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Mathematical Methods 2

Practice Paper C

Time: 3 hours

Candidates may use any non programmable, non graphical calculator which does not have the capability of storing data or manipulating algebraic expressions.

Information for Candidates

This paper follows the most common syllabi of Mathematical Methods used in the United Kingdom Universities for Mathematics, Physics and Engineering degrees.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.

Full marks may be obtained for answers to MOST of the questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 10 questions in this question paper.

The total mark for this paper is 114.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

Non exact answers should be given to an appropriate degree of accuracy.

The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

The maximum number of marks on this paper is 100, which can be obtained without attempting all the questions on the paper.

If the number of marks exceeds 100 then the paper will score maximum marks.

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Topics Examined Under Mathematical Methods 2

- Vector Operators: Gradient, Divergence and Curl
- Index Summation Notation
- Double Integrals in Cartesian and Polar Coordinates
- Triple Integrals in Cartesian, Cylindrical and Spherical Coordinates
- Jacobians
- Volume Integrals
- Surface Integrals
- Line Integrals
- Multiple and Vector Integration in Parametric Form
- Applications of Multiple Integration and Vector Integration
  [Mass, Work, Flux, Pressure, Centre of Mass, Moment of Inertia etc.]
- Green's Theorem and Applications
- Divergence Theorem and Applications
- Stokes' Theorem and Applications

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Question 1

Electric charge q is thinly distributed on the surface of a spherical shell with equation

$$x^2 + y^2 + z^2 = a^2, \ a > 0.$$

Given that $q(x, y) = 2x^2 + y^2$, determine the total charge on the shell. **(10)**

Question 2

The finite region Ω in the x-y plane, is defined by the inequalities

$$x^2 + y^2 \le 1 \quad \text{and} \quad x + y \ge 1.$$

Determine the value of

$$\int_{\Omega} x^3 y \ dx\,dy.$$ **(7)**

Question 3

Use index summation notation to prove the validity of the following vector identity

$$\nabla\left[\mathbf{A}\cdot\mathbf{B}\right] \equiv (\mathbf{B}\cdot\nabla)\mathbf{A} + (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{B}\wedge(\nabla\wedge\mathbf{A}) + \mathbf{A}\wedge(\nabla\wedge\mathbf{B}),$$

where $\mathbf{A} = \mathbf{A}(x, y, z)$ and $\mathbf{B} = \mathbf{B}(x, y, z)$ are smooth vector functions. **(8)**

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Question 4

A solid sphere has equation

$$x^2 + y^2 + z^2 = 1.$$

The region defined by the double cone with Cartesian equation

$$3z^2 \geq x^2 + y^2,$$

is bored out of the sphere.

Determine the volume of the remaining solid. **(7)**

Question 5

The finite region on the x-y plane satisfies

$$4x^4 + 4y^4 \leq \pi^2 - 8x^2y^2 \quad \text{and} \quad 6x^2 + 6y^2 \geq \pi.$$

Find the value of the following integral.

$$\int_R \cos\left(x^2 + y^2\right) \, dx\, dy.$$ **(7)**

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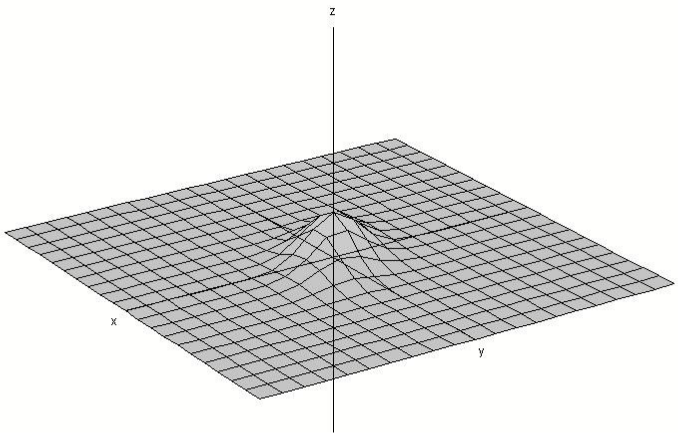
Question 6



The figure above shows the graph of a "hill", modelled by the function $z = f(x,y)$, defined in the entire x-y plane by

$$z = e^{-\left(\frac{5}{4}x^2 - xy + 2y^2\right)}.$$

Use the transformation equations

$$x = u + 2v \text{ and } y = u - v$$

to show that the volume of the "hill" is $\frac{2\pi}{3}$. **(10)**

You may assume without proof that $\int_{-\infty}^{\infty} e^{-s^2}\, ds = \sqrt{\pi}$.

Question 7

$$\mathbf{F}(x,y,z) \equiv y\,\mathbf{i} + x^2\mathbf{j} + z\,\mathbf{k}.$$

Find the magnitude of the flux through the surface with parametric equations

$$\mathbf{r}(u,v) = u\,\mathbf{i} + v\,\mathbf{j} + (u+v)\mathbf{k}, \quad 0 \le u \le 1, \ 1 \le v \le 4 .$$

All integrations must be carried out in parametric. **(10)**

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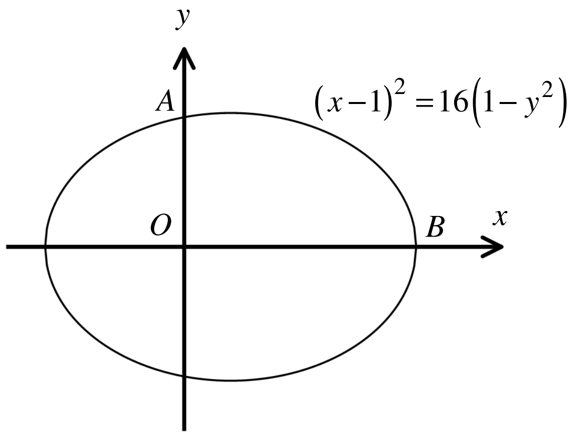
Question 8



The figure above shows the ellipse with equation

$$(x-1)^2 = 16(1-y^2).$$

The ellipse meets the positive x and y axes at the points A and B, respectively, as shown in the figure.

The elliptic path C is the clockwise section from A to B.

Determine the value of each of the following line integrals.

a) $\displaystyle\int_C \left[\left(x^2 + xy\right) \, dx + \left(y^2 + \tfrac{1}{2}x^2\right) \, dy \right].$ **(7)**

b) $\displaystyle\int_C \left[y^3 \, dx + \tfrac{1}{16}(x-1)^3 \, dy \right].$ **(12)**

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Question 9

$$I = \int_0^{\pi} \int_0^{\sin x} 4y \;\; dy\, dx \,.$$

a) Determine the exact value of I . **(4)**

b) Verify the answer of part **(a)** by reversing the order of integration. **(9)**

Question 10

$$\mathbf{A} = 2\mathbf{i} - \mathbf{j} + (4y-3)\mathbf{k} \,.$$

The vector field $\mathbf{A}$ exist around the surface S with Cartesian equation

$$x^2 + y^2 + z^2 = 1, \; z \geq 0 \,.$$

a) Determine the flux of $\mathbf{A}$ through S , where the normal unit field to S is denoted by $\hat{\mathbf{n}}$, such that $\hat{\mathbf{n}} \cdot \mathbf{k} \geq 0$. **(7)**

b) Obtain the answer of part **(a)** by using the Divergence Theorem. **(7)**

c) Use Stokes' Theorem to get an expression for the flux of $\mathbf{A}$ through S , as a line integral, and hence verify the answer of part **(a)**. **(9)**
