IYGB - MP2 PAPER B - QUESTION 1

USING THE STANDARD FORMULA FOR THE DERIVATIVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] \quad \text{WITH} \quad f(x) = \sin x$$

$$\frac{d}{dx}(\sin x) = \lim\left[\frac{\sin(x+h) - \sin x}{h}\right]$$

USING THE IDENTITY $\sin(A+B) \equiv \sin A\cos B + \cos A\sin B$

$$\frac{d}{dx}(\sin x) = \lim_{h \to 0}\left[\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\sin x\cos h - \sin x + \cos x\sin h}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\sin x(\cos h - 1) + \cos x\sin h}{h}\right]$$

$$= \lim_{h \to 0}\left[\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$= \sin x \times 0 + \cos x \times 1$$

$$= \cos x$$

IYGB - MP2 PAPER B - QUESTION 2

$$u_n = \frac{1}{1-u_{n-1}}, \quad u_1 = 2$$

START GENERATING SOME TERMS, LOOKING FOR A PATTERN

$$u_1 = 2$$

$$u_2 = \frac{1}{1-u_1} = \frac{1}{1-2} = -1$$

$$u_3 = \frac{1}{1-u_2} = \frac{1}{1-(-1)} = \frac{1}{2}$$

$$u_4 = \frac{1}{1-u_3} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$u_5 = -1$$

ETC

THUS WE HAVE A PATTERN WITH THE FOLLOWING VALUES

2	u_1	u_4	u_7	u_{10}	u_{13}	u_{16}	u_{19}
-1	u_2	u_5	u_8	u_{11}	u_{14}	u_{17}	u_{20}
$\frac{1}{2}$	u_3	u_6	u_9	u_{12}	u_{15}	u_{18}	
TOTAL	1.5	1.5	1.5	1.5	1.5	1.5	1

$$\therefore \sum_{n=1}^{20} u_n = (6 \times 1.5) + 1 = 10$$

IYGB, MP2 PAPER B — QUESTION 3

USING THE POINT GIVEN $\left(\frac{2}{5}, -\frac{2}{3}\right)$

- $x = \frac{1}{t+a}$

$\frac{2}{5} = \frac{1}{t+a}$

$\frac{5}{2} = t+a$

- $y = \frac{1}{t-a}$

$-\frac{2}{3} = \frac{1}{t-a}$

$-\frac{3}{2} = t-a$

ADDING

$2t = 1$

$t = \frac{1}{2}$ & $\frac{5}{2} = \frac{1}{2} + a$

$a = 2$

FINDING THE GRADIENT FUNCTION, GIVEN THAT $x = \frac{1}{t+2}$ & $y = \frac{1}{t-2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\frac{1}{(t-2)^2}}{-\frac{1}{(t+2)^2}} = +\frac{(t+2)^2}{(t-2)^2} = \left(\frac{t+2}{t-2}\right)^2$$

FINALLY THE GRADIENT AT P, IE WHERE $t = \frac{1}{2}$

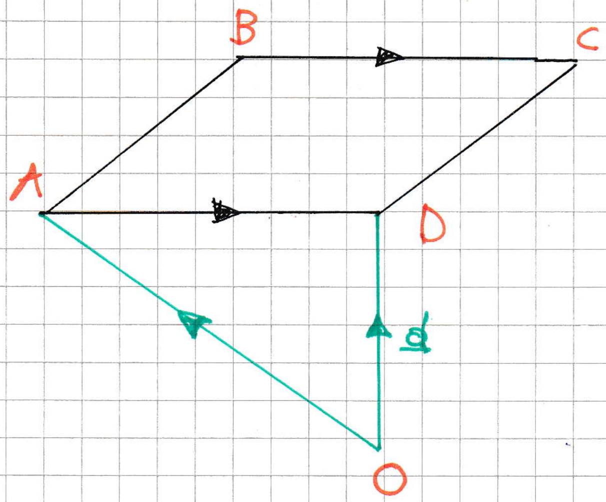
$$\frac{dy}{dx} = \left(\frac{\frac{1}{2}+2}{\frac{1}{2}-2}\right)^2 = \left(\frac{\frac{5}{2}}{-\frac{3}{2}}\right)^2 = \left(-\frac{5}{3}\right)^2 = \frac{25}{9}$$

IYGB - MP2 PAPER B - QUESTION 4

a) LOOKING AT THE DIAGRAM

$$\Rightarrow \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$\Rightarrow \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$$
PARALLEL
$$\Rightarrow \underline{d} = \underline{a} + (\underline{c} - \underline{b})$$
$$\Rightarrow \underline{d} = \underline{a} + \underline{c} - \underline{b}$$
$$\Rightarrow \underline{d} = (1,1,2) + (4,0,1) - (2,1,5)$$
$$\Rightarrow \underline{d} = (3,0,-2)$$

$\therefore D(3,0,-2)$ //



A(1,1,2)
B(2,1,5)
C(4,0,1)

b) E(1,2,12) // B(2,1,5) D(3,0,-2)

BY SEQUENCING / INSPECTION / MIDPOINT FORMULA

x: 1 $\xleftarrow{-1}$ 2 $\xleftarrow{-1}$ 3

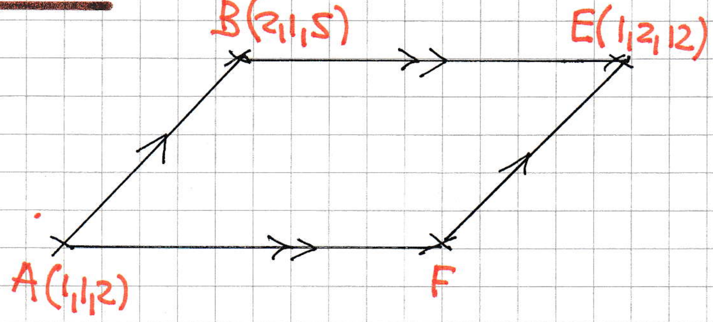
y: 2 $\xleftarrow{+1}$ 1 $\xleftarrow{+1}$ 0

z: 12 $\xleftarrow{+7}$ 5 $\xleftarrow{+7}$ -2

E B D

IYGB - MP2 PAPER B - QUESTION 4

c) AS IN PART (a) OR INSPECTION



(B) to (E)

$2 \xrightarrow{-1} 1$
$1 \xrightarrow{+1} 2$
$5 \xrightarrow{+7} 12$

(A) to (F)

$1 \xrightarrow{-1} 0$
$1 \xrightarrow{+1} 2$
$2 \xrightarrow{+7} 9$

$\therefore F(0,2,9)$

d) $F(0,2,9)$ $C(4,0,1)$ $B(2,1,5)$

MIDPOINT OF FC $= \left(\frac{0+4}{2}, \frac{2+0}{2}, \frac{9+1}{2}\right) = (2,1,5)$ WHICH IS B

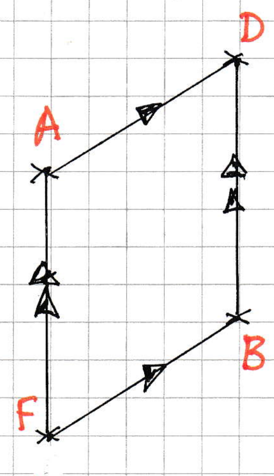
e)

$\overrightarrow{AD} = \underline{d} - \underline{a} = (3,0,-2) - (1,1,2) = (2,-1,-4)$

$\overrightarrow{FB} = \underline{b} - \underline{f} = (2,1,5) - (0,2,9) = (2,-1,-4)$

$\overrightarrow{FA} = \underline{a} - \underline{f} = (1,1,2) - (0,2,9) = (1,-1,-7)$

$\overrightarrow{BD} = \underline{d} - \underline{b} = (3,0,-2) - (2,1,5) = (1,-1,-7)$



REFERRING TO THE CALCULATIONS & THE DIAGRAM

OPPOSITE SIDES ARE PARALLEL (AND EQUAL)

INDEED ADBF IS A PARALLELOGRAM

IYGB – MP2 PAPER B – QUESTION 5

a) REARRANGE THE EQUATION FIRST

$$\Rightarrow \frac{(x+2y)^2}{4x-y} + y = 3x+2$$

$$\Rightarrow (x+2y)^2 + y(4x-y) = (3x+2)(4x-y)$$

$$\Rightarrow x^2+4xy+4y^2+4xy-y^2 = 12x^2-3xy+8x-2y$$

$$\Rightarrow -11x^2+3y^2+11xy-8x+2y=0$$

$$\Rightarrow 11x^2-3y^2-11xy+8x-2y=0$$

DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow 22x - 6y\frac{dy}{dx} - 11y - 11x\frac{dy}{dx} + 8 - 2\frac{dy}{dx} = 0$$

$$\Rightarrow 22x - 11y + 8 = 11x\frac{dy}{dx} + 6y\frac{dy}{dx} + 2\frac{dy}{dx}$$

$$\Rightarrow (11x+6y+2)\frac{dy}{dx} = 22x-11y+8$$

$$\Rightarrow \frac{dy}{dx} = \frac{22x-11y+8}{6y+11x+2}$$

IE $k=11$

b) FIRSTLY WHEN $x=2$

$$\Rightarrow 11\times2^2 - 3y^2 - 11\times2y + 8\times2 - 2y = 0$$

$$\Rightarrow 44 - 3y^2 - 22y + 16 - 2y = 0$$

$$\Rightarrow 0 = 3y^2+24y-60$$

$$\Rightarrow y^2+8y-20=0$$

$$\Rightarrow (y+10)(y-2)=0$$

$$\Rightarrow y = \begin{cases} 2 \\ -10 \end{cases}$$

IE $(2,2)$ & $(2,-10)$

IYGB - FP2 PAPER B - QUESTION 5

FINALLY WE HAVE

$$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{22\times2 - 11\times2 + 8}{6\times2 + 11\times2 + 2} = \frac{44-22+8}{12+22+2} = \frac{30}{36} = \frac{5}{6}$$

$$\left.\frac{dy}{dx}\right|_{(2,-10)} = \frac{22\times2 - 11(-10) + 8}{6(-10) + 11\times2 + 2} = \frac{44+110+8}{-60+22+2} = \frac{162}{-36} = -\frac{9}{2}$$

IYGB - MP2 PAPER B - QUESTION 6

FIRST FIND THE SUM OF THE FIRST 600 INTEGERS USING $S_n = \frac{1}{2}n(n+1)$

$$S_{600} = \frac{1}{2} \times 600 \times 601 = \underline{180300}$$

NEXT FIND THE SUM OF THE MULTIPLES OF 3, BETWEEN 1 & 600

$$3 + 6 + 9 + \dots + 597 + 600$$

$$= 3(1 + 2 + 3 + \dots + 199 + 200)$$

$$= 3S_{200} = 3 \times \frac{1}{2} \times 200 \times 201$$

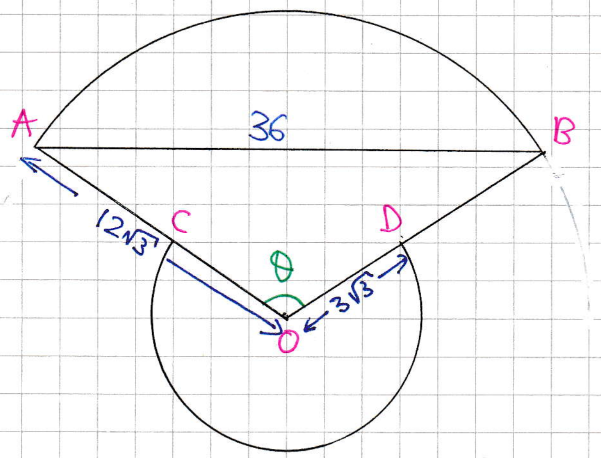
$$= \underline{60300}$$

HENCE THE REQUIRED SUM IS

$$180300 - 60300 = \underline{120000} \quad //$$

IYGB - MP2 PAPER B - QUESTION 7

a) BY THE COSINE RULE ON $\overset{\Delta}{AOB}$, OR SIMPLE TRIGONOMETRY BY SPLITTING $\overset{\Delta}{AOB}$ INTO 2 RIGHT ANGLED TRIANGLES



$\Rightarrow |AB|^2 = |OA|^2 + |OB|^2 - 2|OA||OB|\cos\theta$

$\Rightarrow 36^2 = (12\sqrt{3})^2 + (12\sqrt{3})^2 - 2(12\sqrt{3})(12\sqrt{3})\cos\theta$

$\Rightarrow 1296 = 432 + 432 - 864\cos\theta$

$\Rightarrow 864\cos\theta = -432$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$ // AS REQUIRED

b) AREA OF "TOP SECTOR" AOB

$$\text{"}\tfrac{1}{2}r^2\theta^c\text{"} = \frac{1}{2}(12\sqrt{3})^2 \times \frac{2\pi}{3} = 144\pi$$

AREA OF "BOTTOM SECTOR" (THROWING AREA), COD

$$\text{"}\tfrac{1}{2}r^2\theta^c\text{"} = \frac{1}{2}(3\sqrt{3})^2 \times \left(2\pi - \frac{2\pi}{3}\right) = 18\pi$$

TOTAL AREA

$$144\pi + 18\pi = 162\pi$$

c) ARC LENGTH $\overset{\frown}{AB}$ = "$r\theta^c$" $= 12\sqrt{3} \times \frac{2\pi}{3} = 8\sqrt{3}\pi$

ARC LENGTH $\overset{\frown}{CD}$ = "$r\theta^c$" $= 3\sqrt{3} \times \left(2\pi - \frac{2\pi}{3}\right) = 4\sqrt{3}\pi$

$\therefore$ TOTAL PERIMETER IS GIVEN BY

$$8\sqrt{3}\pi + 4\sqrt{3}\pi + 2(12\sqrt{3} - 3\sqrt{3}) = 12\sqrt{3}\pi + 18\sqrt{3}$$
$$= 6\sqrt{3}(2\pi + 3)$$

// AS REQUIRED

IYGB – MP2 PAPER B – QUESTION 8

MULTIPLY ACROSS & TIDY

$\Rightarrow 2\cos\theta\cos2\theta = \cos\theta + \sin\theta$

$\Rightarrow 2\cos\theta(\cos^2\theta - \sin^2\theta) = \cos\theta + \sin\theta$

$\Rightarrow 2\cos\theta(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \cos\theta + \sin\theta$ difference of squares

AS $\cos\theta + \sin\theta \neq 0$, WE MAY DIVIDE IT THROUGH

$\Rightarrow 2\cos\theta(\cos\theta - \sin\theta)\cancel{(\cos\theta + \sin\theta)} = \cancel{(\cos\theta + \sin\theta)}$

$\Rightarrow 2\cos\theta(\cos\theta - \sin\theta) = 1$

$\Rightarrow 2\cos^2\theta - 2\cos\theta\sin\theta = 1$

$\Rightarrow 2\left(\frac{1}{2} + \frac{1}{2}\cos2\theta\right) - \sin2\theta = 1$ $\quad \{\cos2\theta \equiv 2\cos^2\theta - 1\}$

$\Rightarrow \cancel{1} + \cos2\theta - \sin2\theta = \cancel{1}$

$\Rightarrow \cos2\theta = \sin2\theta$ DIVIDE BY $\cos2\theta$

$\Rightarrow 1 = \tan2\theta$

- $2\theta = \frac{\pi}{4} \pm n\pi$
- $\theta = \frac{\pi}{8} \pm \frac{n\pi}{2}$

$\therefore \theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

IYGB - MP2 PAPER B - QUESTION 9

a) $ff(49) = f(f(49)) = f(2+\sqrt{49}) = f(9)$

$= 2+\sqrt{9} = 5$

b) LET $y = f(x)$

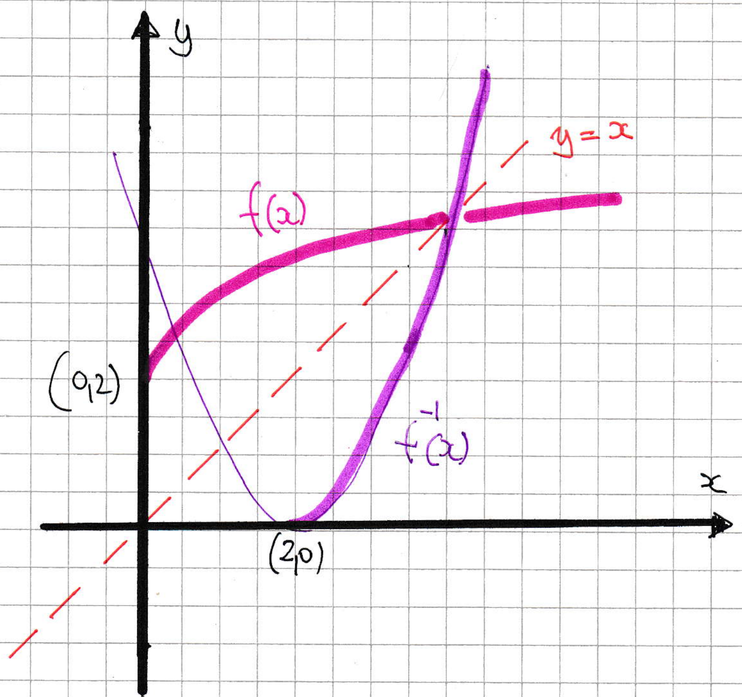
$\Rightarrow y = 2+\sqrt{x}$

$\Rightarrow y-2 = \sqrt{x}$

$\Rightarrow (y-2)^2 = x$

$\therefore f^{-1}(x) = (x-2)^2$

c)



d) SOLVING $f^{-1}(x) = x$, INSTEAD OF $f(x) = f^{-1}(x)$

$\Rightarrow (x-2)^2 = x$

$\Rightarrow x^2-4x+4 = x$

$\Rightarrow x^2-5x+4 = 0$

$\Rightarrow (x-4)(x-1) = 0$

$\therefore x = \cancel{1} \quad (x>2)$

$ 4$

IYGB - MP2 PAPER B - QUESTION 10

START BY REGROUPING TERMS

$$\Rightarrow e^{x}\frac{dy}{dx} + y^{2} = xy^{2}$$

$$\Rightarrow e^{x}\frac{dy}{dx} = xy^{2} - y^{2}$$

$$\Rightarrow e^{x}\frac{dy}{dx} = y^{2}(x-1)$$

SEPARATE THE VARIABLES

$$\Rightarrow e^{x}\,dy = y^{2}(x-1)\,dx$$

$$\Rightarrow \frac{1}{y^{2}}\,dy = \frac{x-1}{e^{x}}\,dx$$

$$\Rightarrow \int y^{-2}\,dy = \int (x-1)e^{-x}\,dx$$

INTEGRATION BY PARTS

$x-1$	1
$-e^{-x}$	e^{-x}

$$\Rightarrow -y^{-1} = -e^{-x}(x-1) - \int -e^{-x}\,dx$$

$$\Rightarrow -\frac{1}{y} = -e^{-x}(x-1) + \int e^{-x}\,dx$$

$$\Rightarrow -\frac{1}{y} = -e^{-x}(x-1) - e^{-x} + C$$

$$\Rightarrow \frac{1}{y} = e^{-x}(x-1) + e^{-x} + C$$

$$\Rightarrow \frac{1}{y} = xe^{-x} \cancel{-e^{-x}} + \cancel{e^{-x}} + C$$

$$\Rightarrow \frac{1}{y} = xe^{-x} + C$$

IYGB - MP2 PAPER B - QUESTION 10

APPLY THE BOUNDARY CONDITION $x=1$ $y=e$

$$\frac{1}{e} = 1 \times e^{-1} + C$$

$$\frac{1}{e} = \frac{1}{e} + C$$

$$C = 0$$

$$\therefore \quad \frac{1}{y} = xe^{-x}$$

$$y = \frac{1}{xe^{-x}}$$

$$y = \frac{1}{x}e^{x}$$

AS REQUIRED

IYGB MP2 PAPER B - QUESTION 11

IT IS GIVEN THAT $\frac{dV}{dt} = 0.0008$ (FROM THE UNITS m^3/s)

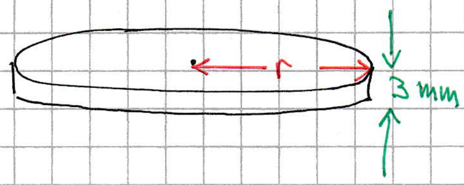
$$\Longrightarrow \frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$\Longrightarrow \frac{dr}{dt} = \frac{1}{\pi r \times 0.006} \times 0.0008$$

$$\Longrightarrow \frac{dr}{dt} = \frac{2}{15\pi r}$$

$$\Longrightarrow \left.\frac{dr}{dt}\right|_{r=6} = \frac{2}{15\pi \times 6}$$

$$\Longrightarrow \left.\frac{dr}{dt}\right|_{r=6} = \frac{1}{45\pi}$$



3 mm = 0.003 m

$V = \pi r^2 \times 0.003$

$\frac{dV}{dr} = \pi r \times 0.006$

IYGB - MP2 PAPER B - QUESTION 12

a) BY THE INVERSE RULE

$\Rightarrow y = \arcsin x$

$\Rightarrow \sin y = x$

$\Rightarrow x = \sin y$

$\Rightarrow \frac{dx}{dy} = \cos y$

$\Rightarrow \frac{dx}{dy} = \pm\sqrt{1-\sin^2 y}$

BUT $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$ so $0 \leqslant \cos y \leqslant 1 \Rightarrow \frac{dx}{dy} = +\sqrt{1-\sin^2 y}$

$\Rightarrow \frac{dx}{dy} = \sqrt{1-\sin^2 y}$

$\Rightarrow \frac{dx}{dy} = \sqrt{1-x^2}$ ← AS $\sin y = x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ // AS REQUIRED

b) DIFFERENTIATION BY THE PRODUCT RULE

$y = x\arcsin 2x \Rightarrow \frac{dy}{dx} = 1 \times \arcsin 2x + x \times \frac{1}{\sqrt{1-4x^2}} \times 2$ ← FROM $2x$

$(2x)^2$

$\Rightarrow \frac{dy}{dx} = \arcsin 2x + \frac{2x}{\sqrt{1-4x^2}}$

NOW WHEN $x = \frac{1}{4}$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}} = \arcsin\frac{1}{2} + \frac{2\times\frac{1}{4}}{\sqrt{1-4\left(\frac{1}{4}\right)^2}} = \frac{\pi}{6} + \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\pi}{6} + \frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{3} = \frac{1}{6}\left(\pi + 2\sqrt{3}\right)$$

IYGB - MP2 PAPER B - QUESTION 13

PROCEED BY SUBSTITUTION $u^2 = ax - 1$

$$\left[u = +(ax-1)^{\frac{1}{2}} \right]$$

$$2u\frac{du}{dx} = a$$

$$2u\,du = a\,dx$$

$$dx = \frac{2u}{a}\,du$$

$$x = \frac{2}{a} \longmapsto u = 1$$

$$x = \frac{17}{a} \longmapsto u = 4$$

TRANSFORMING THE INTEGRAL

$$\int_{\frac{2}{a}}^{\frac{17}{a}} \frac{2ax}{\sqrt{ax-1}}\,dx = \int_1^4 \frac{\cancel{2a}x}{\cancel{u}}\left(\frac{2\cancel{u}}{\cancel{a}}\,du\right)$$

$$= \int_1^4 4x \quad du$$

$$= \int_1^4 \frac{4ax}{a}\,du$$

$$= \frac{4}{a}\int_1^4 ax \quad du$$

$$= \frac{4}{a}\int_1^4 u^2+1\,du$$

$$= \frac{4}{a}\left[\frac{1}{3}u^3 + u\right]_1^4$$

$$= \frac{4}{a}\left[\left(\frac{64}{3}+4\right) - \left(\frac{1}{3}+1\right)\right]$$

$$= \frac{4}{a} \times 24$$

$$= \frac{96}{a}$$