# IYGB – MP1 PAPER J – QUESTION 1

a) START BY FINDING THE GRADIENT, USING $A(-4,5)$ & $B(0,4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-5}{0-(-4)} = \frac{-1}{4} = -\frac{1}{4}$$

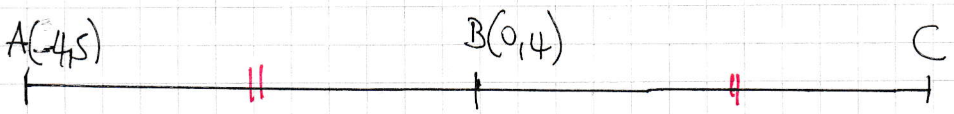
THE REQUIRED EQUATION IS

$$y - y_0 = m(x - x_0)$$
$$y - 5 = -\frac{1}{4}(x+4)$$
$$4y - 20 = -x - 4$$
$$x + 4y = 16$$

OR SIMPLY USING $(0,4)$

$$y = -\frac{1}{4}x + 4$$

b) LOOKING AT THE DIAGRAM




EVIDENTLY B MUST BE THE MIDPOINT OF AC

$x$: $-4 \xrightarrow{+4} \quad \xrightarrow{+4} 4$

$y$: $5 \xrightarrow{-1} 4 \xrightarrow{-1} 3$

$\therefore$ $C(4,3)$

# IYGB - MP1 PAPER J - QUESTION 2

a) $5\sqrt{2} \times 4\sqrt{3} - 6\sqrt{24} = 5 \times 4 \times \sqrt{2} \times \sqrt{3} - 6\sqrt{4}\sqrt{6}$

$= 20\sqrt{6} - 6 \times 2\sqrt{6}$

$= 20\sqrt{6} - 12\sqrt{6}$

$= 8\sqrt{6}$

b) $\dfrac{3+\sqrt{6}}{\sqrt{3}} = \dfrac{(3+\sqrt{6})\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{3\sqrt{3}+\sqrt{18}}{3} = \dfrac{3\sqrt{3}+\sqrt{9}\sqrt{2}}{3}$

$= \dfrac{3\sqrt{3}+3\sqrt{2}}{3} = \sqrt{3}+\sqrt{2}$

ALTERNATIVE

$\dfrac{3+\sqrt{6}}{\sqrt{3}} = \dfrac{3}{\sqrt{3}} + \dfrac{\sqrt{6}}{\sqrt{3}} = \dfrac{3\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{\dfrac{6}{3}} = \dfrac{3\sqrt{3}}{3} + \sqrt{2}$

$= \sqrt{3}+\sqrt{2}$

# IYGB – MP1 PAPER I – QUESTION 3

a) USING THE STANDARD EXPANSION FORMULA

$$\Rightarrow (2+x)^5 = \binom{5}{0}(2)^5(x)^0 + \binom{5}{1}(2)^4(x)^1 + \binom{5}{2}(2)^3(x)^2 + \binom{5}{3}(2)^2(x)^3 + \binom{5}{4}(2)^1(x)^4 + \binom{5}{5}(2)^0(x)^5$$

$$\Rightarrow (2+x)^5 = (1\times32\times1) + (5\times16\times x) + (10\times8\times x^2) + (10\times4\times x^3) + (5\times2\times x^4) + (1\times1\times x^5)$$

$$\Rightarrow (2+x)^5 = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$$

b) REPLACE $x$ WITH $-x^2$ IN THE ABOVE EXPANSION

$$\Rightarrow [2+(-x^2)]^5 = 32 + 80(-x^2) + 80(-x^2)^2 + 40(-x^2)^3 + 10(-x^2)^4 + (-x^2)^5$$

$$\Rightarrow (2-x^2)^5 = 32 - 80x^2 + 80x^4 - 40x^6 + 10x^8 - x^{10}$$

c) NEED TO CREATE $1.99^5$ FROM $(2-x^2)^5$

I.E $\quad 2 - x^2 = 1.99$

$0.01 = x^2$

$x = \pm 0.1 \quad$ (BOTH ARE O.K TO USE)

SUBSTITUTE INTO THE ANSWER OF PART (b)

$$\Rightarrow [2-(0.1)^2]^5 = 32 - 80(0.1)^2 + 80(0.1)^4 - 40(0.1)^6 + \text{"VERY SMALL NUMBERS"}$$

$$\Rightarrow 1.99^5 = 32 - 0.8 + 0.008 - \underbrace{\cancel{0.00004}}_{\text{TOO SMALL}} + \dots$$

$$\Rightarrow 1.99^5 \approx 31.208$$

# IYGB - MP1 PAPER J - QUESTION 4

a) START BY FINDING THE CO-ORDINATES OF A

$$y = 6x - x^2$$
$$y = x(6-x)$$
$$\therefore A(6,0)$$

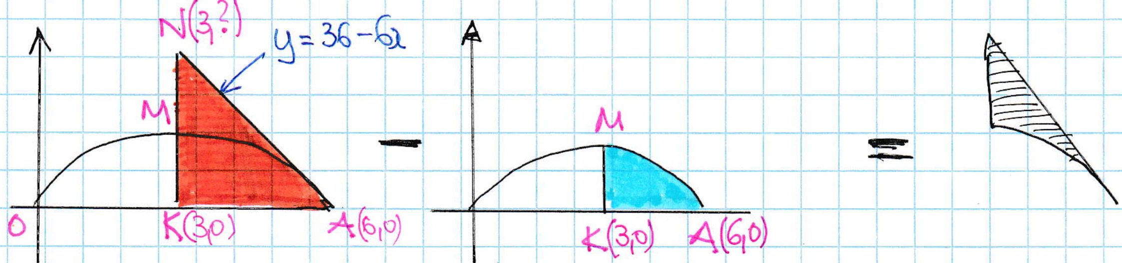
FIND THE GRADIENT AT A

$$y = 6x - x^2$$
$$\frac{dy}{dx} = 6 - 2x$$
$$\left.\frac{dy}{dx}\right|_{x=6} = 6 - 2\times6 = -6 \quad \leftarrow \text{TANGENT GRADIENT}$$

EQUATION OF TANGENT

$$y - y_0 = m(x - x_0)$$
$$y - 0 = -6(x-6)$$
$$y = -6x + 36$$
$$y = 36 - 6x$$

b)




$$y = 36 - 6\times3$$
$$y = 18$$
$$\therefore N(3,18)$$

$$\text{AREA} = \frac{1}{2}|KN||KA|$$
$$= \frac{1}{2}\times18\times3 = 27$$

$$\int_3^6 6x - x^2 \, dx$$
$$= \left[3x^2 - \frac{1}{3}x^3\right]_3^6$$
$$= (108-72) - (27-9)$$
$$= 18$$

$$= 9$$

# IYGB - MP1 PAPER J - QUESTION 5

MANIPULATE THE LEFT HAND SIDE

$$\frac{2^{288} + 2^{285}}{9} = \frac{2^{285} \times 2^{3} + 2^{285}}{9}$$

$$= \frac{8 \times 2^{285} + 2^{285}}{9}$$

$$= \frac{9 \times 2^{285}}{9}$$

$$= 2^{285}$$

$\therefore \ k = 285$

# IYGB - MP1 PAPER I - QUESTION 6

a) FIND THE GRADIENT FUNCTION FOR THE QUADRATIC - USE IT AT $R(4,10)$

$$y = x^2 - 6x + 18$$

$$\frac{dy}{dx} = 2x - 6$$

$$\left.\frac{dy}{dx}\right|_{x=4} = 2\times4-6 = 2$$

EQUATION OF TANGENT AT R

$$y - y_0 = m(x - x_0)$$
$$y - 10 = 2(x-4)$$

WHEN $x=0$

$$y - 10 = -8$$
$$y = 2$$

$\therefore Q(0,2)$

EQUATION OF NORMAL AT R

$$y - y_0 = m(x - x_0)$$
$$y - 10 = -\frac{1}{2}(x-4)$$

WHEN $x=0$

$$y - 10 = 2$$
$$y = 12$$

$P(0,12)$

b) AS THERE IS A RIGHT ANGLE AT R (NORMAL & TANGENT), PQ MUST BE A DIAMETER

$\therefore$ MIDPOINT OF PQ IS $(0,7)$ ← CENTRE

LENGTH PQ IS 10, SO $r=5$

$$\therefore (x-0)^2 + (y-7)^2 = 5^2$$
$$x^2 + (y-7)^2 = 25$$

# IYGB - MP1 PAPER I - QUESTION 7

## REMOVING THE FRACTIONAL PARTS

$$\Rightarrow \frac{3+\sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

$$\Rightarrow 3+\sin^2\theta = 3\cos\theta(\cos\theta - 2)$$

$$\Rightarrow 3+\sin^2\theta = 3\cos^2\theta - 6\cos\theta$$

## USING $\sin^2\theta \equiv 1-\cos^2\theta$

$$\Rightarrow 3+(1-\cos^2\theta) = 3\cos^2\theta - 6\cos\theta$$

$$\Rightarrow 4-\cos^2\theta = 3\cos^2\theta - 6\cos\theta$$

$$\Rightarrow 0 = 4\cos^2\theta - 6\cos\theta - 4$$

$$\Rightarrow 2\cos^2\theta - 3\cos\theta - 2 = 0$$

$$\Rightarrow (2\cos\theta + 1)(\cos\theta - 2) = 0$$

$$\Rightarrow \cos\theta = \begin{cases} \cancel{2} \\ -\frac{1}{2} \end{cases}$$

## FINALLY WE OBTAIN

$$\arccos\left(-\tfrac{1}{2}\right) = 120°$$

$$\Rightarrow \left(\begin{array}{l} \theta = 120° \pm 360n \\ \theta = 240° \pm 360n \end{array}\right. \qquad n = 0,1,2,3,\ldots$$

$$\therefore \theta = 120°, 240°$$

# IYGB - MP1 PAPER I - QUESTION 8

MANIPULATING THE DIFFERENCE OF SQUARES $a^2 - b^2 \equiv (a-b)(a+b)$

$$\Longrightarrow f(n) = 5^{2n} - 1$$
$$= (5^n)^2 - 1^2$$
$$= (5^n - 1)(5^n + 1)$$

NOW CONSIDER THE FOLLOWING ARGUMENT

$\Longrightarrow 5^n$ IS AN ODD INTEGER AS IT IS A POWER OF 5

I.E. 5, 25, 125, 625, 3125, 15625, ...

$\Longrightarrow 5^n + 1$ & $5^n - 1$ ARE BOTH EVEN

BUT FURTHER TO THIS $5^n - 1$ & $5^n + 1$ ARE TWO CONSECUTIVE EVEN NUMBERS, SO ONE OF THEM WILL BE A MULTIPLE OF 4

LET $5^n - 1 = 2a \qquad a \in \mathbb{N}$

$5^n + 1 = 4b \qquad b \in \mathbb{N}$

(OR THE OTHER WAY ROUND)

HENCE WE OBTAIN

$$f(n) = (5^n - 1)(5^n + 1) = 2a \times 4b = 8ab$$

INDEED A MULTIPLE OF 8

# IYGB - MP1 PAPER I - QUESTION 9

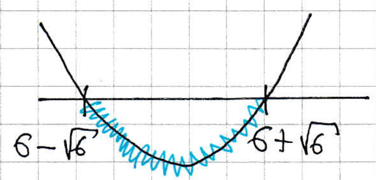
a) AS THE SOLUTIONS ARE SURDS, COMPLETE THE SQUARE OR USE THE QUADRATIC FORMULA

$$f(x)=0 \implies x^2-12x+30=0$$
$$\implies (x-6)^2-6^2+30=0$$
$$\implies (x-6)^2-36+30=0$$
$$\implies (x-6)^2=6$$
$$\implies x-6=\pm\sqrt{6}$$
$$\implies x = \begin{cases} 6+\sqrt{6} \\ 6-\sqrt{6} \end{cases}$$

b) I) USING PART (a)

$$x^2-12+30<0$$

$$C.V = \begin{cases} 6+\sqrt{6} \\ 6-\sqrt{6} \end{cases}$$




$$6-\sqrt{6} < x < 6+\sqrt{6}$$

II) USING PART (b I)

$$6-\sqrt{6} < n < 6+\sqrt{6}$$

$$3.5505... < n < 8.4494...$$

$$\therefore n = 4,5,6,7,8 \qquad \text{OR} \qquad 4 \le n \le 8$$
$$n \in \mathbb{N}$$

# IYGB - MP1 PAPER I - QUESTION 10

## SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 2x + k \\ y = x^2 - 8x + 1 \end{array}\right\} \Rightarrow x^2 - 8x + 1 = 2x + k$$

$$\Rightarrow x^2 - 10x + 1 - k = 0$$

$$\Rightarrow x^2 - 10x + (1-k) = 0$$

## IF TANGENT WE ARE LOOKING FOR REPEATED ROOTS...

$$b^2 - 4ac = 0 \Rightarrow (-10)^2 - 4 \times 1 \times (1-k) = 0$$

$$\Rightarrow 100 - 4(1-k) = 0$$

$$\Rightarrow 100 - 4 + 4k = 0$$

$$\Rightarrow 4k = -96$$

$$\Rightarrow k = -24$$

## FINALLY IF $k = -24$

$$x^2 - 10x + (1-k) = 0$$

$$x^2 - 10x + 25 = 0$$

$$(x-5)^2 = 0$$

$$x = 5$$

USING $y = 2x - 24$ GIVES $-14$

$\therefore$ CONTACT POINT $(5, -14)$

# IYGB - MP1 PAPER I - QUESTION 11

$f(x) = x^3 - 6x^2 + 10x - 3$

$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ REPRESENTS A TRANSLATION BY

2 UNITS TO THE "LEFT" $\Rightarrow$ "$f(x+2)$"

3 UNITS "UPWARDS" $\Rightarrow$ "$f(x)+3$"

HENCE WE HAVE

$f(x+2)+3 = \left[(x+2)^3 - 6(x+2)^2 + 10(x+2) \cancel{-3}\right] \cancel{+3}$

$= (x+2)^3 - 6(x+2)^2 + 10(x+2)$

$= x^3 + 6x^2 + 12x + 8 - 6(x^2+4x+4) + 10x + 20$

$= x^3 \cancel{+6x^2} + 12x + 8$
$\cancel{-6x^2} - 24x - 24$
$10x + 20$

$x^3 - 2x + 4$

$(x+2)(x+2)^2$
$= (x+2)(x^2+4x+4)$
$= x^3 + 4x^2 + 4x$
$+ 2x^2 + 8x + 8$
$= x^3 + 6x^2 + 12x + 8$

$\therefore y = x^3 - 2x + 4$

FINALLY THE $y$ INTERCEPT, WHEN $x=0$

$(0,4)$

# IYGB - MP1 PAPER I - QUESTION 12

a) MANIPULATE AS FOLLOWS

$$\Rightarrow \frac{1}{2} \times 4^{3x+1} = 600^{600}$$

$$\Rightarrow \log_{10}\left(\frac{1}{2} \times 4^{3x+1}\right) = \log_{10} 600^{600}$$

$$\Rightarrow \log_{10}\left(\frac{1}{2}\right) + \log_{10} 4^{3x+1} = 600 \log_{10} 600$$

$$\Rightarrow \log_{10}\left(\frac{1}{2}\right) + (3x+1)\log_{10} 4 = 600 \log_{10} 600$$

$$\Rightarrow (3x+1)\log_{10} 4 = 600 \log_{10} 600 - \log\left(\frac{1}{2}\right)$$

$$\Rightarrow 3x+1 = \frac{600 \log_{10} 600 - \log_{10}(0.5)}{\log_{10} 4}$$

$$\Rightarrow 3x+1 = 2769.145607\ldots$$

$$\Rightarrow x = 922.7152024\ldots$$

$$x \simeq 923$$

b) PROCEED USING THE RULES OF LOGS

$$\Rightarrow \log_3 (2y+5) = 1 - \log_3 y$$

$$\Rightarrow \log_3 (2y+5) + \log_3 y = 1$$

$$\Rightarrow \log_3 [y(2y+5)] = \log_3 3$$

$$\Rightarrow \log_3 [2y^2 + 5y] = \log_3 3$$

$$\Rightarrow 2y^2 + 5y = 3$$

# IYGB - MP1 PAPER I - QUESTION 12

$$\Rightarrow \quad 2y^2 + 5y - 3 = 0$$

$$\Rightarrow \quad (2y - 1)(y + 3) = 0$$

$$\Rightarrow \quad y = \begin{cases} \frac{1}{2} \\ \cancel{-3} \end{cases}$$

AS THIS MAKES THE LOGARITHMIC ARGUMENT NEGATIVE

$$\therefore \quad y = -\frac{1}{2}$$

# IYGB – MP1 PAPER I – QUESTION 13

a) SUBSTITUTE $x=1$ TO GET $k$

$$\Rightarrow 1^3 + \left(2-\tfrac{1}{5}k\right)\times 1^2 - (2k+1)\times 1 + 20 = 0$$

$$\Rightarrow \cancel{1} + 2 - \tfrac{1}{5}k - 2k \cancel{-1} + 20 = 0$$

$$\Rightarrow 22 = \tfrac{11}{5}k$$

$$\Rightarrow 110 = 11k$$

$$\Rightarrow k = 10$$

b) PUT $k=10$ INTO THE EQUATION

$$\Rightarrow x^3 + \left(2-\tfrac{1}{5}k\right)x^2 - (2k+1)x + 20 = 0$$

$$\Rightarrow x^3 - 21x + 20 = 0$$

AS $x=1$ IS A SOLUTION, THEN $(x-1)$ MUST BE A FACTOR

BY LONG DIVISION OR MANIPULATION

$$\Rightarrow x^3 - 21x + 20 = 0$$

$$\Rightarrow x^2(x-1) + x(x-1) - 20(x-1) = 0$$

$$\Rightarrow (x-1)(x^2+x-20) = 0$$

$$\Rightarrow (x-1)(x+5)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} 1 \\ 4 \\ -5 \end{cases}$$