]\mathbf{j}.$$

Evaluate the line integral

$$\oint_C \mathbf{F}\cdot\mathbf{dr},$$

where C is the curve

$$\{(x,y): y=3,\ -2 \le x \le 2\} \cup \{(x,y): y = x^2-1,\ -2 \le x \le 2\},$$

traced in an anticlockwise direction. **(10)**

Question 8

Find a simplified expression for the surface area cut out of the sphere with equation

$$x^2+y^2+z^2=a^2,\ a>0,$$

when it is intersected by the cylinder with equation

$$x^2+y^2=ax,\ a>0.$$ **(14)**

Question 9

Evaluate the surface integral

$$\int_S \mathbf{F} \cdot \mathbf{dS},$$

where S is the surface represented parametrically by

$$\mathbf{r}(u,v) = \begin{bmatrix} u+v \\ u-v \\ u \end{bmatrix}, \quad 0 \le u \le 2, \quad 0 \le v \le 3,$$

and $\mathbf{F}$ is the vector field

$$x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}.$$

All integrations must be carried out in parametric. **(10)**

Question 10

The surface S has Cartesian equation

$$(z-1)^2 = x^2 + y^2, \ 1 \le z \le 3.$$

a) Sketch the graph of S. **(2)**

b) Evaluate $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ zx^2 & xy^2 & yz^2 \end{vmatrix}$. **(1)**

c) Given that $\mathbf{F} = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$, evaluate the integral

$$\int_S \mathbf{F} \cdot \mathbf{dS}.$$ **(10)**
