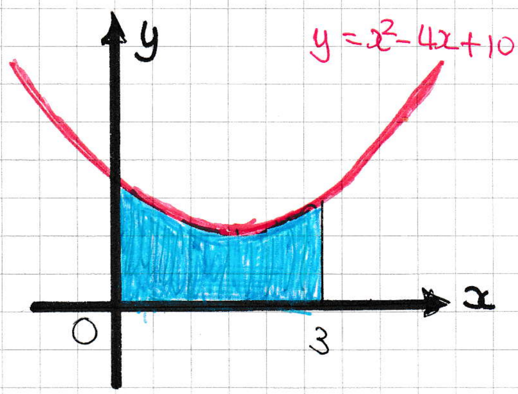
IYGB – MP1 PAPER C – QUESTION 1



$$\text{AREA} = \int_0^3 x^2 - 4x + 10 \, dx$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 10x \right]_0^3$$

$$= \left(\frac{1}{3} \times 3^3 - 2 \times 3^2 + 10 \times 3 \right) - \cancel{\left(\frac{1}{3} \times 0^3 - 2 \times 0^2 + 10 \times 0 \right)}$$

$$= 9 - 18 + 30$$

$$= 21$$

IYGB - MP1 PAPER C - QUESTION 2

a) COLLECTING ALL THE RELEVANT INFORMATION FIRST

- $+2x^2 + \dots \Longrightarrow$ ∪
- $x=0 \quad y=4 \Longrightarrow (0,4)$
- $y=0$

$2x^2-9x+4=0$

$(2x-1)(x-4)=0$

$x = \begin{cases} 1/2 \\ 4 \end{cases} \Longrightarrow \begin{matrix}(\frac{1}{2},0)\\(4,0)\end{matrix}$

- $y = 2x^2-9x+4$

$\frac{1}{2}y = x^2 - \frac{9}{2}x + 2$

$\frac{1}{2}y = (x-\frac{9}{4})^2 - \frac{81}{16} + 2$

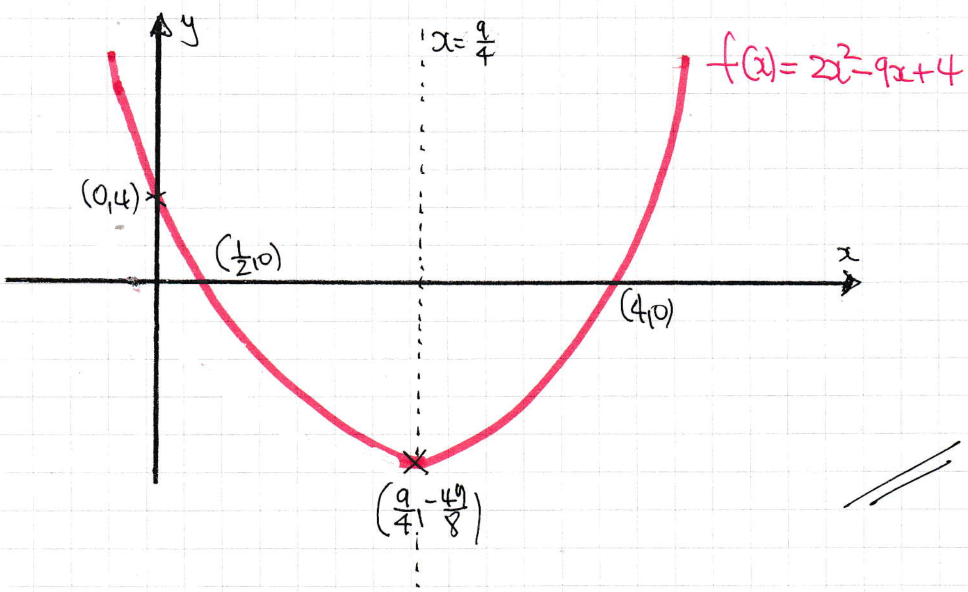
$y = 2(x-\frac{9}{4})^2 - \frac{81}{8} + 4$

$y = 2(x-\frac{9}{4})^2 - \frac{81}{8} + \frac{32}{8}$

$y = 2(x-\frac{9}{4})^2 - \frac{49}{8}$

$\therefore$ ∪ $(\frac{9}{4}, -\frac{49}{8})$

PRODUCING THE SKETCH

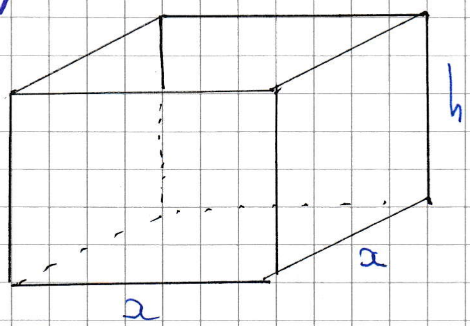


b) LOOKING AT THE GRAPH ABOVE

$f(x) > 0 \Longrightarrow x < \frac{1}{2}$ OR $x > 4$

IYGB - MP1 PAPER C - QUESTION 3

a)



CONSTRAINT ON VOLUME

$V = 500$

$x^2h = 500$

$x(xh) = 500$

$xh = \frac{500}{x}$

$A = x^2 + 4xh$

$A = x^2 + 4\left(\frac{500}{x}\right)$

$A = x^2 + \frac{2000}{x}$ // AS REQUIRED

b) $A = x^2 + 2000x^{-1}$

$\Rightarrow \frac{dA}{dx} = 2x - 2000x^{-2}$

STATIONARY $\Rightarrow \frac{dA}{dx} = 0$

$\Rightarrow 2x - 2000x^{-2} = 0$

$\Rightarrow 2x - \frac{2000}{x^2} = 0$

$\Rightarrow 2x = \frac{2000}{x}$

$\Rightarrow 2x^3 = 2000$

$\Rightarrow x^3 = 1000$

$\Rightarrow x = 10 \text{ m}$ //

c) • $A = x^2 + \frac{2000}{x}$

$A_{MIN} = 10^2 + \frac{2000}{10}$

$A_{MIN} = 300 \text{ m}^2$ //

• $\frac{dA}{dx} = 2x - 2000x^{-2}$

$\frac{d^2A}{dx^2} = 2 + 4000x^{-3}$

$\left.\frac{d^2A}{dx^2}\right|_{x=10} = 6 > 0$

INDEED A MINIMUM //

IYGB - MP1 PAPER C - QUESTION 4

$$\tan(3x-75)^\circ = \tan 450^\circ, \quad 300^\circ \leq x < 500$$

SETTING UP A GENERAL SOLUTION IN DEGREES

$$\Longrightarrow \quad 3x - 75^\circ = 450^\circ \pm 180n \qquad n = 0,1,2,3,\ldots$$

$$\Longrightarrow \quad 3x = 525^\circ \pm 180n$$

$$\Longrightarrow \quad x = 175^\circ \pm 60n$$

COLLECTING THE SOLUTIONS IN THE REQUIRED INTERVAL

$$x = \ldots \cancel{175^\circ}, \cancel{235^\circ}, \cancel{295^\circ}, 355^\circ, 415^\circ, 475^\circ, \cancel{535^\circ}, \ldots$$

$$x = 355^\circ, 415^\circ, 475^\circ$$

IYGB - MP1 PAPER C - QUESTION 5

LET THE EQUATION OF THE CIRCLE IN EXPANDED FORM BE

$$x^2+y^2+Ax+By=C$$

- $(-1,0) \Rightarrow (-1)^2+0^2+A(-1)+B(0)=C$
$$\Rightarrow 1-A=C$$

- $(7,0) \Rightarrow 7^2+0^2+7A+B(0)=C$
$$\Rightarrow 49+7A=C$$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} C=1-A \\ C=49+7A \end{array}\right\} \Rightarrow 49+7A=1-A$$
$$\Rightarrow 8A=-48$$
$$\Rightarrow A=-6 \quad \& \quad C=7$$

SUBSTITUTE THESE VALUES IN AND TRY THE POINT $(3,8)$

$$\Rightarrow x^2+y^2-6x+By=7$$
$$\Rightarrow 3^2+8^2-6(3)+B(8)=7$$
$$\Rightarrow 9+64-18+8B=7$$
$$\Rightarrow 8B=-48$$
$$\Rightarrow B=-6$$

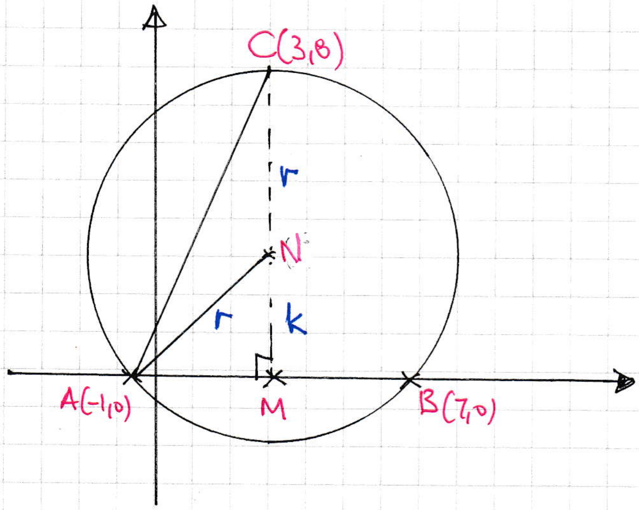
FINALLY WE HAVE THE EQUATION

$$\Rightarrow x^2+y^2-6x-6y=7$$
$$\Rightarrow (x-3)^2-9+(y-3)^2-9=7$$
$$\Rightarrow (x-3)^2+(y-3)^2=25$$

$\therefore$ CENTRE AT $(3,3)$, RADIUS 5

IYGB - MP1 PAPER C - QUESTION 5

ALTERNATIVE USING CIRCLE GEOMETRY



- $M(3,0)$ MIDPOINT OF AB (CIRCLE THEOREM)
- $N(3,k)$ THE CENTRE OF THE CIRCLE HAS x CO-ORDINATE 3 (CIRCLE THEOREM)
- DISTANCE OF AC IS $\sqrt{(8-0)^2+(3+1)^2}$
  $\sqrt{64+16} = \sqrt{80}$

LOOKING AT $\hat{AMN}$

$$|AM|^2 + |MN|^2 = |AN|^2$$
$$4^2 + k^2 = r^2$$
$$16 + k^2 = r^2$$

LOOKING AT $\hat{AMC}$

$$|AM|^2 + |MC|^2 = |AC|^2$$
$$4^2 + (r+k)^2 = (\sqrt{80})^2$$
$$16 + (r+k)^2 = 80$$
$$(r+k)^2 = 64$$
$$r+k = +8$$

COMBINING EQUATIONS

$$\left.\begin{array}{l} 16+k^2=r^2 \\ r+k=8 \end{array}\right\} \Rightarrow r = 8-k$$

$$\Rightarrow 16+k^2 = (8-k)^2$$
$$\Rightarrow 16+\cancel{k^2} = 64-16k+\cancel{k^2}$$
$$\Rightarrow 16k = 48$$
$$\Rightarrow k=3$$
$$\Rightarrow r=5$$

$$\therefore N(3,\overset{k}{3}) \ \& \ r=5$$

IYGB - MP1 PAPER C - QUESTION 6

a) MANIPULATING THE EXPONENTIAL AS FOLLOWS

$$2^{3x+4} = 2^{3\left(x+\frac{4}{3}\right)} \quad \text{ie } "f\left(x+\tfrac{4}{3}\right)"$$

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -\frac{4}{3} \\ 0 \end{pmatrix}$

b) CREATING AN ENLARGEMENT AS FOLLOWS

$$2^{3x+4} = 2^{3x} \times 2^{4} = 16 \times 2^{3x} \quad \text{ie } "16f(x)"$$

$\therefore$ STRETCH PARALLEL TO THE y AXIS BY SCALE FACTOR 16

IYGB - MP1 PAPER C - QUESTION 7

IF THE POSITIVE INTEGER n, IS NOT DIVISIBLE BY 3, THEN IT MUST BE OF ONE OF THE FOLLOWING FORMS

- $n = 3k+1, \ k \in \mathbb{N}$
- $n^2 - 1 = (3k+1)^2 - 1$
  $= 9k^2 + 6k + 1 - 1$
  $= 9k^2 + 6k$
  $= 3(3k^2 + 2k)$

  I.E. DIVISIBLE BY 3

- $n = 3k+2, \ k \in \mathbb{N}$
- $n^2 - 1 = (3k+2)^2 - 1$
  $= 9k^2 + 12k + 4 - 1$
  $= 9k^2 + 12k + 3$
  $= 3(3k^2 + 4k + 1)$

  I.E. DIVISIBLE BY 3

HENCE, BY EXHAUSTION, THE RESULT HOLDS //

IYGB – MP1 PAPER C – QUESTION 8

a) WHEN $n=0$ (START), $V=40$

$\therefore$ £40,000

b) USING LOGARITHMS OF ANY BASE INCLUDING NATURAL

$$V = 40\left(1+\frac{r}{100}\right)^n \Rightarrow \ln V = \ln\left[40\left(1+\frac{r}{100}\right)^n\right]$$

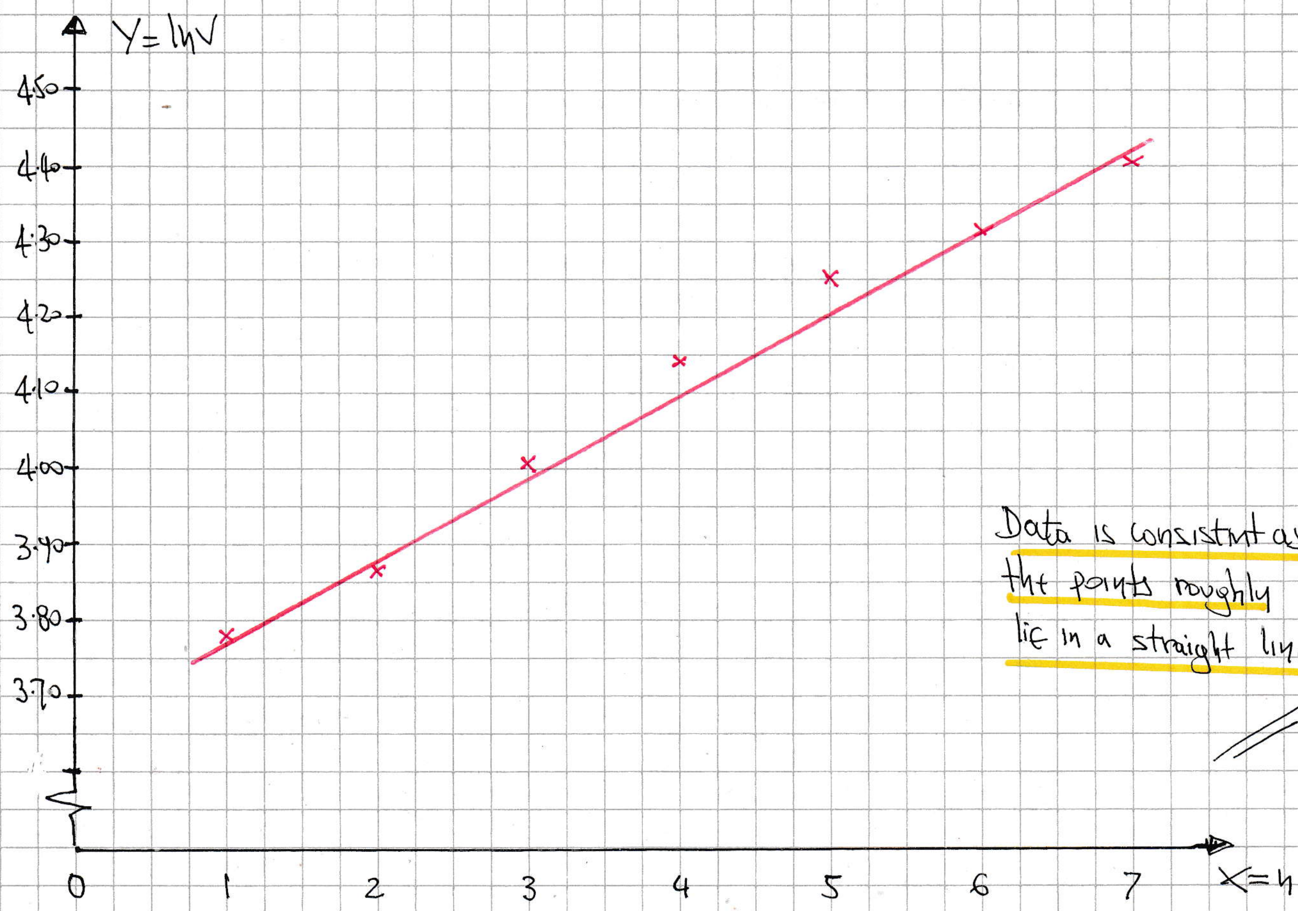
$$\Rightarrow \ln V = \ln 40 + \ln\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow \ln V = \ln 40 + n\ln\left(1+\frac{r}{100}\right)$$

$$\Rightarrow \ln V = n\ln\left(1+\frac{r}{100}\right) + \ln 40$$

Y   X   GRADIENT   "Y INTERCEPT"

$X = n$	1	2	3	4	5	6	7
$Y = \ln V$	3.78	3.87	4.01	4.14	4.20	4.32	4.41



Data is consistent as the points roughly lie in a straight line

IYGB — MP1 PAPER C — QUESTION 8

c) THE ANNUAL % GROWTH IS PART OF THE GRADIENT

USING TWO POINTS ON THE LINE $(2, 3.88)$ & $(7, 4.42)$

$$\Rightarrow \text{GRADIENT} = \frac{4.42 - 3.88}{7 - 2}$$

$$\ln\left(1 + \frac{r}{100}\right) = 0.108$$

$$1 + \frac{r}{100} = e^{0.108}$$

$$1 + \frac{r}{100} = 1.11404\ldots$$

$$100 + r = 111.404\ldots$$

$$r = 11.1404$$

$$r \simeq 11$$

i.e. 11%

d) USING THE FORMULA NOW

$$V = 40\left(1 + \frac{11}{100}\right)^n$$

$$V = 40 \times 1.11^{10}$$

$$V = 113.576\ldots$$

$$\therefore V \simeq £113.576\ldots$$

$$V \simeq £114{,}000$$

NOT RELIABLE AS THERE IS NO EVIDENCE THAT THIS TREND WILL CONTINUE

IYGB – MP1 PAPER C – QUESTION 9

- IF $f(x) = \frac{1}{x^3}$ THEN $f(x+h) = \frac{1}{(x+h)^3}$

- $f(x+h) - f(x) = \frac{1}{(x+h)^3} - \frac{1}{x^3} = \frac{x^3 - (x+h)^3}{x^3(x+h)^3}$

$$= \frac{\cancel{x^3} - (\cancel{x^3} + 3x^2h + 3xh^2 + h^3)}{x^3(x+h)^3}$$

$$= -\frac{3x^2h + 3xh^2 + h^3}{x^3(x+h)^3}$$

FROM THE FORMAL DEFINITION OF THE DERIVATIVE AS A LIMIT

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h \to 0}\left[\left[f(x+h) - f(x)\right] \div h\right]$$

$$= \lim_{h \to 0}\left[-\frac{3x^2h + 3xh^2 + h^3}{x^3(x+h)^3} \times \frac{1}{h}\right]$$

$$= \lim_{h \to 0}\left[-\frac{(3x^2 + 3xh + h^2)\cancel{h}}{x^3(x+h)^3} \times \frac{1}{\cancel{h}}\right]$$

$$= \lim_{h \to 0}\left[-\frac{3x^2 + 3xh + h^2}{x^3(x+h)^3}\right]$$

TAKING LIMITS YIELDS

$$= -\frac{3x^2}{x^3 \times x^3} = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

IYGB - MP1 PAPER C - QUESTION 10

a) $A(-4,-7) \quad B(4,9)$

$\Rightarrow$ GRAD AB $= \dfrac{9-(-7)}{4-(-4)} = \dfrac{16}{8} = 2$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 9 = 2(x-4)$

$\Rightarrow y - 9 = 2x - 8$

$\Rightarrow y = 2x + 1$

b) SOLVING SIMULTANEOUSLY

$\left.\begin{array}{l} y = 2x+1 \\ y = \frac{1}{2}x + 4 \end{array}\right\} \Rightarrow 2x+1 = \frac{1}{2}x+4$

$\Rightarrow 4x + 2 = x + 8$

$\Rightarrow 3x = 6$

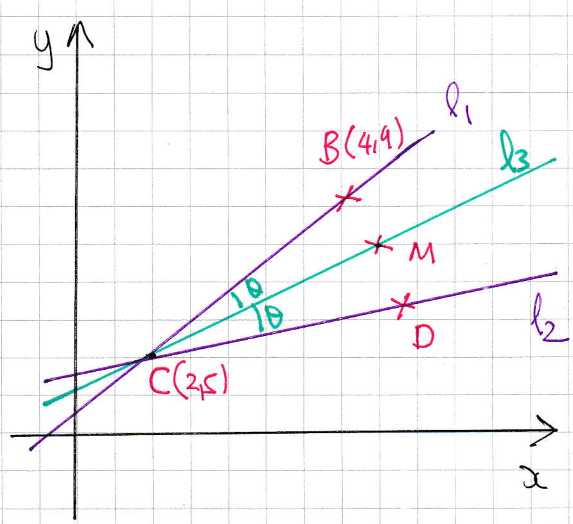
$\Rightarrow x = 2$

$\Rightarrow y = 5$

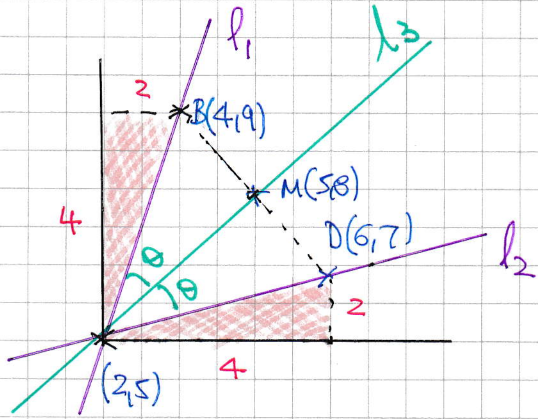
$\therefore C(2,5)$

c) DRAW A DIAGRAM

- FIND A POINT D ON l_2 SO THAT $|BC| = |DC|$
- FIND THE MIDPOINT OF BD, CALL IT M
- FIND THE GRADIENT MC
- DETERMINE THE EQUATION OF THE ANGLE BISECTOR l_3



IYGB - MP1 PAPER C - QUESTION 10



BY INSPECTION

D(6,7)

M(5,8)

$$\text{GRAD MC} = \frac{8-5}{5-2} = 1$$

$\therefore$ l_3 HAS GRADIENT 1

$$\therefore \quad y-5 = 1(x-2)$$

$$y = x+3$$

IYGB — MP1 PAPER C — QUESTION 11

a) USING THE STANDARD FORMULA FOR EXPANDING $(1+2x)^n$

$$\Rightarrow (1+ax)^n = 1 + \frac{n}{1}(ax)^1 + \frac{n(n-1)}{1\times 2}(ax)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(ax)^3 + \dots$$

$$\Rightarrow (1+2x)^7 = 1 + \frac{7}{1}(2x)^1 + \frac{7\times 6}{1\times 2}(2x)^2 + \frac{7\times 6\times 5}{1\times 2\times 3}(2x)^3 + \dots$$

$$\Rightarrow (1+2x)^7 = 1 + 14x + 21(4x^2) + 35(8x^3) + \dots$$

$$\Rightarrow (1+2x)^7 = 1 + 14x + 84x^2 + 280x^3 + \dots$$

b) LOOKING THE EXPRESSION GIVEN, PROCEED AS FOLLOWS

$$(3+4x-4x^2)(1+2x)^6 = -(4x^2-4x-3)(1+2x)^6$$
$$= -(2x+1)(2x-3)(1+2x)^6$$
$$= -(2x-3)(2x+1)^7$$
$$= (3-2x)(1+2x)^7$$

USING PART (a)

$$(3+4x-4x^2)(1+2x)^6 = (3-2x)(1+14x+84x^2+280x^3+\dots)$$
$$= 3 + 42x + 252x^2 + 840x^3 + \dots$$
$$\quad\quad -2x - 28x^2 - 168x^3 + \dots$$

$$= 3 + 40x + 224x^2 + 672x^3 + \dots$$

ALTERNATIVE TO PART (b)

$$(1+2x)^6 = 1 + \frac{6}{1}(2x)^1 + \frac{6\times 5}{1\times 2}(2x)^2 + \frac{6\times 5\times 4}{1\times 2\times 3}(2x)^3 + \dots$$

$$(1+2x)^6 = 1 + 12x + 15(4x^2) + 20(8x^3) + \dots$$

IYGB - MP1 PAPER C - QUESTION 11

$$\Rightarrow (1+2x)^6 = 1 + 12x + 60x^2 + 160x^3 + \dots$$

NOW MULTIPLY THE EXPANSION BY THE GIVEN QUADRATIC

$$(3+4x-4x^2)(1+2x)^6 = (3+4x-4x^2)(1+12x+60x^2+160x^3+\dots)$$

$$\begin{aligned} = 3 + 36x &+ 180x^2 + 480x^3 + \dots \\ 4x &+ 48x^2 + 240x^3 + \dots \\ &- 4x^2 - 48x^3 - \dots \end{aligned}$$

$$= 3 + 40x + 224x^2 + 672x^3 + \dots$$

AS BEFORE

IYGB - MP1 PAPER C - QUESTION 12

$$P = \frac{125ka^t}{k+2a^t}, \quad t \geqslant 0$$

P = POPULATION (NUMBER)
t = TIME (IN YEARS)

$t=0, \quad P=100$
$t=5, \quad P=200$

a) USING $t=0$, $P=100$ IN THE ABOVE FORMULA

$$\Rightarrow 100 = \frac{125k \times a^0}{k+2 \times a^0}$$

$$\Rightarrow 100 = \frac{125k}{k+2}$$

$$\Rightarrow 100(k+2) = 125k$$

$$\Rightarrow 100k + 200 = 125k$$

$$\Rightarrow 200 = 25k$$

$$\Rightarrow \underline{k = 8}$$

b) USING $t=5$, $P=200$ IN THE REVISED FORMULA

$$\Rightarrow P = \frac{125 \times 8 \times a^t}{8 + 2 \times a^t}$$

$$\Rightarrow 200 = \frac{125 \times 8 \times a^5}{8 + 2 \times a^5}$$

$$\Rightarrow 200 = \frac{1000a^5}{8+2a^5} \quad \div 200$$

$$\Rightarrow 1 = \frac{5a^5}{8+2a^5}$$

$$\Rightarrow 8 + 2a^5 = 5a^5$$

$$\Rightarrow 8 = 3a^5$$

$$\Rightarrow a^5 = \frac{8}{3}$$

$$\Rightarrow a = \sqrt[5]{8/3} = 1.216728684\ldots \qquad \therefore \underline{a \approx 1.217}$$

IYGB - MP1 PAPER C - QUESTION 12

c) STARTING AGAIN WITH A YET REVISED VERSION OF THE FORMULA

$$P = \frac{125 \times 8 \times a^t}{8+2a^t} \Longrightarrow P = \frac{\overset{500}{\cancel{1000}}a^t}{{}_4\cancel{8+2a^t}} \qquad [a = 1.216720684\ldots]$$

$$\Longrightarrow P = \frac{500a^t}{4+a^t}$$

$$\Longrightarrow 400 = \frac{500a^t}{4+a^t}$$

$$\div 100$$

$$\Longrightarrow 4 = \frac{5a^t}{4+a^t}$$

$$\Longrightarrow 16 + 4a^t = 5a^t$$

$$\Longrightarrow 16 = a^t$$

$$\Longrightarrow \log 16 = \log a^t$$

$$\Longrightarrow \log 16 = t \log a$$

$$\Longrightarrow t = \frac{\log 16}{\log a} = \frac{\log 16}{\log(1.2167\ldots)} = 14.13390\ldots$$

$$\therefore t \approx 14.13$$

d) LOOKING AT THE FORMULA $P = \frac{1000a^t}{8+2a^t}$ WHICH CAN BE REDUCED

TO $P = \frac{500a^t}{4+a^t}$

DIVIDE TOP & BOTTOM OF THE FRACTION BY a^t TO GIVE

$$P = \frac{\frac{500a^t}{a^t}}{\frac{4}{a^t} + \frac{a^t}{a^t}} \Longrightarrow P = \frac{500}{\frac{4}{a^t}+1}$$

AS t GETS VERY LARGE a^t ALSO GETS VERY LARGE (AS $a > 1$)

SO $\frac{4}{a^t}$ BECOMES PRACTICALLY ZERO, WHICH MEANS THE FORMULA GIVES $P = \frac{500}{1}$

AS THE POPULATION STARTS FROM 100 THE LIMITING VALUE IS 500 & CANNOT EXCEED IT