IYGB – MP1 PAPER G – QUESTION 1

$$f(x) \equiv x^3 - 19x + k \quad , \quad x \in \mathbb{R}$$

a) PASSES THROUGH THE ORIGIN $(0,0)$

$$\Longrightarrow 0 = 0^3 - 19 \times 0 + k$$
$$\Longrightarrow k = 0$$

b) MEETS THE y AXIS AT $y=5$, IE $(0,5)$

$$\Longrightarrow 5 = 0^3 - 19 \times 0 + k$$
$$\Longrightarrow k = 5$$

c) MEETS THE x AXIS AT $x=2$, IE $(2,0)$

$$\Longrightarrow 0 = 2^3 - 19 \times 2 + k$$
$$\Longrightarrow 0 = 8 - 38 + k$$
$$\Longrightarrow k = 30$$

d) PASSES THROUGH $(-1,-7)$

$$\Longrightarrow -7 = (-1)^3 - 19(-1) + k$$
$$\Longrightarrow -7 = -1 + 19 + k$$
$$\Longrightarrow -7 = 18 + k$$
$$\Longrightarrow k = -25$$

IYGB - MP1 PAPER G - QUESTION 2

(a) $(3-\sqrt{8})^2 = 3^2 - 2\times3\times\sqrt{8} + \sqrt{8}^2$

$= 9 - 6\sqrt{8} + 8$

$= 17 - 6\sqrt{8}$

$= 17 - 6\times2\sqrt{2}$

$= 17 - 12\sqrt{2}$

$\sqrt{8} = \sqrt{4}\sqrt{2}$
$\sqrt{8} = 2\sqrt{2}$

b) $\dfrac{\sqrt{63}}{3} + \dfrac{14}{\sqrt{7}} = \dfrac{\sqrt{9}\sqrt{7}}{3} + \dfrac{14\sqrt{7}}{\sqrt{7}\sqrt{7}}$

$= \dfrac{3\sqrt{7}}{3} + \dfrac{14\sqrt{7}}{7}$

$= \sqrt{7} + 2\sqrt{7}$

$= 3\sqrt{7}$

IYGB - MP1 PAPER G - QUESTION 3

MANIPULATE AS FOLLOWS

$$\Rightarrow \frac{5\sin\theta - 2\cos\theta}{\sin\theta} = 3$$

$$\Rightarrow 5\sin\theta - 2\cos\theta = 3\sin\theta$$

$$\Rightarrow 2\sin\theta = 2\cos\theta$$

$$\Rightarrow \sin\theta = \cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$$

$$\Rightarrow \tan\theta = 1$$

$\arctan(1) = 45°$

$\theta = 45° \pm 180n \quad n = 0,1,2,3,\dots$

$\theta = 45°, 225°$

IYGB - MP1 PAPER G - QUESTION 4

a) USING $y - y_0 = m(x - x_0)$

$$\Rightarrow y + 3 = \frac{1}{3}(x - 10)$$
$$\Rightarrow 3y + 9 = x - 10$$
$$\Rightarrow 3y - x = -19$$
$$\Rightarrow x - 3y - 19 = 0$$

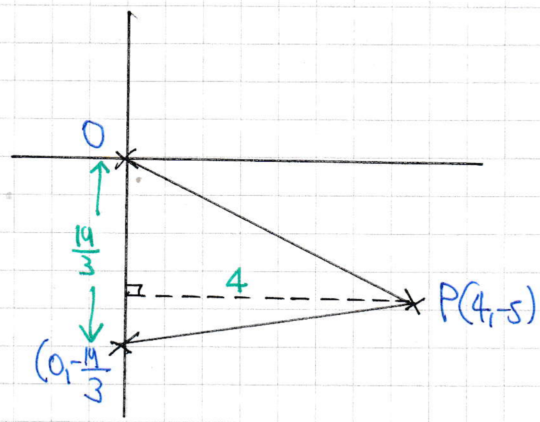
b) FIND EQUATION OF l_2 & SOLVE EQUATIONS

$$\Rightarrow l_2: \quad y = -2x + 3$$

BY SUBSTITUTION INTO l_1

$$\Rightarrow x - 3(-2x+3) - 19 = 0$$
$$\Rightarrow x + 6x - 9 - 19 = 0$$
$$\Rightarrow 7x = 28$$
$$\Rightarrow x = 4$$
$$\Rightarrow y = -5 \qquad \therefore P(4,-5)$$

c) ON l_1, $x=0$, YIELDS $-3y - 19 = 0 \Rightarrow y = -\frac{19}{3}$



REQUIRED AREA $= \frac{1}{2} \times \frac{19}{3} \times 4$

$$= 2 \times \frac{19}{3}$$
$$= \frac{38}{3}$$

IYGB – MP1 PAPER G – QUESTION 5

a) COMPLETING THE SQUARE

$$f(x) = x^2 - 2x - 4 = (x-1)^2 - 1^2 - 4$$
$$= (x-1)^2 - 1 - 4$$
$$= (x-1)^2 - 5$$

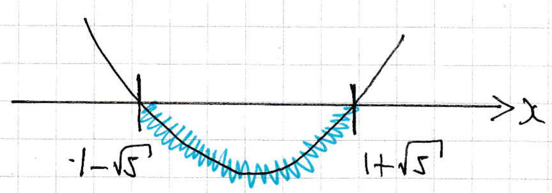
b) USING PART (a) $f(x) = 0$

$$\Rightarrow (x-1)^2 - 5 = 0$$
$$\Rightarrow (x-1)^2 = 5$$
$$\Rightarrow x - 1 = \pm\sqrt{5}$$
$$\Rightarrow x = \begin{cases} 1+\sqrt{5} \\ 1-\sqrt{5} \end{cases}$$

c) TIDYING UP THE INEQUALITY

$$\Rightarrow 2(3x-4) - (x+6)(x-2) > 0$$
$$\Rightarrow 6x - 8 - (x^2 - 2x + 6x - 12) > 0$$
$$\Rightarrow \cancel{6x} - 8 - x^2 + 2x - \cancel{6x} + 12 > 0$$
$$\Rightarrow -x^2 + 2x + 4 > 0$$
$$\Rightarrow x^2 - 2x - 4 < 0$$

USING PART (a) & (b)



$\therefore \; 1-\sqrt{5} < x < 1+\sqrt{5}$

IYGB - MP1 PAPER G - QUESTION 6

a) $y = 8 + 2x - x^2$

- WHEN $x=0$

$y = 8$

$\therefore P(0,8)$

- WHEN $y=0$

$0 = 8 + 2x - x^2$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = \begin{cases} -2 & \leftarrow Q(-2,0) \\ 4 & \leftarrow R(4,0) \end{cases}$

b) DIFFERENTIATING

$\frac{dy}{dx} = 2 - 2x$

$\left.\frac{dy}{dx}\right|_{\substack{x=0 \\ \uparrow \\ P}} = 2 - 2\times 0 = 2 \quad \leftarrow$ tangent gradient.

P(0,8)

$\therefore$ EQUATION OF TANGENT : $y = 2x + 8$

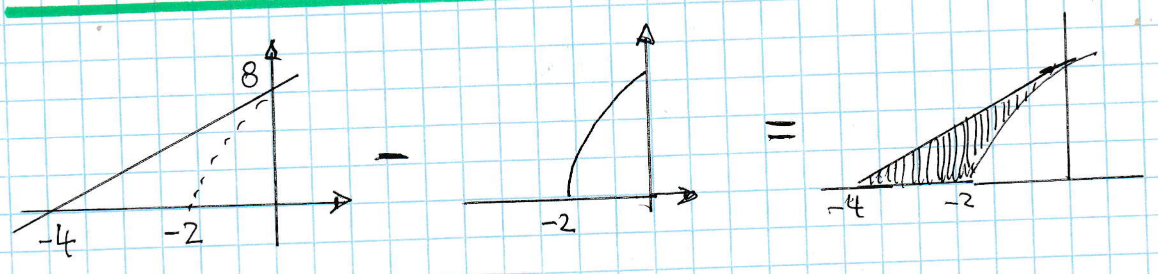
c) THE x INTERCEPT OF THE TANGENT IS GIVEN BY $y=0$

$\Rightarrow 0 = 2x + 8$

$\Rightarrow -8 = 2x$

$\Rightarrow x = -4$

LOOKING AT THE "PICTORIAL" EQUATION BELOW



IYGB – MP1 PAPER G – QUESTION 6

AREA OF TRIANGLE

$\frac{1}{2} \times 4 \times 8 = 16$

AREA "UNDER THE CURVE" BETWEEN −2 & 0

$$\int_{-2}^{0} 8 + 2x - x^2 \, dx = \left[8x + x^2 - \frac{1}{3}x^3 \right]_{-2}^{0}$$

$$= (0 + 0 - 0) - \left(-16 + 4 + \frac{8}{3}\right)$$

$$= 16 - 4 - \frac{8}{3}$$

$$= \frac{28}{3}$$

THE REQUIRED AREA $= 16 - \frac{28}{3} = \frac{20}{3}$

AS REQUIRED

IYGB – MP1 PAPER G – QUESTION 7

a) USING THE STANDARD BINOMIAL EXPANSION FORMULA

$$(1-2x)^{11} = 1 + \frac{11}{1}(-2x)^1 + \frac{11\times10}{1\times2}(-2x)^2 + \frac{11\times10\times9}{1\times2\times3}(-2x)^3 + \frac{11\times10\times9\times8}{1\times2\times3\times4}(-2x)^4 + \dots$$

$$= 1 - 22x + 220x^2 - 1320x^3 + 5280x^4 + \dots$$

b) WORKING AS FOLLOWS

$$1 - 2x = \frac{14}{15}$$

$$\frac{1}{15} = 2x$$

$$x = \frac{1}{30}$$

USING $x = \frac{1}{30}$ IN THE EXPANSION OF PART (a)

$$\left(1 - 2\times\frac{1}{30}\right)^{11} \simeq 1 - 22\left(\frac{1}{30}\right) + 220\left(\frac{1}{30}\right)^2 - 1320\left(\frac{1}{30}\right)^3 + 5280\left(\frac{1}{30}\right)^4$$

$$\left(\frac{14}{15}\right)^{11} \simeq 1 - \frac{11}{15} + \frac{11}{45} - \frac{11}{225} + \frac{22}{3375}$$

$$\left(\frac{14}{15}\right)^{11} \simeq \frac{1582}{3375}$$

AS REQUIRED

c)

$$\boxed{\text{PERCENTAGE ERROR} = \left|\frac{\text{ACTUAL ERROR}}{\text{ACTUAL ANSWER}}\right| \times 100}$$

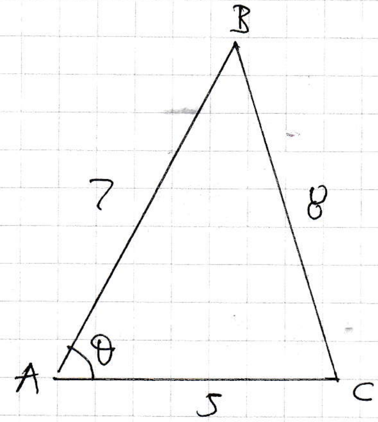
$$= \left|\frac{\frac{1582}{3375} - \left(\frac{14}{15}\right)^{11}}{\left(\frac{14}{15}\right)^{11}}\right| \times 100$$

$$= 0.122\ \%$$

IYGB - MP1 PAPER G - QUESTION 8

METHOD A

BY THE COSINE RULE



$$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos\theta$$

$$8^2 = 7^2 + 5^2 - 2\times7\times5\cos\theta$$

$$64 = 49 + 25 - 70\cos\theta$$

$$70\cos\theta = 10$$

$$\cos\theta = \frac{1}{7}$$

NOW IF $\cos\theta = \frac{1}{7}$

$$\sin\theta = +\sqrt{1-\cos^2\theta} \quad (\theta \text{ A PHYSICAL ANGLE})$$

$$\sin\theta = +\sqrt{1-\frac{1}{49}}$$

$$\sin\theta = \sqrt{\frac{48}{49}}$$

$$\sin\theta = \frac{\sqrt{16}\sqrt{3}}{7}$$

$$\sin\theta = \frac{4\sqrt{3}}{7}$$

HENCE THE AREA IS GIVEN BY

$$\frac{1}{2}|AB||AC|\sin\theta = \frac{1}{2}\times5\times\cancel{7}\times\frac{4}{\cancel{7}}\sqrt{3} = 10\sqrt{3}$$

METHOD B

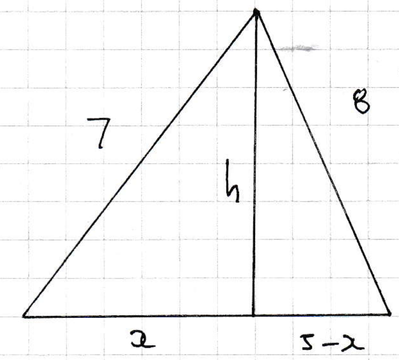
BY HERON FORMULA THE SEMIPERIMETER IS $\frac{1}{2}(7+8+5) = 10$

$$\text{AREA} = \sqrt{10(10-8)(10-7)(10-5)} = \sqrt{10\times2\times3\times5} =$$

$$= \underbrace{\sqrt{5}\sqrt{2}}_{\sqrt{10}}\sqrt{2}\sqrt{3}\sqrt{5} = \sqrt{5}\sqrt{5}\sqrt{2}\sqrt{2}\sqrt{3} = 5\times2\times\sqrt{3} = 10\sqrt{3}$$

IYGB - MP1 PAPER G - QUESTION 9

METHOD C



- $x^2+h^2=49$
- $(5-x)^2+h^2=64$

SUBTRACT

$$x^2-(5-x)^2=-15$$
$$x^2-(25-10x+x^2)=-15$$
$$\cancel{x^2}-25+10x-\cancel{x^2}=-15$$
$$10x=10$$
$$x=1$$

$\therefore$ $x^2+h^2=49$

$$1+h^2=49$$
$$h^2=48$$
$$h=+\sqrt{48}$$
$$h=\sqrt{16}\sqrt{3}$$
$$h=4\sqrt{3}$$

FINALLY WE HAVE

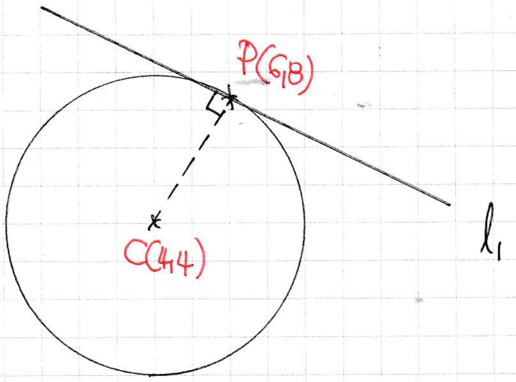
$$\text{AREA}=\frac{1}{2}\times5\times h=\frac{1}{2}\times5\times4\sqrt{3}=10\sqrt{3}$$

IYGB – MP1 PAPER G – QUESTION 9

a) FIND THE GRADIENT PC

- $m_{PC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{6-4} = \frac{4}{2} = 2$
- $m_{l_1} = -\frac{1}{2}$
- $l_1: \; y - y_0 = m(x - x_0)$

$$y - 8 = -\tfrac{1}{2}(x-6)$$
$$2y - 16 = -x + 6$$
$$2y + x = 22$$



b) SOLVING SIMULTANEOUSLY

$$\left.\begin{aligned} l_1&: \; 2y + x = 22 \\ l_2&: \; y = 2x - 14 \end{aligned}\right\} \Rightarrow 2(2x-14) + x = 22$$
$$\Rightarrow 4x - 28 + x = 22$$
$$\Rightarrow 5x = 50$$
$$\Rightarrow x = 10$$
$$\Rightarrow y = 6 \qquad \therefore Q(10,6)$$

c) START BY FINDING THE EQUATION OF THE CIRCLE

$$|CP| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(8-4)^2 + (6-4)^2} = \sqrt{16+4} = \sqrt{20}$$

CIRCLE : $(x-4)^2 + (y-4)^2 = 20$

l_2 : $y = 2x - 14$

SOLVING SIMULTANEOUSLY

$$\Rightarrow (x-4)^2 + (2x-14-4)^2 = 20$$
$$\Rightarrow (x-4)^2 + (2x-18)^2 = 20$$

IYGB - MP1 PAPER G - QUESTION 9

$$\Longrightarrow \left\{\begin{matrix} x^2 - 8x + 16 \\ 4x^2 - 72x + 324 \end{matrix}\right\} = 20$$

$$\Longrightarrow 5x^2 - 80x + 340 = 20$$

$$\Longrightarrow x^2 - 16x + 64 = 0$$

$$\Longrightarrow (x-8)^2 = 0$$

REPEATED ROOT INTERSECTION, SO l_2 IS A TANGENT (AT $x=8$)

& USING $y = 2x - 14$ YIELDS $y = 2$

$\therefore$ $R(8,2)$

IYGB - MP1 - PAPER G - QUESTION 10

$$\boxed{f(x) = x^3 + 2}$$

a) $f(-1) = (-1)^3 + 2 = -1 + 2 = 1$

b) $f(-1+h) = (-1+h)^3 + 2 = (h-1)^3 + 2 = (h-1)(h^2-2h+1) + 2$

$$= h^3 - 2h^2 + h - h^2 + 2h - 1 + 2$$

$$= h^3 - 3h^2 + 3h + 1$$

c) $$f'(-1) = \lim_{h \to 0}\left[\frac{f(-1+h) - f(-1)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{(h^3 - 3h^2 + 3h + 1) - (1)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{h^3 - 3h^2 + 3h}{h}\right]$$

$$= \lim_{h \to 0}\left[h^2 - 3h + 3\right]$$

TAKING THE LIMIT NOW, AS $h \to 0$

$$= 3$$

AS REQUIRED

IYGB - MP1 PAPER G - QUESTION 11

WORKING AS FOLLOWS

$$a^2b^2+4 = (x^{\frac{1}{2}}+x^{-\frac{1}{2}})^2(x^{\frac{1}{2}}-x^{-\frac{1}{2}})^2+4$$

$$= \left[(x^{\frac{1}{2}})^2+2(x^{\frac{1}{2}})(x^{-\frac{1}{2}})+(x^{-\frac{1}{2}})^2\right]\left[(x^{\frac{1}{2}})^2-2(x^{\frac{1}{2}})(x^{-\frac{1}{2}})+(x^{-\frac{1}{2}})^2\right]+4$$

$$= \left[x^1+2x^0+x^{-1}\right]\left[x^1-2x^0+x^{-1}\right]+4$$

$$= (x+2+x^{-1})(x-2+x^{-1})+4$$

$$= \begin{array}{l} x^2 - \cancel{2x} + x^0 \\ \quad + \cancel{2x} - 4 + \cancel{2x^{-1}} \\ \qquad\qquad x^0 - \cancel{2x^{-1}} + x^{-2} \end{array} \quad +4$$

$$= x^2 - 2 + x^{-2} + 4$$

$$= x^2 + 2 + \frac{1}{x^2}$$

$$= x^2 + 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$$

$$= \left(x+\frac{1}{x}\right)^2$$

AS REQUIRED

IYGB - MP1 PAPER G - QUESTION 12

USING A SUBSTITUTION

$$y = 10 - x$$

$$\Rightarrow \log_2 x + 2\log_4(10-x) = 4$$

CHANGING THE BASE

$$\Rightarrow \log_2 x + 2\left(\frac{\log_2(10-x)}{\log_2 4}\right) = 4$$

$$\Rightarrow \log_2 x + 2\left(\frac{\log_2(10-x)}{\log_2 2^2}\right) = 4\log_2 2$$

$$\Rightarrow \log_2 x + 2\left(\frac{\log_2(10-x)}{2\cancel{\log_2 2}}\right) = \log_2 16$$

$$\Rightarrow \log_2 x + 2\left(\frac{\log_2(10-x)}{2}\right) = \log_2 16$$

$$\Rightarrow \log_2 x + \log_2(10-x) = \log_2 16$$

$$\Rightarrow \log_2\left[x(10-x)\right] = \log_2 16$$

$$\Rightarrow x(10-x) = 16$$

$$\Rightarrow 10x - x^2 = 16$$

$$\Rightarrow 0 = x^2 - 10x + 16$$

$$\Rightarrow (x-8)(x-2) = 0$$

$$\Rightarrow x = \begin{cases} 2 \\ 8 \end{cases} \qquad y = \begin{cases} 8 \\ 2 \end{cases}$$

$\therefore$ $(8,2)$ OR $(2,8)$

IYGB – MP1 PAPER G – QUESTION 12

VARIATION

$$\Rightarrow \log_2 x + 2\log_4 y = 4$$

$$\Rightarrow \log_2 x + 2\log_4 y = 4$$

$$\Rightarrow \log_2 x + \frac{2\log_2 y^2}{\log_2 4} = 4$$

$$\Rightarrow \log_2 x + \frac{2\log_2 y}{\log_2 2^2} = 4\log_2 2$$

$$\Rightarrow \log_2 x + \frac{2\log_2 y}{2\cancel{\log_2 2}} = \log_2 16$$

$$\Rightarrow \log_2 x + \log_2 y = \log_2 16$$

$$\Rightarrow \log_2 (xy) = \log_2 16$$

$$\Rightarrow xy = 16$$

BUT $x + y = 10$

$$\Rightarrow xy + y^2 = 10y$$

$$\Rightarrow 16 + y^2 = 10y$$

$$\Rightarrow y^2 - 10y + 16 = 0$$

$$\Rightarrow (y-2)(y-8) = 0$$

$$\Rightarrow y = \begin{cases} 2 \\ 8 \end{cases} \qquad x = \begin{cases} 8 \\ 2 \end{cases}$$

$\therefore$ $(2,8)$ & $(8,2)$